Enoncés

1. Valider

$$0 = 1 \qquad \frac{i}{1+e^{i\pi}} \neq \frac{0}{1} \qquad 2^4 \neq 2\ [4] \qquad \text{Arg}\ i = \pi/2 \qquad \text{Arg}(e^{i\pi} + e^{-i\pi}) = \text{Arg}(e^{-i\pi} - e^{i\pi})\ [2\pi]$$

$$\sqrt{2} > 1 \qquad \text{Re}\, i < \text{Im}\, i \qquad i \leq 2|i| \qquad \text{Arg}\ i \geq 0 \qquad 0 = 1 \text{ et Arg } 0 = 0\,[2\pi] \qquad 1+1=2 \text{ ou } \sqrt{-1} = 0$$

$$(\sqrt{-1})^2 = -1 \Longrightarrow 1+1=2 \qquad \text{si } 1 > 2 \text{ alors } 1/0 > 2/0 \qquad \text{Arg}\ i = \pi/2 \Longleftrightarrow \text{Arg}\ (i/2) = \pi/4$$

$$\forall x \in \mathbf{R},\ \frac{1}{x^2} = (\frac{1}{x})^2 \qquad \exists z \in \mathbf{C},\ z^5 + ez^3 + z^2 + \sqrt{2}z = i\pi \text{ et Re}\, z \neq 0$$

$$\forall z \in \mathbf{C},\ \text{Arg}\, z = \pi/2\,[2\pi] \Longrightarrow \text{Im}\, z = 0 \qquad \forall x \in \mathbf{C},\ \exists y \in \mathbf{C},\ \sqrt{y} = x \qquad \forall x, y \in \mathbf{R},\ x/y = y/x.$$

Quel est le domaine de définition de l'énoncé: $\exists y \in \mathbf{R},\ y \neq x \Longrightarrow 1/y \neq 1/x.$

2. Calculer

a) Evaluer

$$0 = 1 \qquad 4^5 = 4\ [5] \qquad \frac{i}{e^{i\pi} + e^{-i\pi}} = \frac{0}{1} \qquad 2^4 \neq 2\,[4] \qquad \text{Arg}\ i \neq \pi/2\,[2\pi] \qquad \sqrt{2} < 1 \quad |i| \leq 0$$

$$0 = 1 \text{ et Arg } 1 = \text{Arg } 2\,[2\pi] \qquad 1+1=2 \text{ ou } \sqrt{1} = 0$$

$$(\sqrt{1})^2 = -1 \Longrightarrow 1+1=2 \qquad \text{si } 1 > 2 \text{ alors } 0/1 > 0/2 \qquad 1 = 2 \Longleftrightarrow 2 = 4$$

$$\forall x, y \in \mathbf{R},\ xy = yx \qquad \exists x \in \mathbf{R},\ x^2 = x \qquad \forall z \in \mathbf{C},\ \text{Arg}\, z = \pi/2\,[2\pi] \Longrightarrow \text{Im}\, z = 0$$

$$\forall x \in \mathbf{R},\ \exists y \in \mathbf{R},\ y^2 = x \qquad \forall x \in \mathbf{C},\ \exists y \in \mathbf{C},\ y^2 = x.$$

b) Nier

(x et y sont des réels)

$$x = x+1 \qquad x \neq y \qquad |x| > \sqrt{2} \qquad x \leq e^2 \qquad x < 0 \text{ ou } e^x = -2 \qquad x \neq 0 \Longrightarrow \frac{1}{x} = 1$$

$$x > 0 \Longleftrightarrow y < 1 \qquad \forall x \in \mathbf{R},\ x \leq x^2 \qquad \exists x \in \mathbf{R},\ x = x+1 \qquad \forall x \in \mathbf{R},\ \exists n \in \mathbf{N},\ x \geq n \text{ et } x < n+1.$$

c) **Formuler** la contraposée puis la réciproque

$$\forall x, y \in \mathbf{R},\ \sin x = \sin y \Longrightarrow x = y \qquad \forall a, b \in \mathbf{N},\ a^3 \neq b^3 \Longrightarrow a \neq b$$

$$\forall u, v \in \mathbf{R},\ e^u < e^v \Longrightarrow u < v \qquad \forall x, y \in \mathbf{N},\ x \leq y \Longrightarrow \cos x \leq \cos y\,.$$