First order traffic flow models: intersection modelling, network modelling, applications

J.P. Lebacque
M.M. Khoshyaran

INRETS-GRETIA
ETC (Economics traffic clinic)

OUTLINE

1. Traffic modelling (objectives, state of art)
2. First order models, second order models
3. A fast review of the LWR model on a line
4. Local supply and demand, numerical schemes
5. Boundary conditions, intersection models
6. The LWR model on a network
7. Conclusions and next steps

Traffic flow modelling Objectives

Modelling tools are useful for traffic engineering tasks:

- Implementation of on-line control strategies
- Off-line evaluation of control strategies: ramp metering, speed control, collective route guidance via VMS, intersection control, externalities etc.
- Prediction and estimation of the traffic state
- New infrastructure construction etc.

Traffic flow modelling approaches

Two main types of modelling approaches:

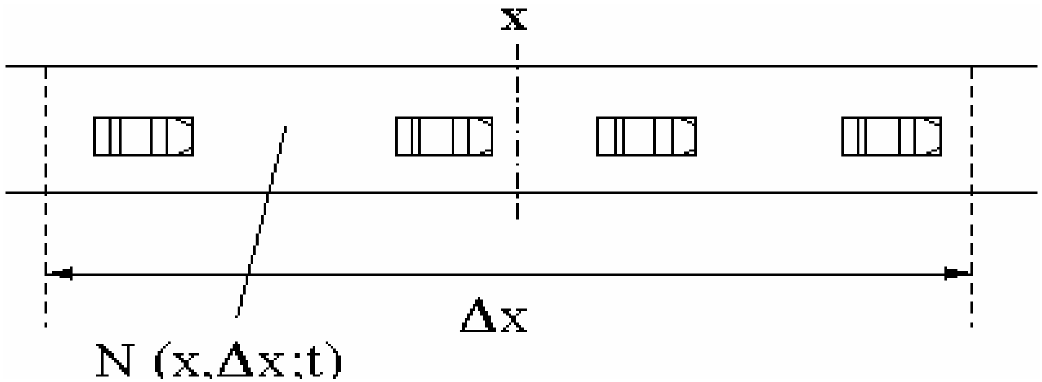
1. Microscopic (evaluation, simulation)
   a. follow-the-leader,
   b. cellular automata,
   c. multi-agents
2. Macroscopic (evaluation, control):
   a. First order Modelling
   b. Second order Modelling

Macroscopic traffic description

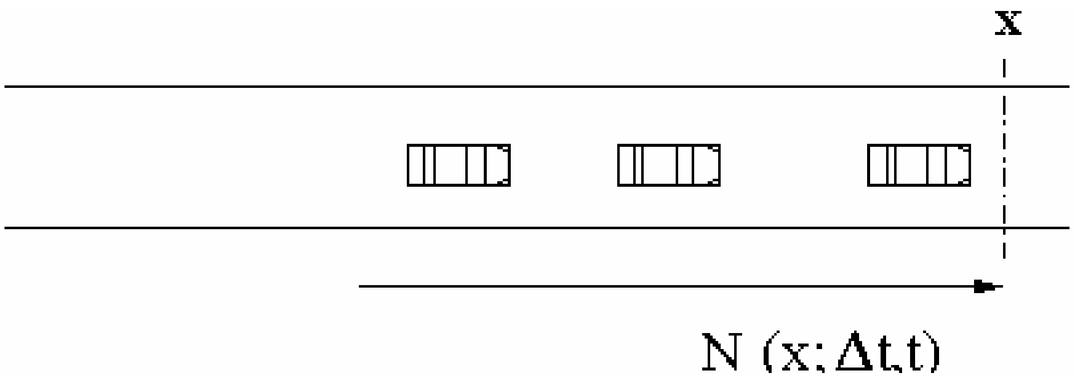
- Hydrodynamic analogy
- Continuum hypothesis: traffic state can be described by functions of location x and time t
- Variables:
  - Density $\rho(x,t)$ (or $K(x,t)$)
  - Flow $q(x,t)$
  - Velocity $v(x,t)$

Definitions

- Density

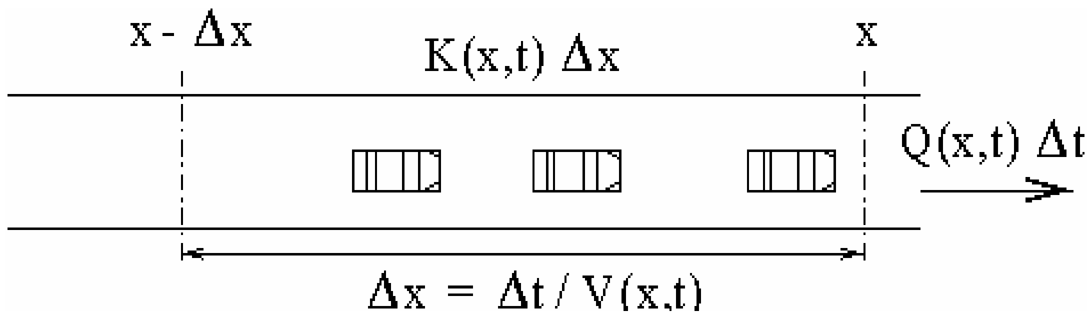


- Flow

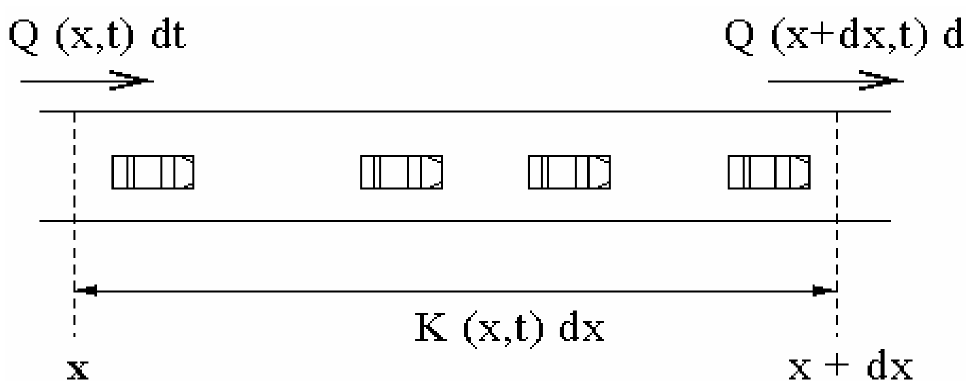


Definitions (2)

- Speed



- Conservation



- Only for space-time scales > 100 meters x 5 seeconds

Macroscopic traffic description

- Limiting factor of the continuum hypothesis:
  - Avogadro number $N = 6.025\ 10^{23}$ (number of molecules per 22.4 liters of gas under normal conditions)
  - Maximum (jam) density on highways, as communicated by operators: 180 vh / km x lane

Macroscopic approaches: Basic equations

1- Continuity Equation:

$$\partial_t \rho(x,t) + \partial_x q(x,t) = 0$$

2- Volume-Density-speed relationship:

$$q(x,t) = \rho(x,t)\, v(x,t)$$

3- Fundamental Diagram (equilibrium)

$$v(x,t) = V_e(\rho(x,t))$$ where V_e monotone decreasing function

First order

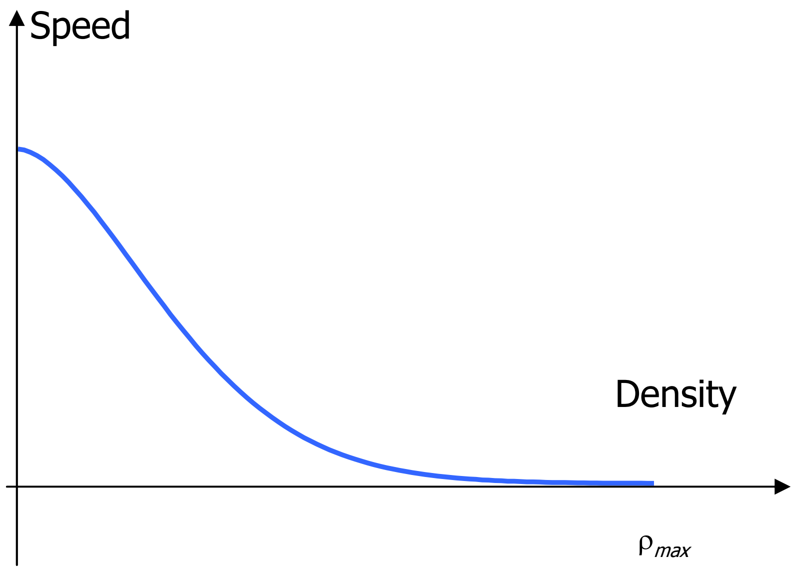
4- *Momentum equation*

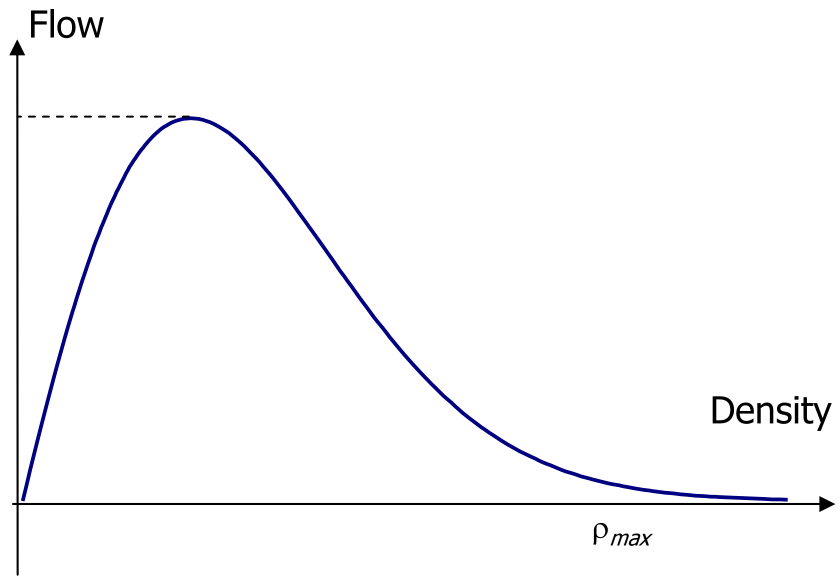
$$dv(x,t)/dt = \partial_t v(x,t) + v(x,t)\partial_x v(x,t) = G(\rho(x,t), v(x,t))$$

Second order

Fundamental diagram, equilibrium

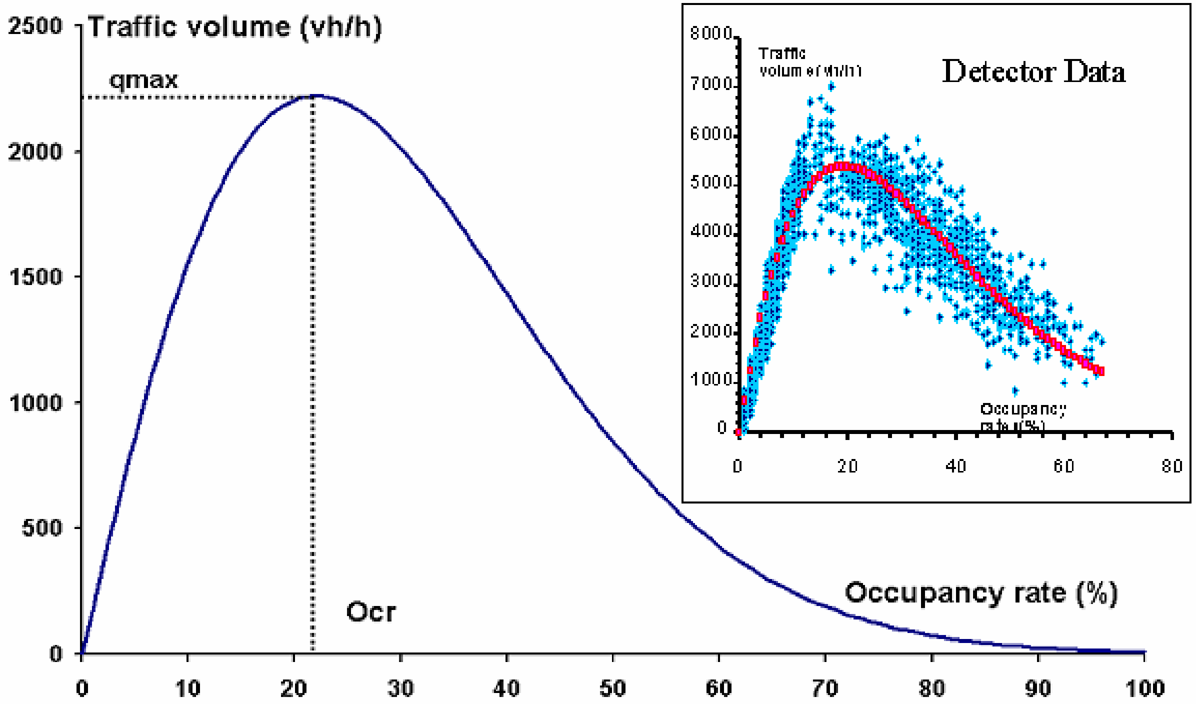
- No less than 25 FDs in the literature (TRB 165)
- Example of fundamental diagram





