Exercise n°4 : minimization under equality constraints

Exercise 1 : Let the following function :

$$f(x,y) = x^4 + y^4 + 4xy \tag{1}$$

1.1. Prove that f has a global minimum on $\mathbb{R}^2$.

1.2. Compute the hessian matrix of f and determine if the function f is convex on all $\mathbb{R}^2$.

1.3. Determine the critical points of f and give their nature : local minimum, local maximum, saddle-point.

1.4. At which points does f reach its global minimum ? What is the minimal value of f ? Explain why f cannot be convex.

Exercise 2 :

We deal with the following problem :

$$\sup_{x^2+y^2=1} xy. \tag{2}$$

2.1. Justify the existence of a solution of (2).

2.2. Solve with the help of Lagrange multipliers.

2.3. Give a geometric interpretation of the results.