Wave propagation in random media

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Our motivation to give this lecture is twofold. On one hand the theory of waves propagating in randomly layered media has been intensively studied during the last thirty years and the results are scattered among numerous papers. It is now in a stable state, in particular in the extremely interesting regime of separation of scales as introduced by G. Papanicolaou and his co-authors. On the other hand the time reversal experiments conducted since the nineties, with ultrasonic waves by M. Fink and his group in Paris, or in the context of ocean acoustics by W. Kuperman and his collaborators, has attracted a lot of attention due to the surprising effects of refocusing and enhanced resolution by multiple scattering of the waves. These experiments, have opened the door to numerous potential applications, in particular in the domains of imaging and communications. A quantitative mathematical analysis is crucial in the understanding of these phenomena and for the development of their applications.

Wave propagation in random media is a vast field where a lot of work in various regimes have been done. The first part of this lecture focuses on wave propagation in one-dimensional random media. We will review the asymptotic theory for random differential equations, including homogenization theory and diffusion approximation. We will apply this theory to study the transmission of waves through a random mediium. We shall also give an analysis of wave propagation in a random waveguide, where transverse spatial effects are important. Finally we will address wave propagation in the parabolic regime, in which the semi-classical analysis of the Schrödinger equation with a random potential plays an important role.

1. Wave propagation in one-dimensional random media: effective medium theory.
   Homogenization in random media.

2. Wave propagation in one-dimensional random media: the coherent wave front, the incoherent wave fluctuations, time reversal.
   Diffusion approximation, asymptotic theory for random differential equations.

3. Wave propagation and time reversal in a random waveguide.

4. Wave propagation and time reversal in the parabolic regime.
   Semi-classical analysis of the Schrödinger equation with a random potential.