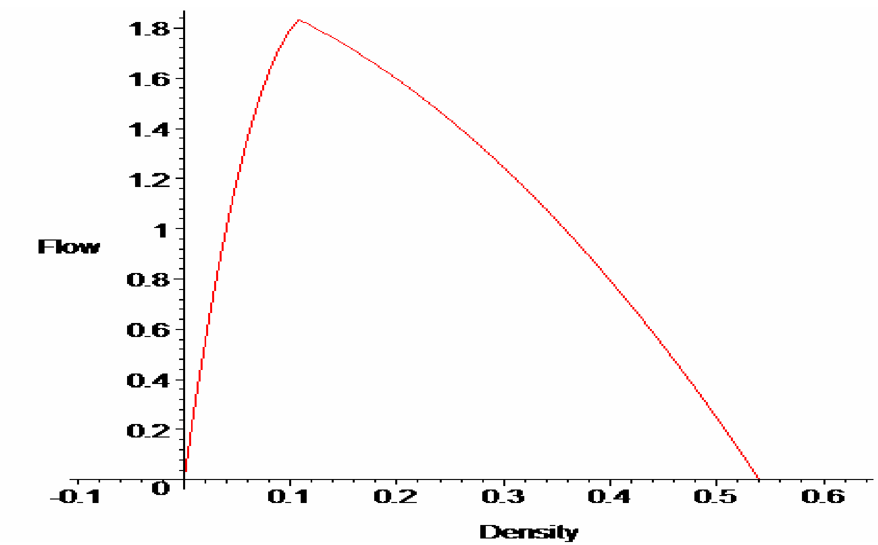
1^{st} vs 2^{nd} order models

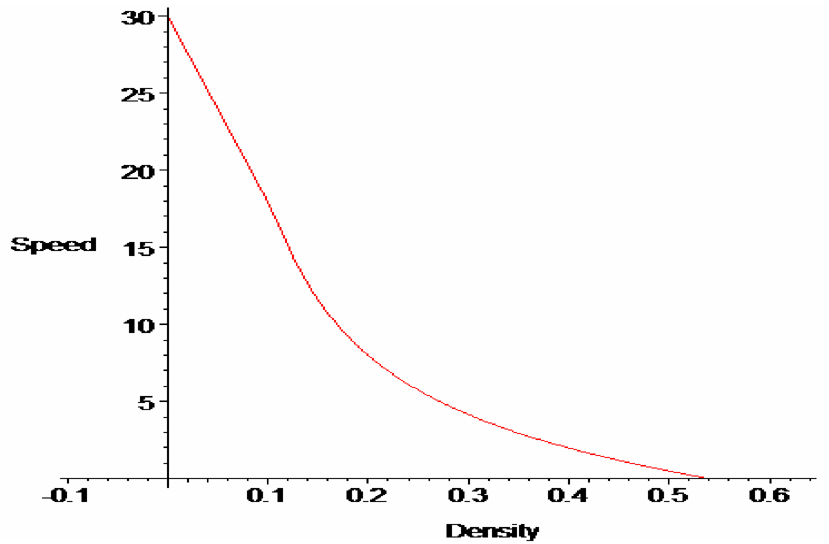
- 1^{st} order: assumed at equilibrium (ρ, v on the fundamental diagram)
- 2^{nd} order: out of equilibrium (ρ, v points are not on the fundamental diagram)
- Both are dynamic (term “equilibrium” is misleading)



Another example of FD

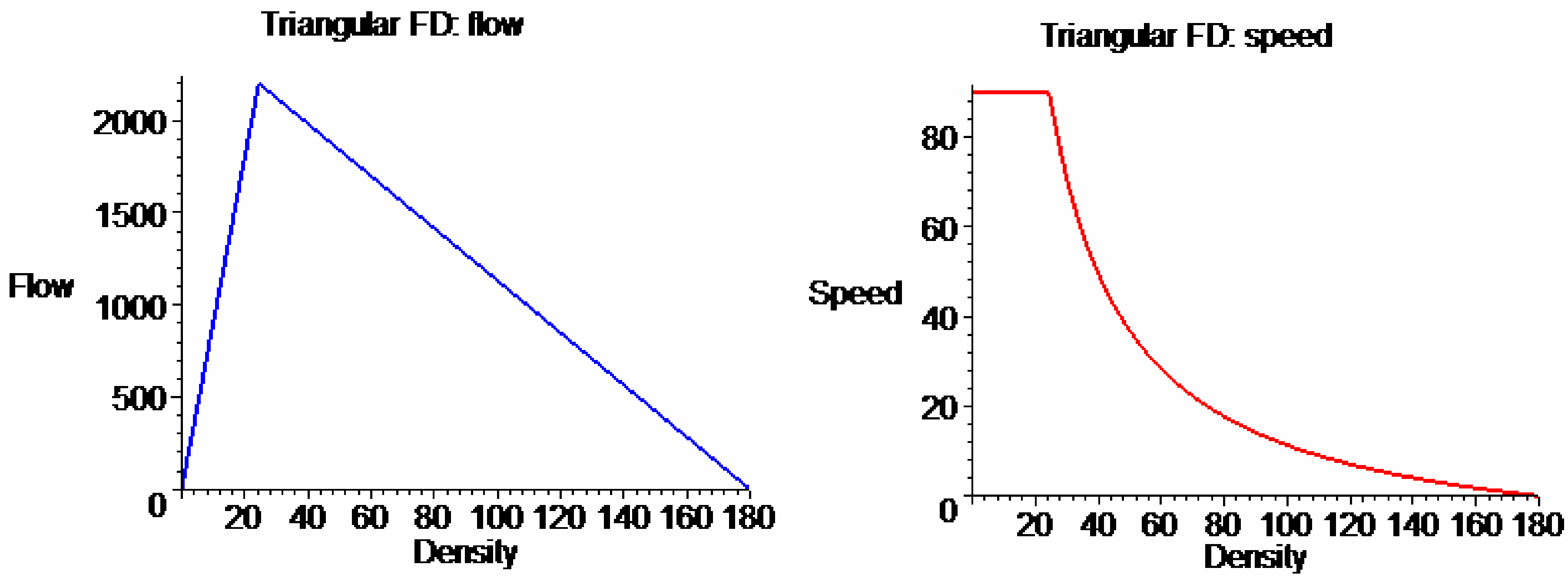
- Cf METACOR, STRADA





Another example still of FD

- Cf Newell, Daganzo



Momentum equation approaches

$$\partial_t V + V\partial_x V = \frac{1}{\tau}\left(V_{eq}(\rho) - V\right) - \frac{1}{\rho} K^2(\rho)\partial_x \rho$$

$$K(\rho) = \sqrt{-\frac{\upsilon}{2\tau} V_e'(\rho)}$$ Payne model (1971):

$$K(\rho) = 0$$ Ross's model 1988

$$K(\rho) = \sqrt{-\rho V_e'(\rho)\exp\left(\frac{1}{a}\left(V_e(\rho) - v\right)\right)}$$ Del Castillo's model 1993

$$K(\rho) = -\rho V_e'(\rho)$$ Zhang's model 1998

Momentum equation approaches(2)

$$\partial_t V + V\partial_x V = \frac{1}{\tau}\left(V_{eq}(\rho) - V\right) - \frac{1}{\rho} K^2(\rho)\partial_x \rho$$

The conservative form of the system (including the CE):

$$\partial_t U + \partial_x F(U) = S(U)$$

$$\text{with} \quad U = \begin{pmatrix} \rho \\ v \end{pmatrix} \quad \text{and} \quad S(U) = \begin{pmatrix} 0 \\ \frac{1}{\tau}\left(V_e(\rho) - v\right) \end{pmatrix}$$

$$F(U) = \begin{pmatrix} v & \rho \\ \frac{1}{\rho} K^2(\rho) & v \end{pmatrix}$$

The dynamic of the system is given by the eigenvalues of *F(U)*

What does it mean?

- **Conservative form**: no mathematically "illegal" derivatives ⇔ no physically meaningless expressions
- **Eigenvalues describe the dynamics**: the characteristic speeds are the propagation speed of information, small perturbations in the flow (linearization)

$$\partial_t V + V\partial_x V = \frac{1}{\tau}\left(V_{eq}(\rho) - V\right) - \frac{1}{\rho} K^2(\rho)\partial_x \rho \qquad \partial_t U + \partial_x F(U) = S(U)$$

Interpretation of the momentum equation

- Traffic acceleration is the result of
  - Relaxation term: traffic state tends towards equilibrium
  - Anticipation term: interaction of a vehicle with surrounding vehicles

$$\partial_t V + V \partial_x V = \frac{1}{\tau}\left(V_{eq}(\rho) - V\right) - \frac{1}{\rho} K^2(\rho) \partial_x \rho$$

Eigenvalues: interpretation

- Special waves:
  - Small perturbations of the traffic flow
  - Self-similar solutions

$$\partial_t U + \partial_x f(U) = 0$$

- The velocity of these special waves is equal to the eigenvalues of

$$A(U) \stackrel{def}{=} \nabla f(U)$$

Momentum equation approaches(3)

System eigenvalues:

$$\lambda_1(U) = v - K(\rho)$$
$$\lambda_2(U) = v + K(\rho)$$

The system is hyperbolic (except ROSS)
The anisotropic character of the traffic is not preserved (except ROSS) due to:

$$\lambda_2(U) = v + K(\rho) > v$$

Daganzo, 1995 claims the « requiem for the second order traffic modelling »
Aw, Rascle 2000 « resurrection of the second order traffic model"

Why is the anisotropic character of traffic not respected?

- If some eigenvalue is > traffic speed, information travels faster than traffic

- Information “catches up” drivers

- Upstream lower density “attracts” vehicles backward $\Rightarrow$ negative speeds

The LWR model

- Introduced by Lighthill, Whitham (1955), Richards (1956)
- Traffic at equilibrium: speed-density points on the fundamental diagram
- A single conservation law (for density; the flux is the flow), a single eigenvalue

The LWR model in a nutshell

- The equations:

$$\left| \begin{array}{ll} \frac{\partial K}{\partial t} + \frac{\partial Q}{\partial x} = 0 & \text{conservation equation} \\ Q = KV & \text{definition of } V \\ V = V_e(K, x) & \text{behavioural equation} \end{array} \right.$$

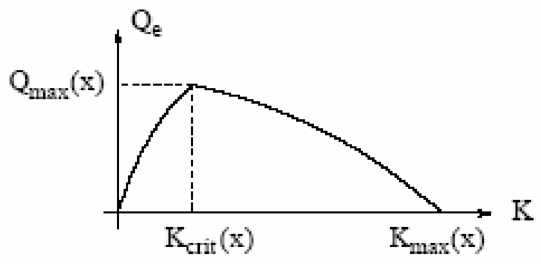
or:

$$\frac{\partial K}{\partial t} + \frac{\partial}{\partial x} Q_e(K, x) = 0$$

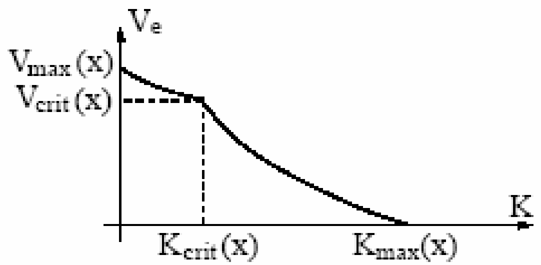
The LWR model (2)

Conventions

Equilibrium flow Q_e :



Equilibrium speed V_e :



Notations:

- Q : Flow
- K : Density
- V : Speed
- V_e : equilibrium speed
- V_e : equilibrium flow

$$Q_e(K,x) \stackrel{def}{=} KV_e(K,x)$$

Analytical solutions (reminder)

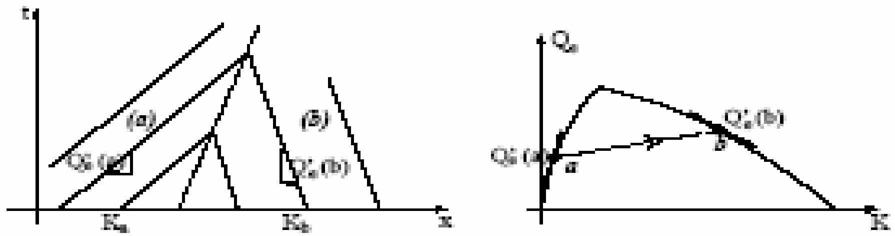
- Description of solutions:

• using characteristics and shockwaves

• characteristics with > 0 slope $\Leftrightarrow$ fluid traffic $K < K_{crit}$

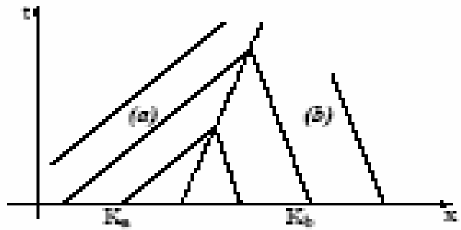
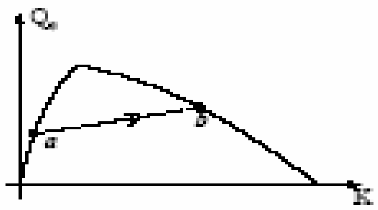
• characteristics with < 0 slope $\Leftrightarrow$ trafic congested traffic $K > K_{crit}$

• coordinates (x, t)



Analytical solutions (2)

- Shock-waves:

Shock-wave velocity:

$$v = \frac{Q_a - Q_b}{K_a - K_b} = \frac{[Q]}{[K]}$$

(Rankine-Hugoniot)

Concave fundamental diagram $\Rightarrow$
Only *deceleration* shockwaves are allowed in *entropy solutions*.

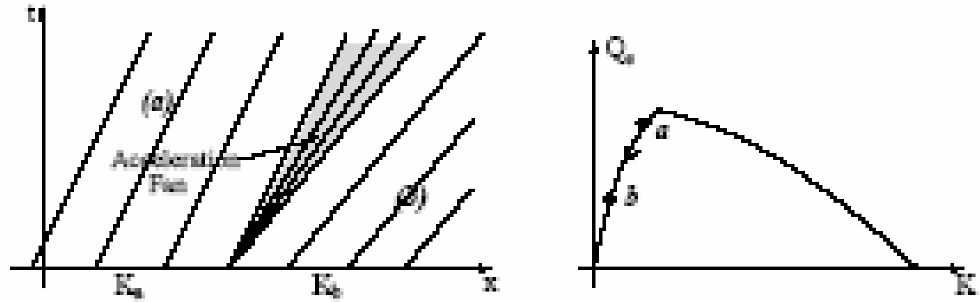
<u>Comment:</u> Experimental fundamental diagrams need *not* be concave.

Analytical solutions (3)

- Rarefaction waves:
  - Characteristic speed (eigenvalue):

$$Q_e'(K)$$

Acceleration ⇔ rarefaction wave ⇔ so-called rarefaction *fan*:



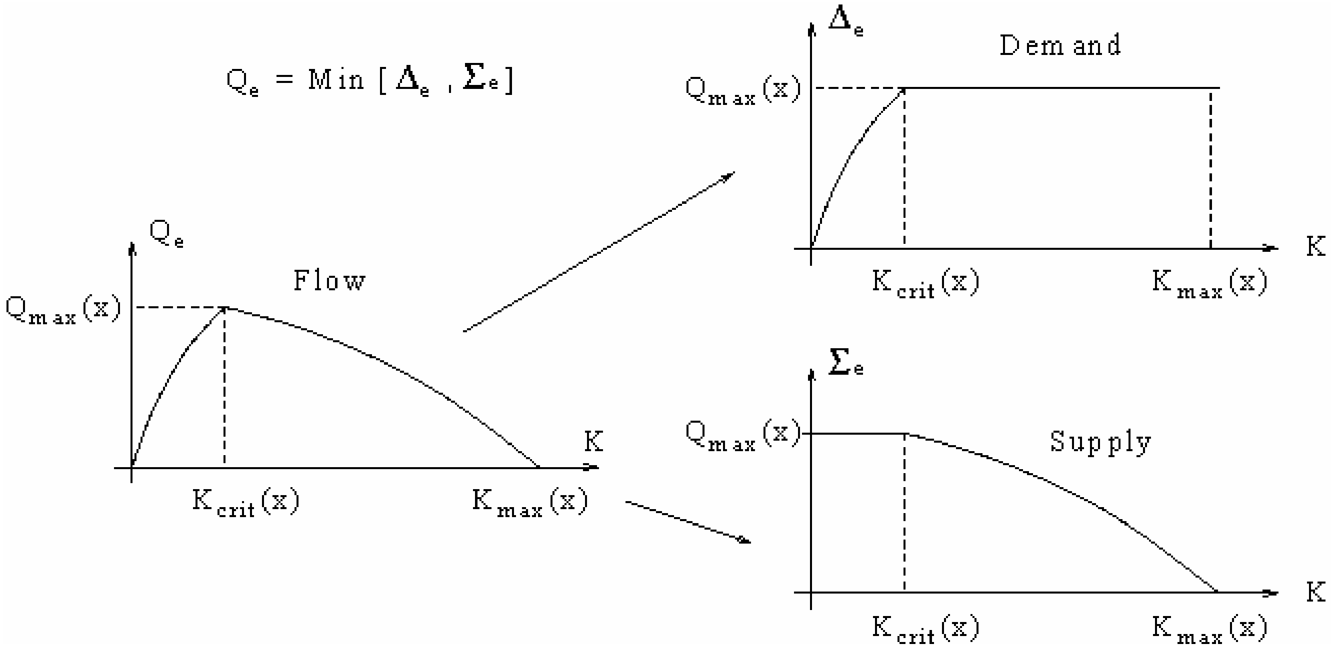
- Entropic solutions are not necessarily physically correct (*boundedness of acceleration*, Lebacque 1997-2002-2003).

- Comments:

- Some intersection models require *boundedness of acceleration*.

The LWR model: supply / demand

- the *equilibrium supply* Σ_e and *demand* Δ_e functions (Lebacque, 1996)



The LWR model: the min formula

- The local supply and demand:

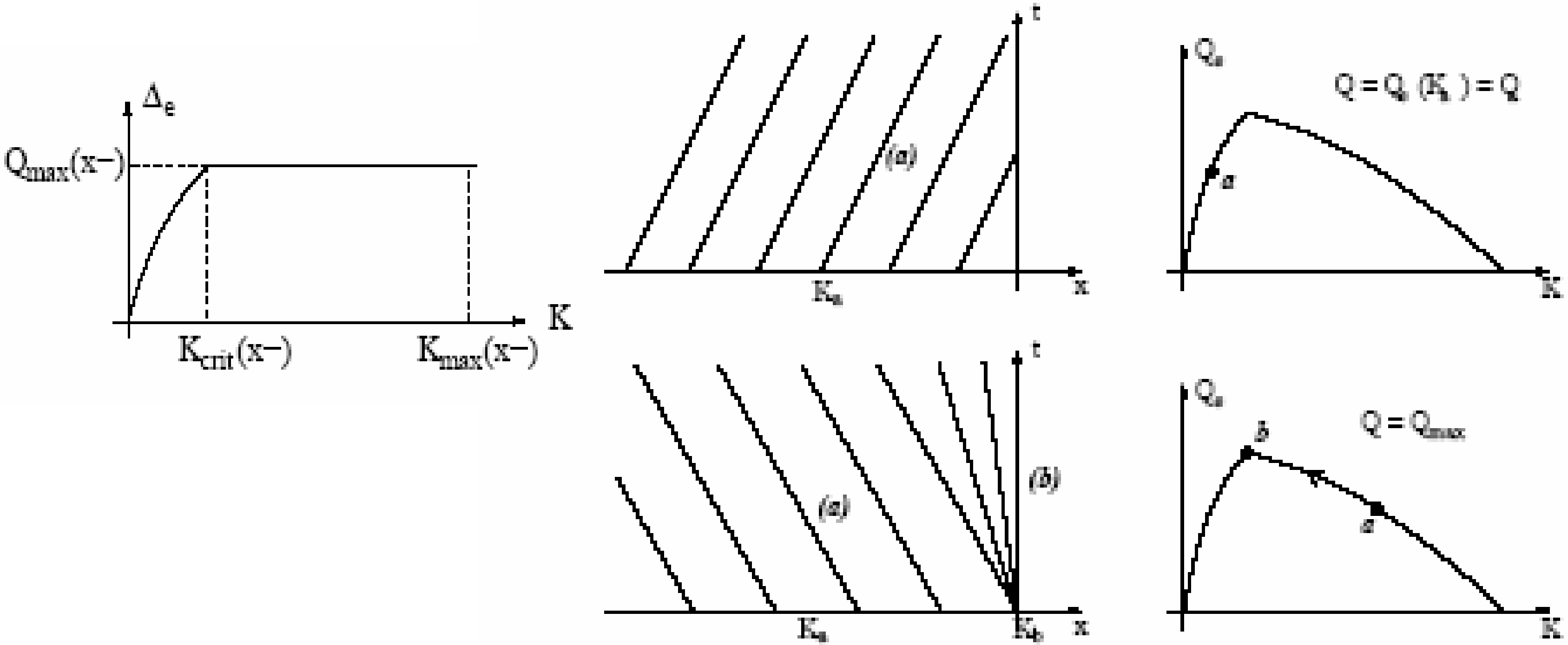
$$\Sigma(x,t) = \Sigma_e\left(K\left(x+,t\right), x+\right)$$
$$\Delta(x,t) = \Delta_e\left(K\left(x-,t\right), x-\right)$$

- The min formula

$$Q\left(x,t\right) = \mathrm{Min}\left[\Sigma\left(x,t\right), \Delta\left(x,t\right)\right]$$

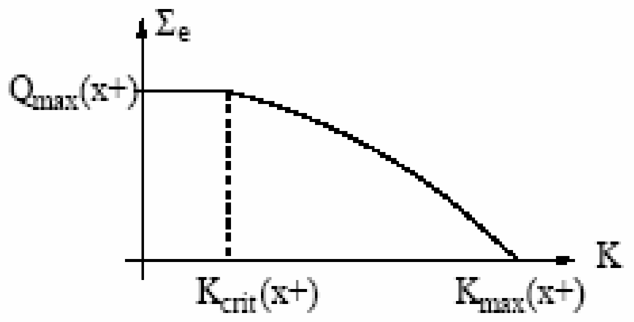
Local traffic demand

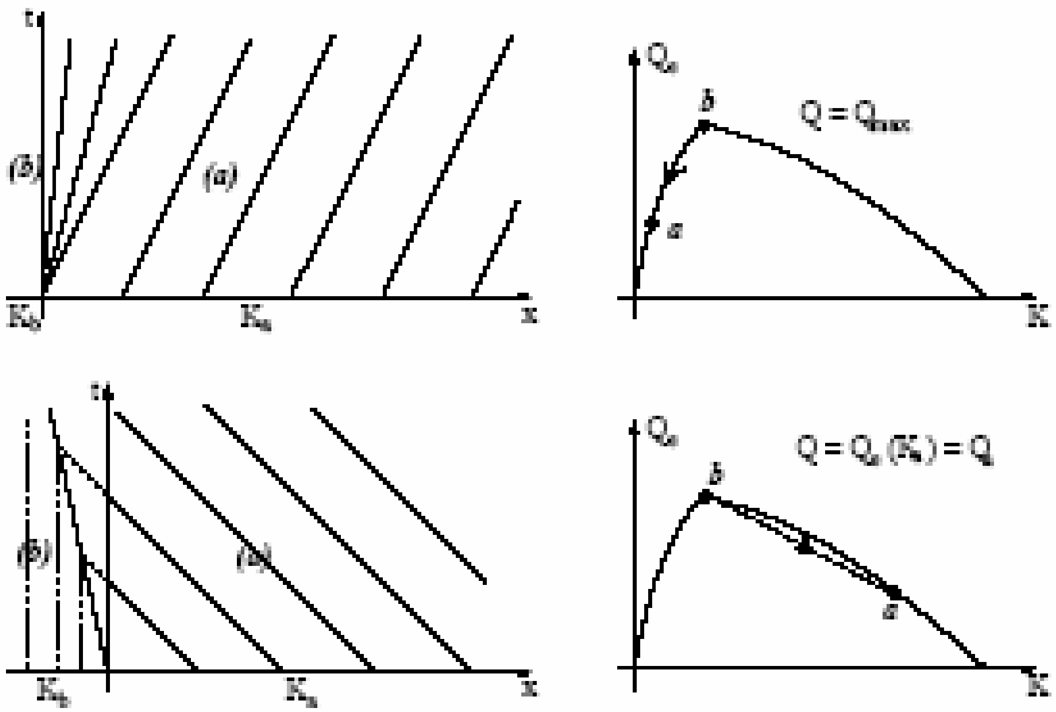
Local Traffic Demand *Demand* at a point x is the greatest possible *outflow* at that point:



Local traffic supply

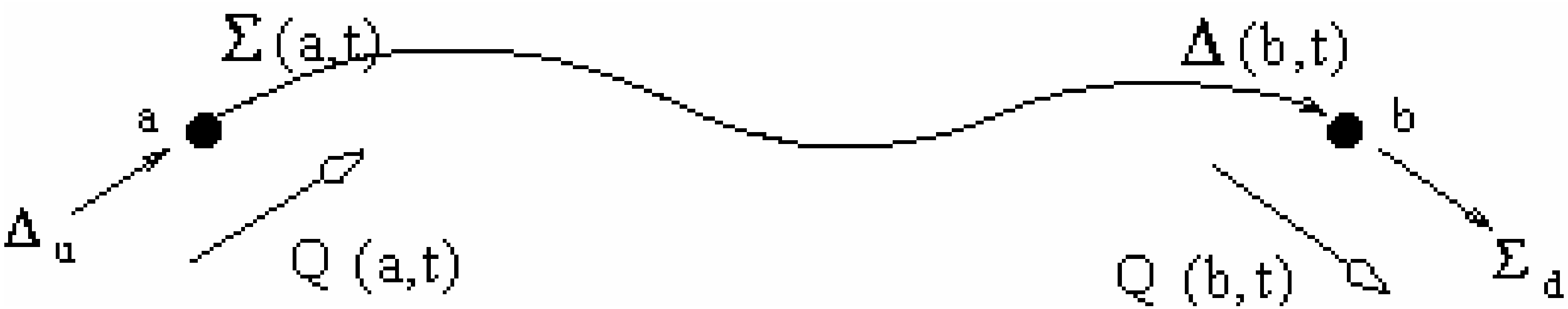
Local Traffic Supply *Supply* at a point x is the greatest possible *inflow* at that point:





Supply-demand boundary conditions

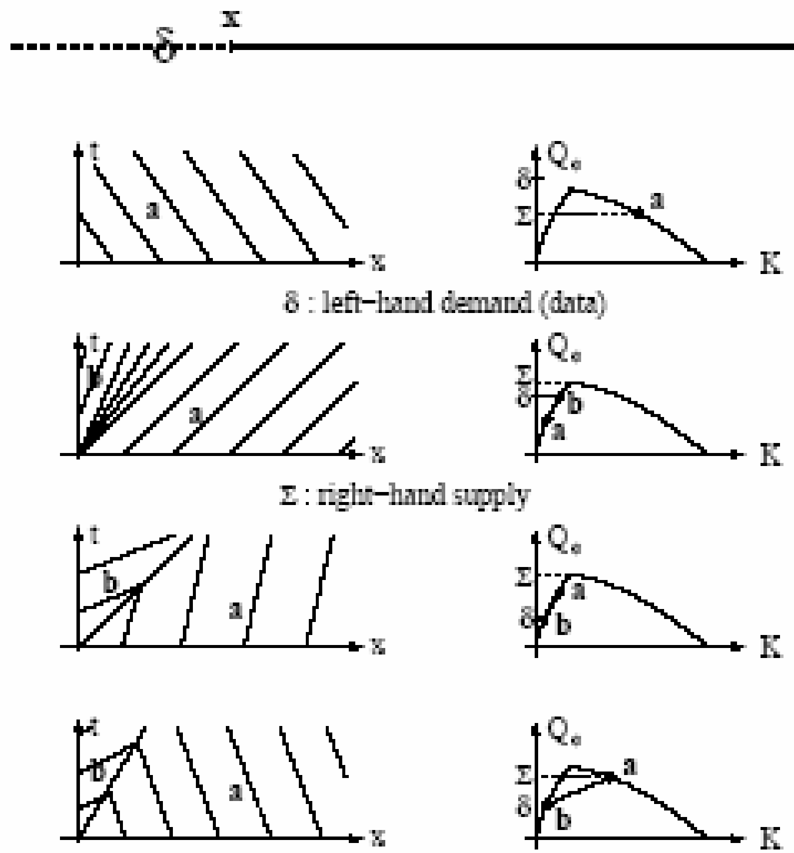
- Link supply : $\Sigma(a,t) = \Sigma_e(K(a+,t),a)$
- Link demand : $\Delta(b,t) = \Delta_e(K(b-,t),b)$
- Min formula : $Q(a,t) = \mathrm{Min}\left[\Delta_u(t), \Sigma(a,t)\right]$
  $Q(b,t) = \mathrm{Min}\left[\Delta(b,t), \Sigma_d(t)\right]$



Upstream boundary condition

- The upstream boundary condition determines the link inflow (the Min formula)

$$Q(a,t) = \mathrm{Min}\left[\Delta_u(t), \Sigma(a,t)\right]$$



Upstream boundary condition

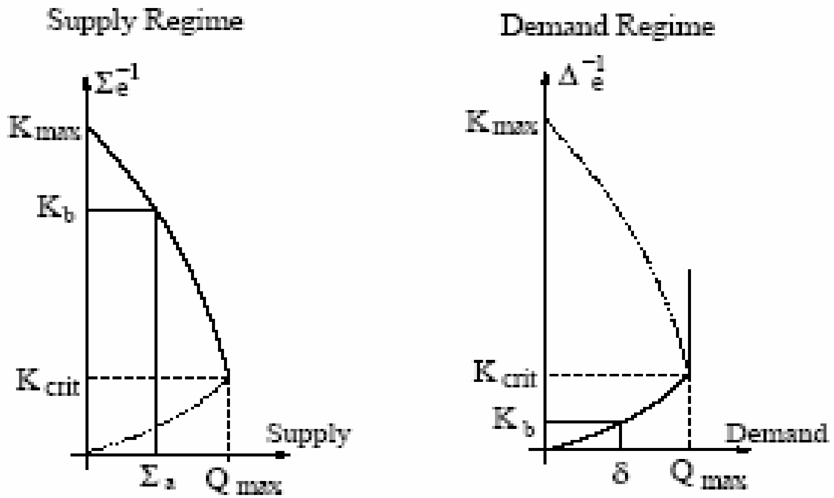
- The upstream boundary condition determines the density at the link entry point

- Symmetric rules apply at the link exit

Let $q = Min[\delta, \Sigma_a]$ be the entry flow

If $\Sigma_a < \delta,\ K_b = \Sigma_e^{-1}(q)$

If $\Sigma_a \geq \delta,\ K_b = \Delta_e^{-1}(q)$

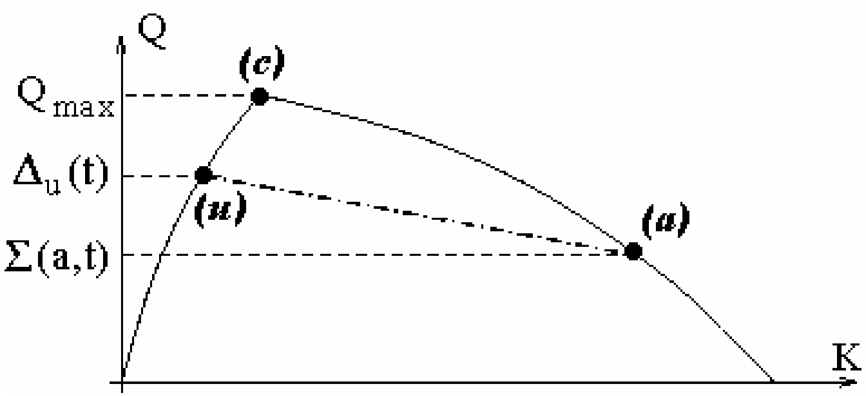


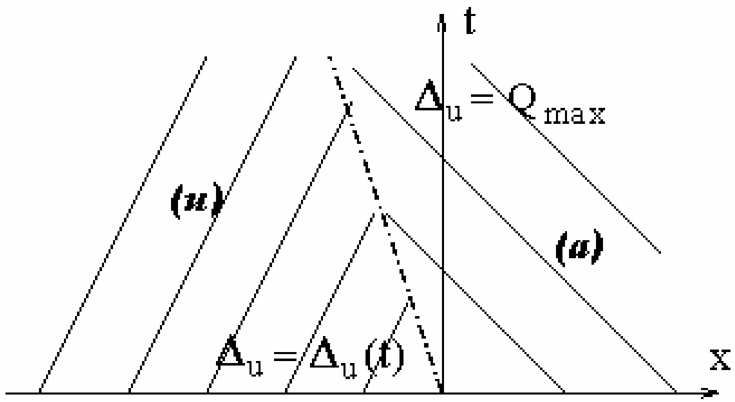
Upstream demand can be modified by boundary conditions

- If link supply $\Sigma_a(t)$ is less than upstream demand ⇨
- The upstream demand is modified:

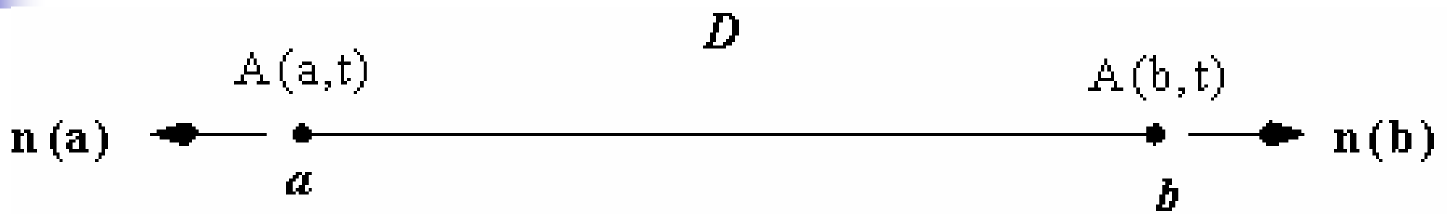
$$\Delta_u(t) \to \Delta_u(t+) = Q_{max}$$

- This fact is fundamental for intersection modeling
- Symmetric result for downstream supply





The BLN (Bardos-Leroux-Nédélec) boundary condition



- Origin: viscosity solutions of the LWR
- Idea: to impose a density-like *A* at the boundary
- Mathematical expression

$$\{\operatorname{sgn}[K(x,t)-\kappa]-\operatorname{sgn}[A(c,t)-\kappa]\}[Q_e(K(c,t),c)-Q_e(\kappa)].n(c)\geq 0$$

$$\forall c\in\partial D=\{a,b\}\quad \text{and}\quad \forall\kappa\geq 0$$

BLN boundary conditions: the density at the boundary cannot be prescribed

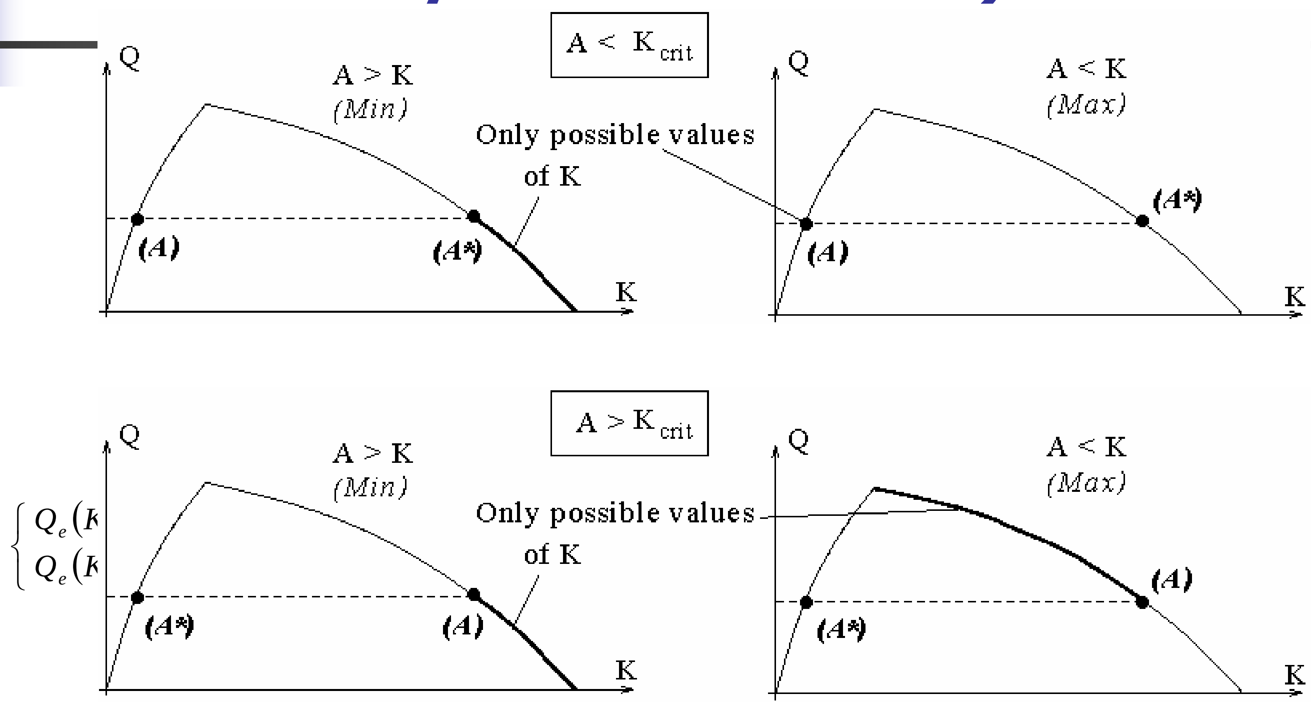
$$\begin{cases} Q_e(K) = \mathrm{Min}_{\kappa \in [A,K]} Q_e(\kappa) & \text{if} \quad K \geq A \\ Q_e(K) = \mathrm{Max}_{\kappa \in [K,A]} Q_e(\kappa) & \text{if} \quad K \leq A \end{cases}$$

- With upstream boundary conditions:

$$A \leq K_{crit} : \quad K \in \{A\} \cup \left[A^*, K_{max}\right]$$
$$A \geq K_{crit} : \quad K \in \left[K_{crit}, K_{max}\right]$$

- *A* cannot be prescribed

Graphical illustration (upstream boundary conditions)



$$\begin{cases} Q_e(K) = \operatorname{Min}_{\kappa \in [A,K]} Q_e(\kappa) & \text{if} \quad K \geq A \\ Q_e(K) = \operatorname{Max}_{\kappa \in [K,A]} Q_e(\kappa) & \text{if} \quad K \leq A \end{cases}$$

$$A \leq K_{crit} \; : \quad K \in \{A\} \cup \left[A^*, K_{max}\right]$$
$$A \geq K_{crit} \; : \quad K \in \left[K_{crit}, K_{max}\right]$$

The BLN and the supply / demand boundary conditions are equivalent

$$A \le K_{crit} \quad : \quad K \in \{A\} \cup \left[A^*, K_{max}\right]$$
$$A \ge K_{crit} \quad : \quad K \in \left[K_{crit}, K_{max}\right]$$

- Cases K ≠ A: $\Leftrightarrow \quad \Sigma_e(K) \le \Delta_e(A)$
  - $A \le K_{crit}$ and $K \in \left[A^*, K_{max}\right)$

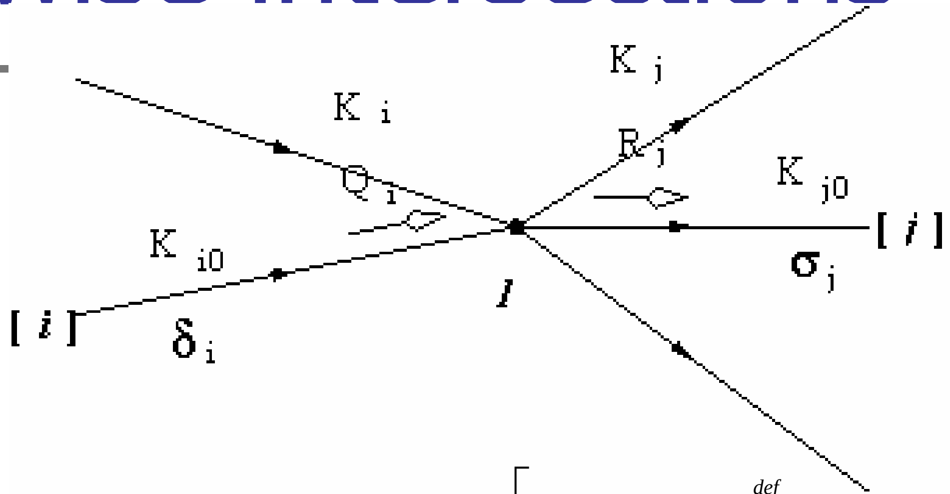
$$\Sigma_e(K) = Q_e(K) \le Q_e(A) = \Delta_e(A)$$

  - $A \ge K_{crit}$ and $K \in \left[K_{crit}, K_{max}\right)$

$$\Sigma_e(K) = Q_e(K) \le Q_{max} = \Delta_e(A)$$

- *A* over-critical in all these cases

Point-wise intersections

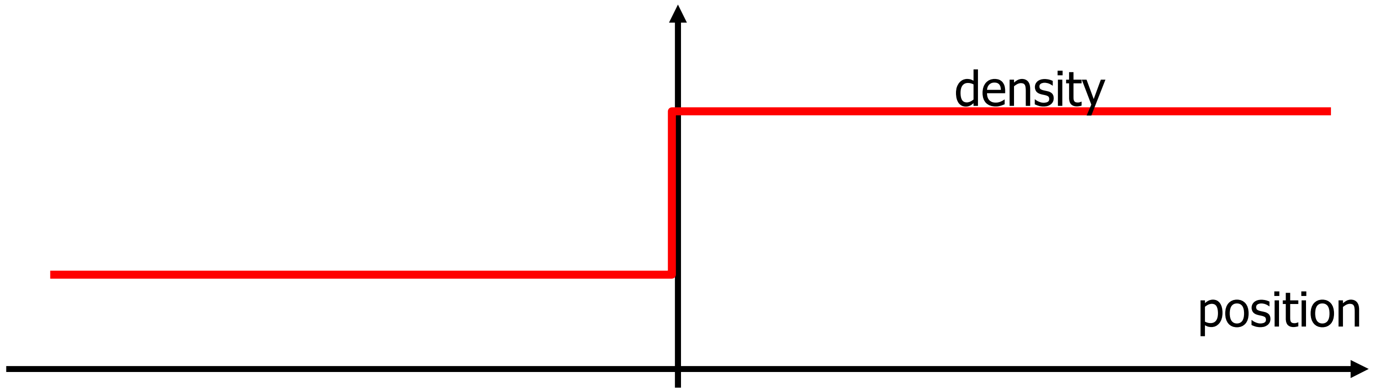


- References: Holden-Risebro 1995, Coclite Piccoli 2002, Lebacque Khoshyaran 1998 2002 2005
- Basic idea: solve the (generalized) Riemann problem for the intersection
- Result: constraints on the through flows

$$\left[\begin{array}{l} Q_i \le \Delta_e\left(K_{i0}\right) \overset{def}{=} \delta_i \\ \quad \text{and} \quad \begin{cases} K_i = \Sigma_e^{-1}\left(Q_i\right) & \text{if} \quad Q_i < \delta_i \\ K_i = K_{i0} & \text{if} \quad Q_i = \delta_i \end{cases} \\ \\ R_j \le \Sigma_e\left(K_{j0}\right) \overset{def}{=} \sigma_j \\ \quad \text{and} \quad \begin{cases} K_j = \Delta_e^{-1}\left(Q_j\right) & \text{if} \quad R_j < \sigma_j \\ K_j = K_{j0} & \text{if} \quad R_j = \sigma_j \end{cases} \end{array}\right.$$

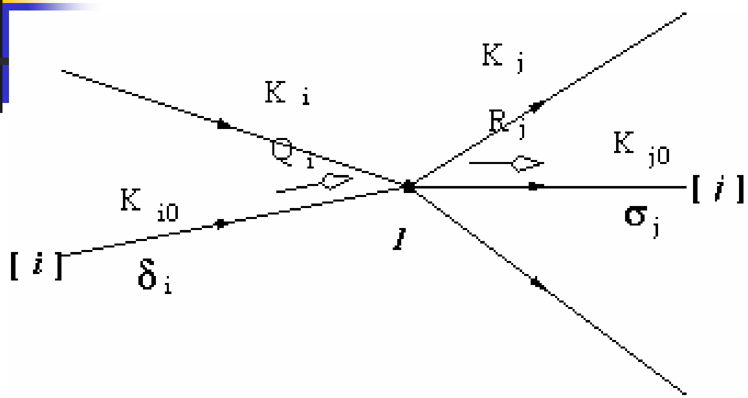
Riemann problem

- It is an archetype for many practical situations
- Initial conditions are piecewise constant
- Solutions are self-similar (waves)



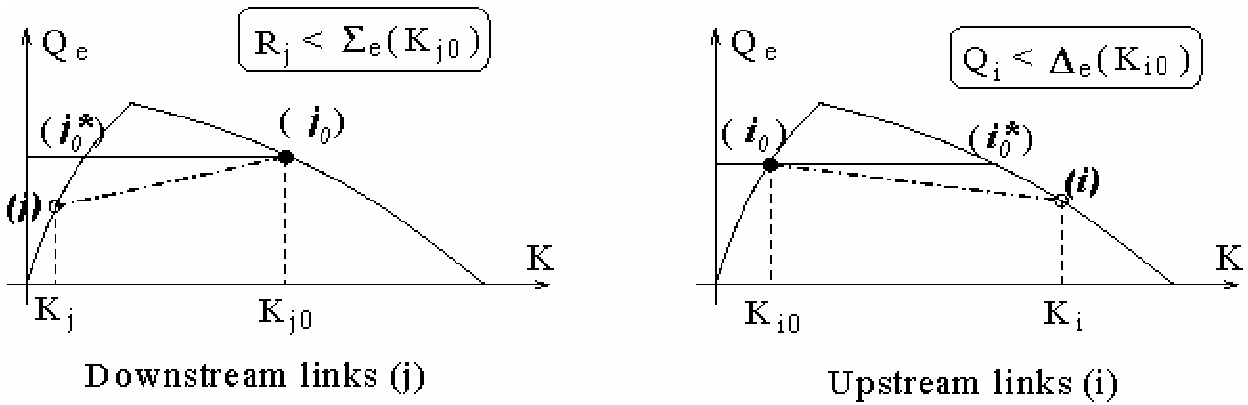
- It is the key for developing numerical methods

Point-wise intersections: flow constraints



$$
\begin{cases}
0 \le Q_i \le \delta_i & \forall i \\
0 \le R_j \le \sigma_j & \forall j \\
\sum_i Q_i = \sum_j R_j &
\end{cases}
$$

- Flow constraints imply that density changes propagate in the right direction



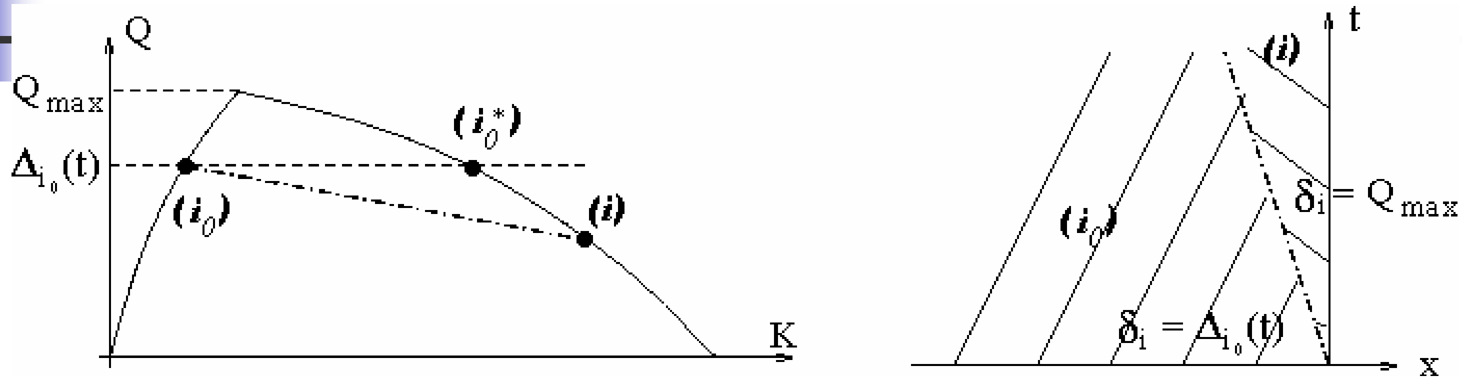
Necessity of intersection models

- Other flow constraints are possible (turning movement proportions, assignment coefficients…)
- Flow constraints do not suffice to determine the flow values
- A behavioral intersection model is necessary

$$(Q, R) = f(\delta, \sigma)$$

- But not all models are consistent

Invariance principle 1



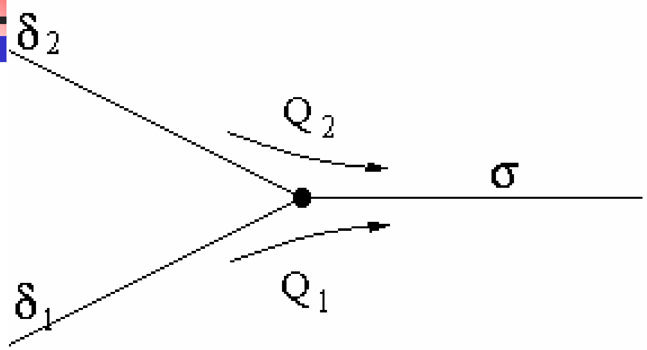
- Upstream link ***i***. If $Q_i(t)<\delta_i(t)$: supply regime at the link exit and $\delta_i(t+) = Q_{max}$
- Downstream link ***j***. If $R_j(t)<\sigma_j(t)$: supply regime at the link exit and $\sigma_j(t+) = Q_{max}$
- The intersection model $(Q,R) = f(\delta,\sigma)$ must be invariant by the transformation

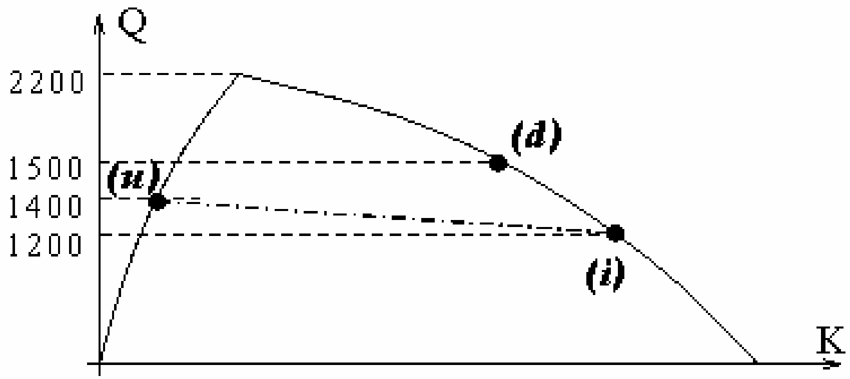
$$\left[\begin{array}{ll} \delta_i \rightarrow Q_{i,max} & \text{if } Q_i < \delta_i \\ \sigma_j \rightarrow R_{j,max} & \text{if } R_j < \sigma_j \end{array}\right.$$

Invariance principle 2

- Another way of stating the invariance principle:
  - The intersection model must be compatible with self-similarity of solutions
  - Riemann problem at the node
  - Lebacque-Khoshyaran 1998-2003-2005

The invariance principle: an example

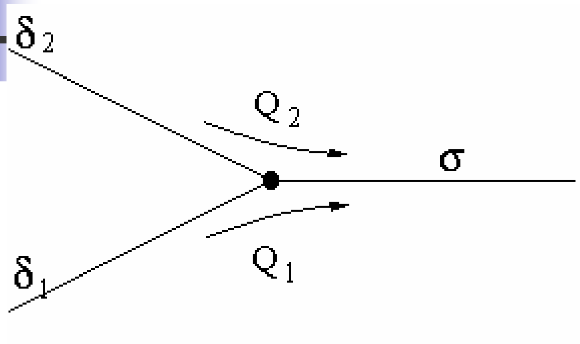


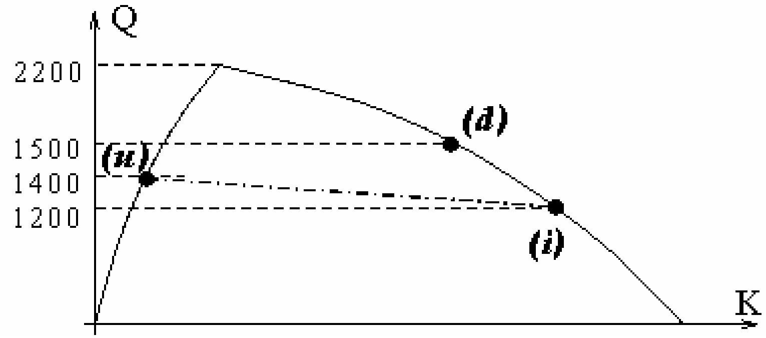


- Distribution scheme:
- Numerical values:
  - σ = 3000 vh / h (2 lanes)
  - δ1 = 2100 vh / h (1 lane)
  - δ2 = 1400 vh / h (1 lane)
  - $Q_{k,max}$ = 2200 vh / h

$$\begin{cases} Q_i = \delta_i & \text{if} \quad \sum_i \delta_i \le \sigma \\ Q_i = \dfrac{\delta_i}{\sum_k \delta_k} & \text{if} \quad \sum_i \delta_i > \sigma \end{cases}$$

The invariance principle: an example (cont'd)





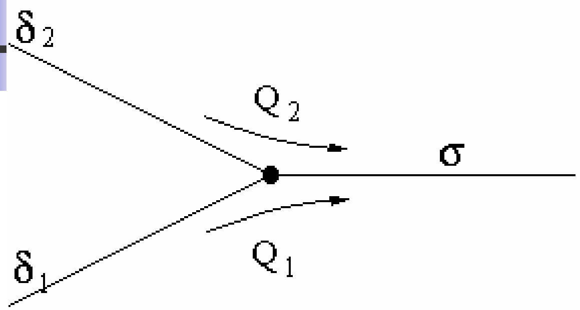
$$Q_1 = \frac{2100}{2100+1400} 3000 = 1800 \text{ vh/h} < 2100 \text{ vh/h} = \delta_1(t)$$

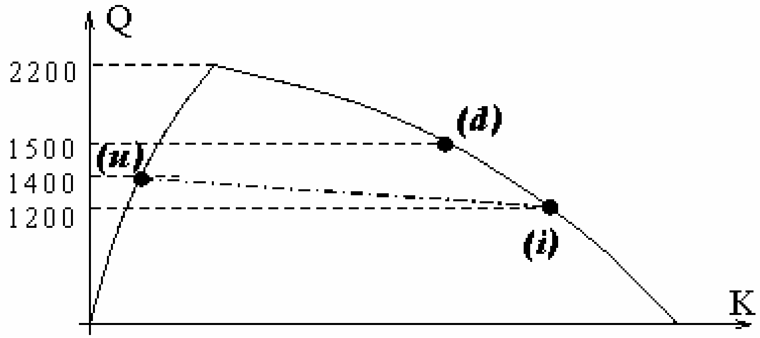
$$Q_2 = \frac{1400}{2100+1400} 3000 = 1200 \text{ vh/h} < 1400 \text{ vh/h} = \delta_2(t)$$

- The flow values calculated by the distribution scheme imply a shift in the upstream demands

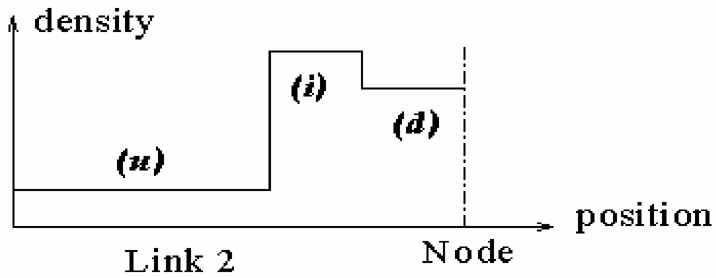
$$\left| \begin{array}{l} \delta_1(t+) = 2200 \text{ vh/h} \\ \delta_2(t+) = 2200 \text{ vh/h} \end{array} \right.$$

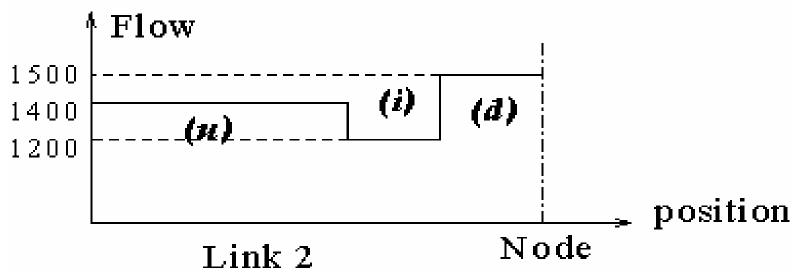
The invariance principle: an example (cont'd)





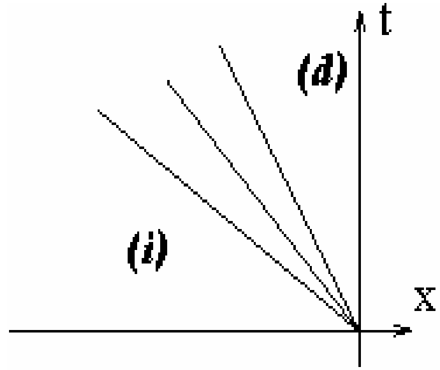
- The new demand values determine new flow values $Q_1 = Q_2 = 1500$ vh/h
- Illustration for link [*2*]
- 3 traffic states



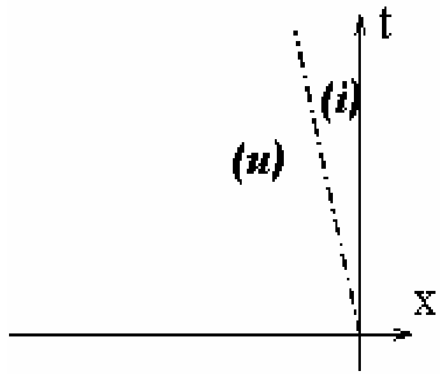


The invariance principle: an example (cont'd)

- States **(*u*)**, **(*i*)**, **(*d*)**



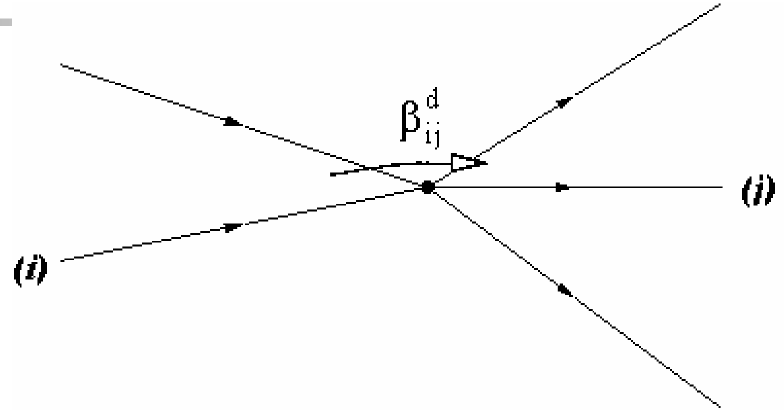
- Velocity of the **(*u*)** → **(*i*)** shockwave < the velocity of the **(*d*)** → **(*i*)** rarefaction wave ⇒
  - The state **(*i*)** must vanish
- Velocity of the **(*u*)** → **(*d*)** shockwave > 0 ⇒
  - state **(*d*)** must vanish at *t+*.
  - ⇒ inconsistency



An example of a node model satisfying the invariance principle: the optimization node model

$$Max \sum_i \Phi_i(Q_i) + \sum_j \Psi_j(R_j)$$

$$\left| \begin{array}{ll} Q_i \leq \delta_i & \forall i \\ R_j \leq \sigma_j & \forall j \\ \sum_i \gamma_{ij} Q_i - R_j = 0 & \forall j \end{array} \right.$$



- γ_{ij} : turning movement coefficients (deduced from the assignment coefficients)
- Constraints:
  - Node inflows less than upstream demands
  - Node outflows less than downstream supplies
  - Conservation of node out-flows

Optimization node model

- The Karush-Kuhn-Tucker optimality conditions yield (***s**$_j$*) coefficient of the outflow (***j***) conservation equation)

$$
\begin{aligned}
&Q_i = \mathrm{Min}\left[\delta_i, \Phi_i'^{-1}\left(-\sum_l \gamma_{il} s_l\right)\right] \\
&R_j = \mathrm{Min}\left[\Psi_j'^{-1}(s_j), \sigma_j\right] \\
&\sum_i \gamma_{ij} Q_i - R_j = 0 \quad \forall j
\end{aligned}
$$

- The in- and out-flows are given by a Min-formula ⇒ The model satisfies the invariance principle

$$
\begin{aligned}
&Q_i = \mathrm{Min}\left[\delta_i, \varphi_i\right] \\
&R_j = \mathrm{Min}\left[\psi_j, \sigma_j\right]
\end{aligned}
$$

Optimization node model (cont'd)

- Interpretation of the criterion:
  - Φ_i : → partial supply of node (for link (i))
  - Ψ_j : → partial demand of node (for link (j))
- Coefficients $\boldsymbol{s_j}$: "node state"

- Other models satisfy the invariance principle (dynamic pointwise, equilibrium)

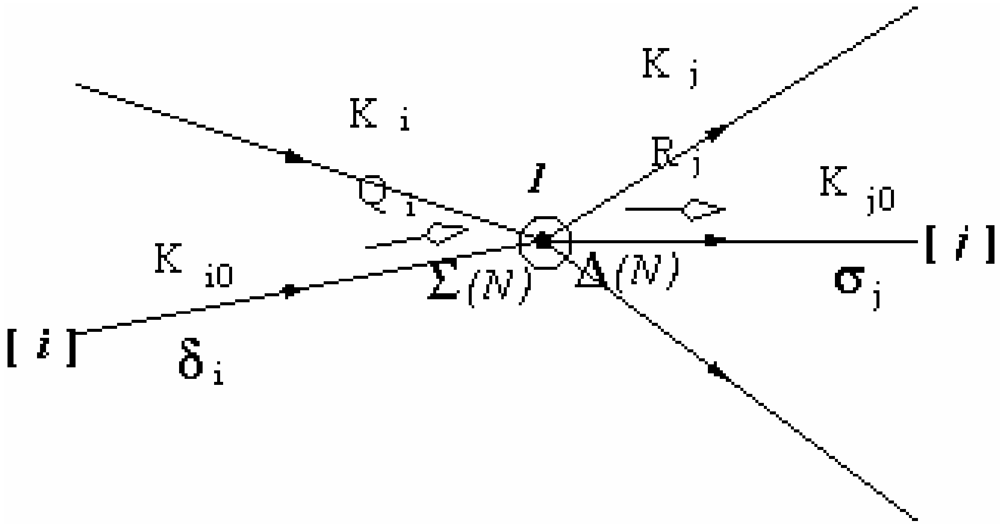
A second example: dynamic node models

- They are characterized by inner state dynamics

$$N = \sum_{ij} N_{ij}$$

$$NO_j = \sum_{i} N_{ij}$$

$$\Sigma_i(N) = \beta_i \, \Sigma(N) \quad \forall i$$



$$\dot{N}_{ij} = \gamma_{ij} Q_i - \frac{N_{ij}}{NO_j} R_j$$

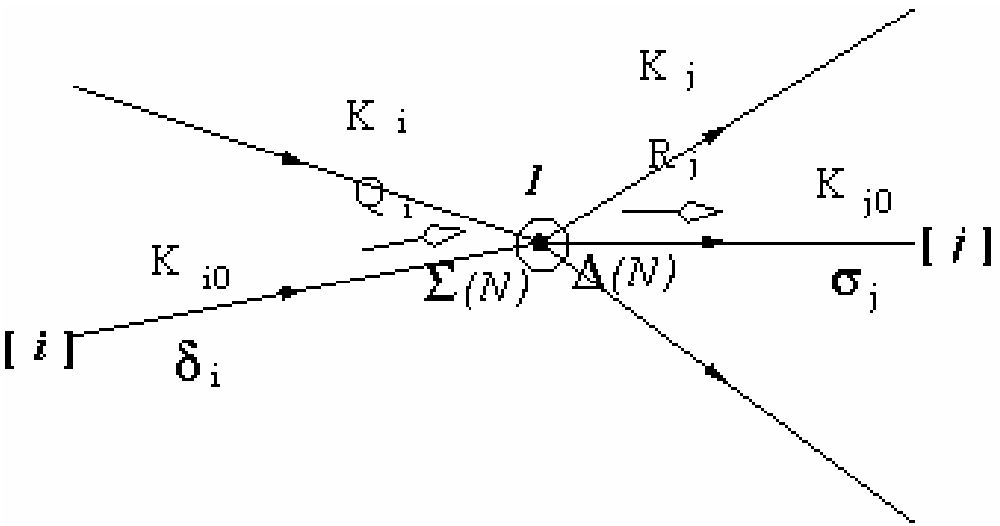
$$Q_i = \mathrm{Min}\left[\delta_i, \beta_i \Sigma(N)\right]$$

$$R_j = \mathrm{Min}\left[\frac{NO_j}{N}\Delta(N), \sigma_j\right]$$

Equilibrium node models

- They are derived from dynamic node models
- Assumption: node time-scale << link time-scale

$$\begin{cases} Q_i = \mathrm{Min}\left[\delta_i, \beta_i \Sigma(N)\right] & \forall i \\ R_j = \mathrm{Min}\left[\dfrac{NO_j}{N}\Delta(N), \sigma_j\right] & \forall j \\ R_j - \gamma_{ij} Q_i = 0 & \forall j \end{cases}$$



Unknowns: N and ratios NO_j/N

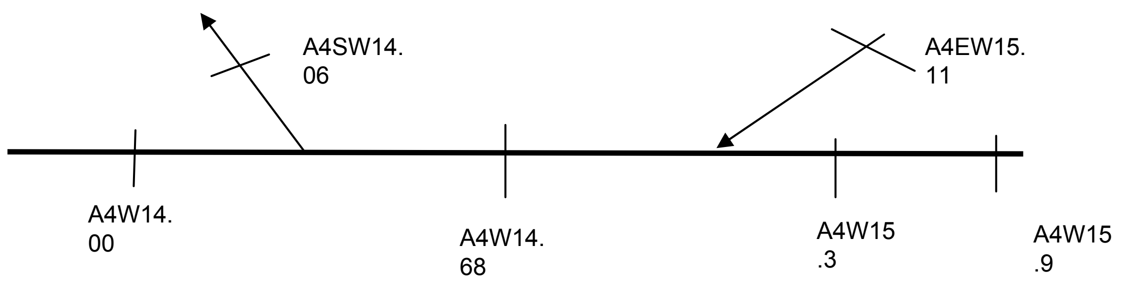
(Node State, no conservation)

Optimization vs Equilibrium node models

- Both models provide node supplies and demands
- Optimization and equilibrium node models are equivalent for
  - Merges
  - FIFO Diverges

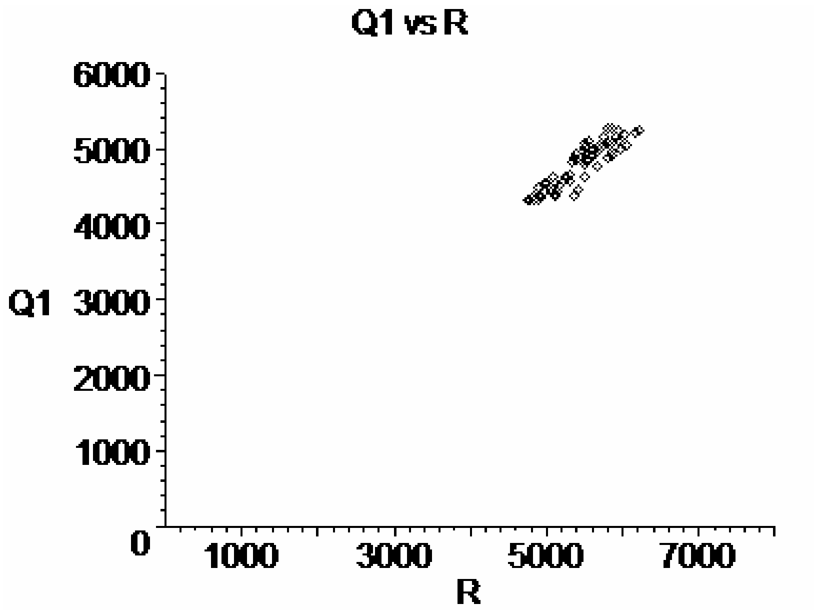
Some numerical results

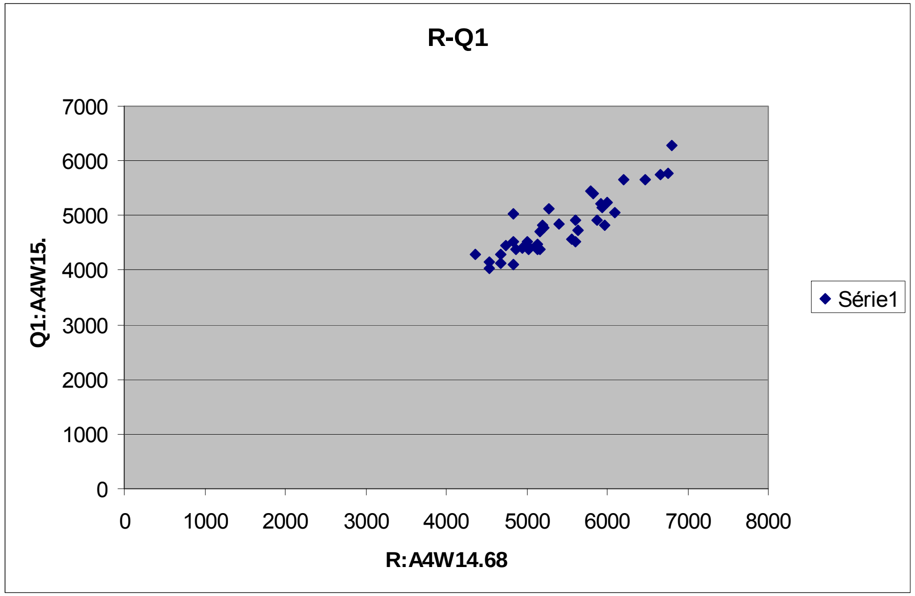
- The site: A4
- The sets of points are bounded by linear constraints ⇒ invariance with respect to aggregation
- Results for the merge



The site (A4 motorway, close to Paris)

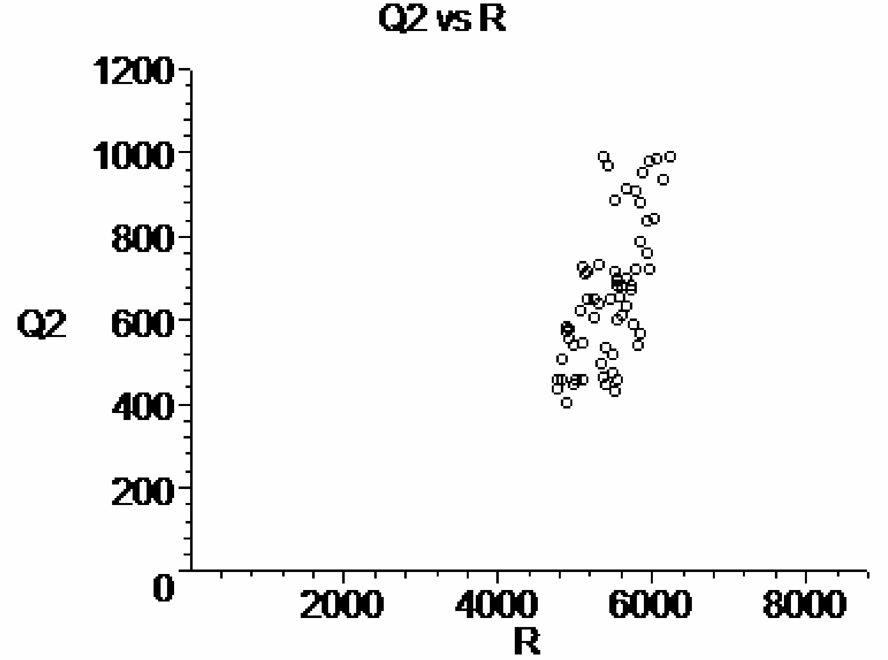
Some numerical results (cont'd): non stochastic scatter of data points

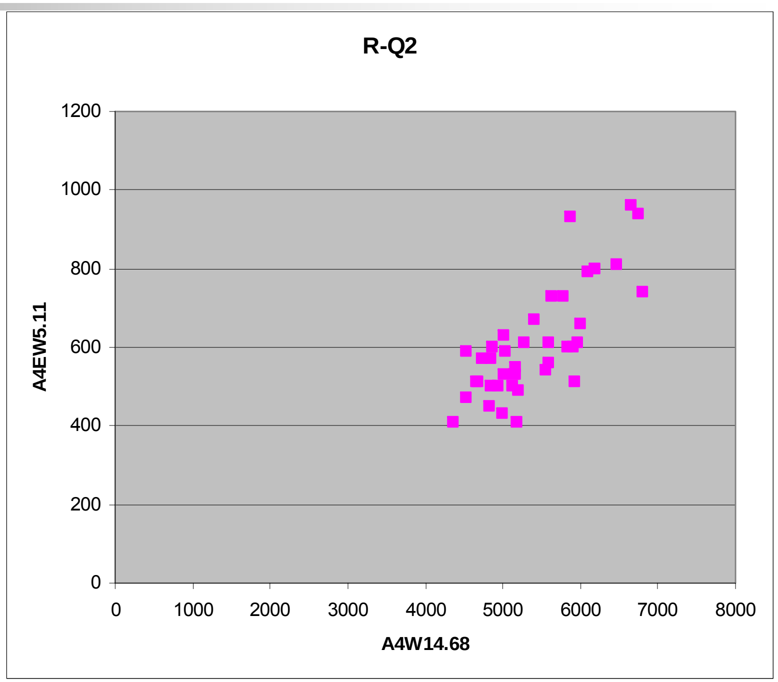




- Motorway: upstream vs downstream flow

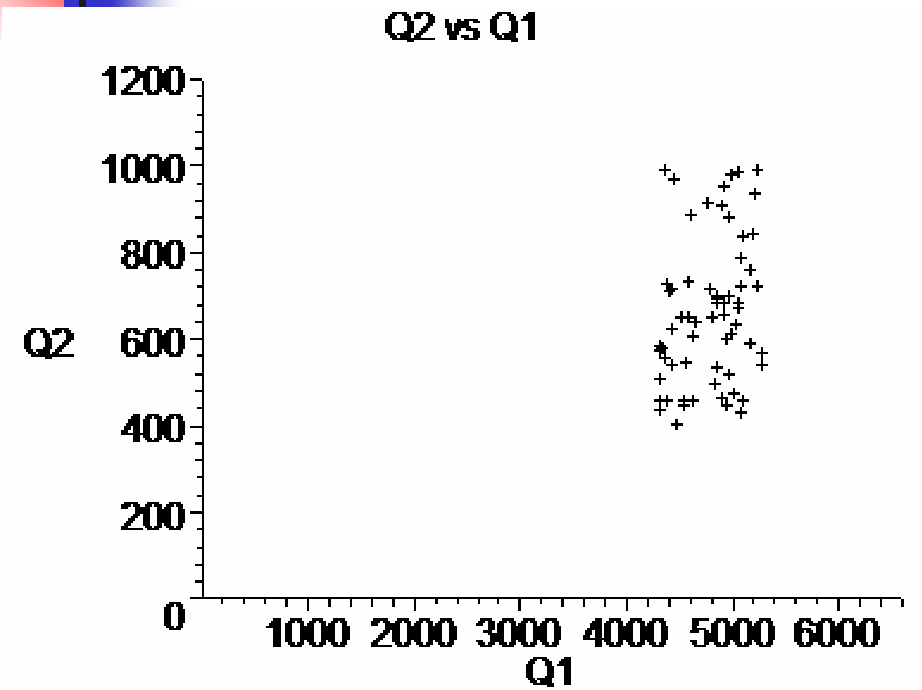
Some numerical results (cont'd)

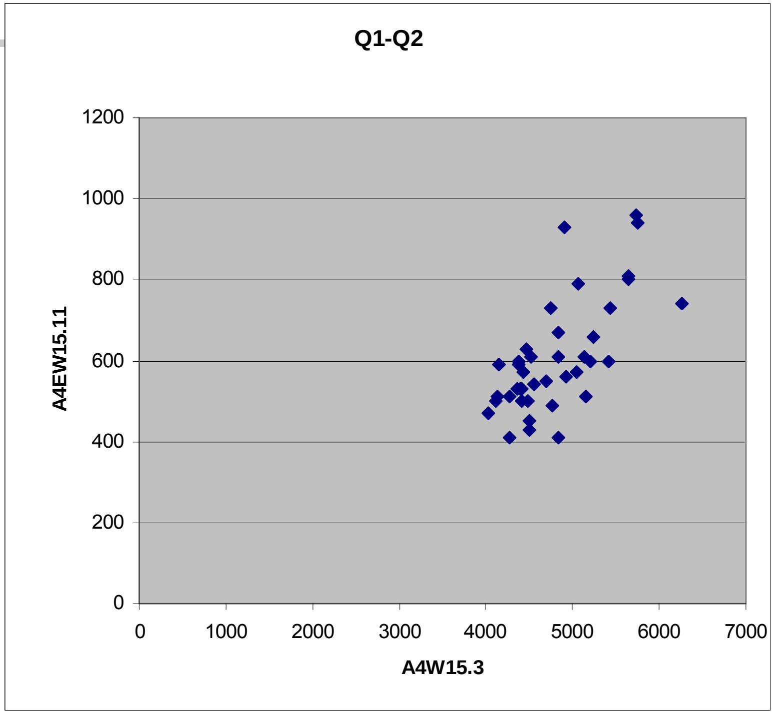




- On ramp flow vs motorway downstream flow

Some numerical results (cont'd)

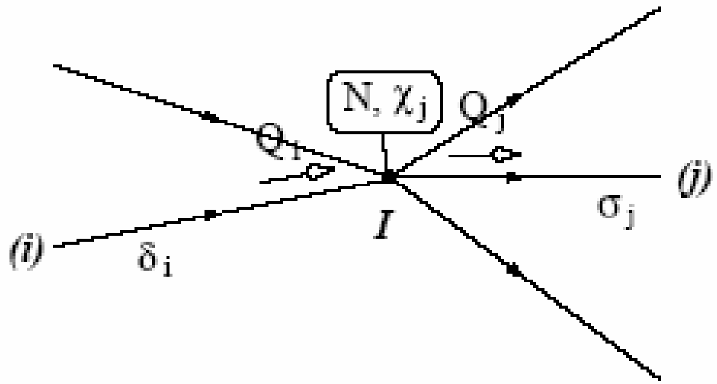




- Onramp vs upstream motorway flow

Physical node models

- Internal state of node:
  - Stored vehicles
  - Node supply/demand functions
  - (Lebacque-Khoshyaran 1998-2002)



$$
\begin{aligned}
N &= \sum_i Q_i - \sum_j Q_j \\
Q_i &= Min\ [\delta_i, \beta_i \Sigma_e(N)] \\
Q_j &= Min\ \left[\chi_j \Delta_e(N), \sigma_j\right]
\end{aligned}
$$

Assume:
fundamental diagram $Q_e(N)$,
and node traffic supply and demand functions
$\Sigma_e(N)$, $\Delta_e(N)$.

β_i: split coefficient for node supply

χ_j: composition coefficient of node traffic

Discretized node models

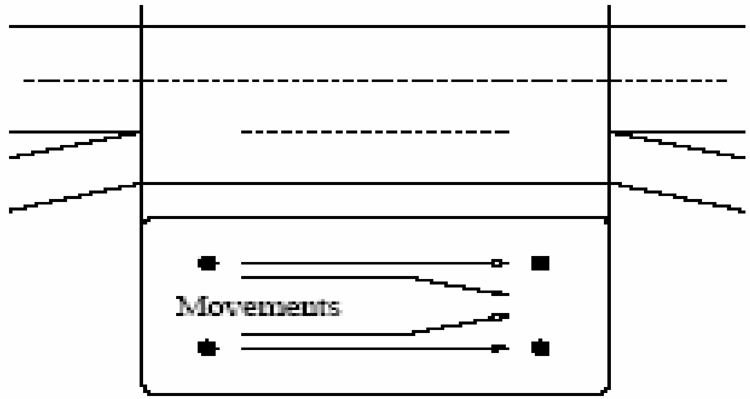
- Exchange zones
  - Generalize cells of Godunov scheme
  - Conflicts are described implicitely
  - Buisson, Lebacque, Lesort 1995-1996
  - Haj-Salem, Lebacque 2003: bounded acceleration

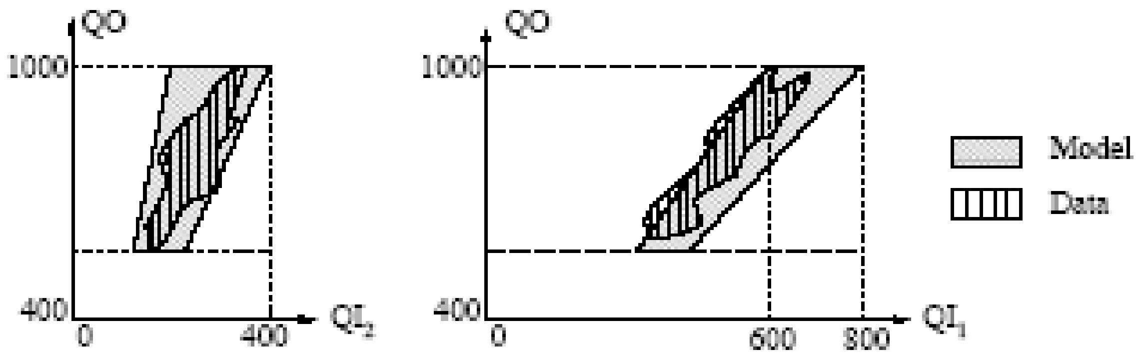
Exchange zones (cont'd)

- Features:
  - global variables N, NI_i, NO_j, N_{ij}^d (movements, per final destination: non FIFO)
  - global supply and demand $\Sigma_e(N)$ and $\Delta_e(N)$
  - partial supplies and demands $\Sigma_i = \beta_i \Sigma_e(N)$, $\Delta_j = (NO_j/N)\Delta_e(N)$

Exchange zones (cont'd)

- Linear partial supply model
- Fits exp. data

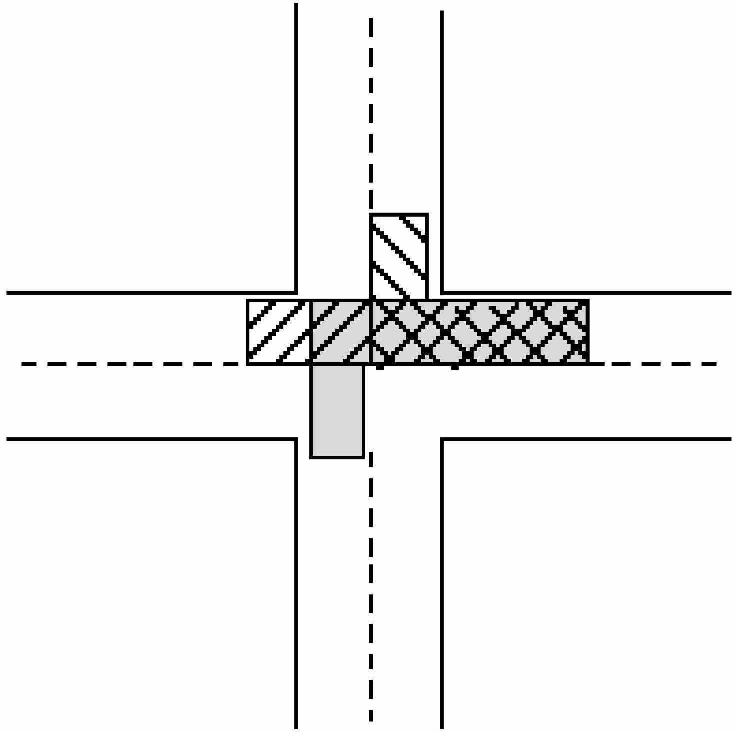




(oversaturated merge, Oltra and Jardin 1998)

The SSMT node

- Models movements with overlapping cells
- SSMT (Lebacque 1984) and METACOR (Haj-Salem 1995)
- Model of priority conflicts (opposing movements)



Extension of the LWR model to networks

- Links: supply/demand boundary conditions for total flow
- Composition (assignment) coefficients are carried by traffic flow
- Node models connect the demand of their upstream nodes to the supplies of their downstream nodes
- Node models must satisfy the invariance principle

Multicommodity flow

- Flow is disaggregated per "commodity" (destination, path, driver category…) *d*

$$K(x,t) = \sum_{d=1\dots D} K^d(x,t) \quad \forall x,t$$

$$Q(x,t) = \sum_{d=1\dots D} Q^d(x,t) \quad \forall x,t$$

- Conservation per commodity (attribute)

$$\frac{\partial K^d}{\partial t} + \frac{\partial Q^d}{\partial x} = 0 \quad \forall d = 1\dots D$$

$$Q^d = K^d V \quad \text{and} \quad K^d = \chi^d K, Q^d = \chi^d Q \quad \forall d$$

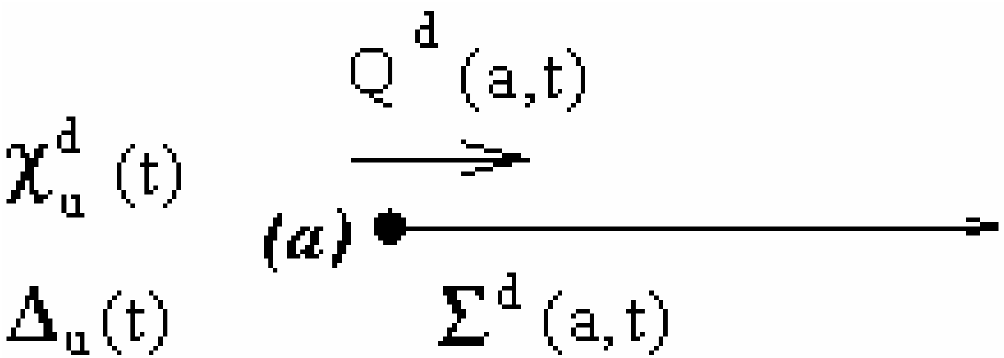
Multicommodity flow: FIFO model

- Principle: all vehicle have the same speed, whichever their attribute
- Composition stays constant along trajectories

$$\frac{\partial \chi^d}{\partial t} + V_e(K)\frac{\partial \chi^d}{\partial x} = 0 \quad \forall d = 1 \ldots D$$

Network boundary data

- Supply, demand for the total flow
- Upstream composition for network inflow



$$\chi^d(a,t) = \chi_u^d(t)$$

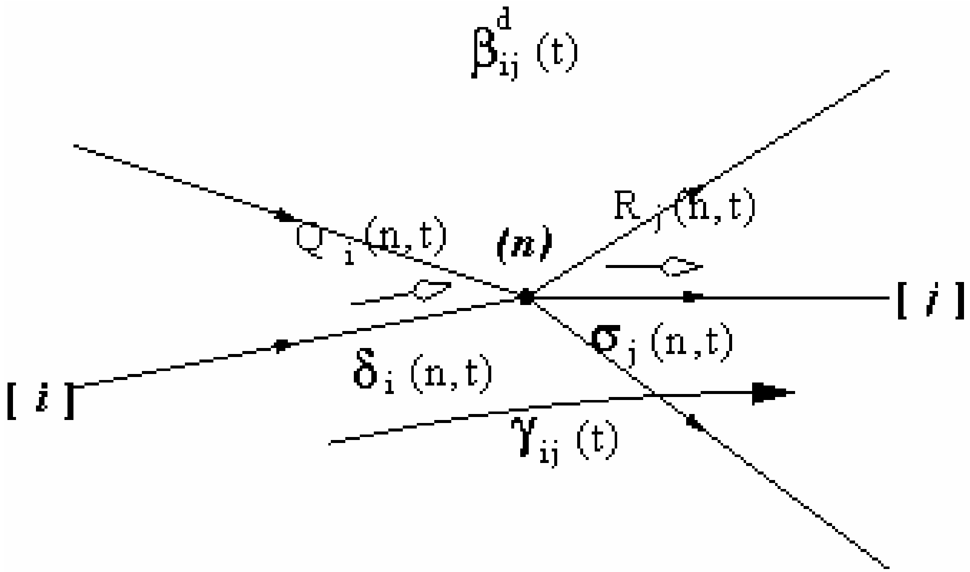
$$Q(a,t) = \mathrm{Min}\left[\Delta_u(t), \Sigma(a,t)\right]$$

$$Q^d(a,t) = \chi^d(a,t)\,Q(a,t)$$

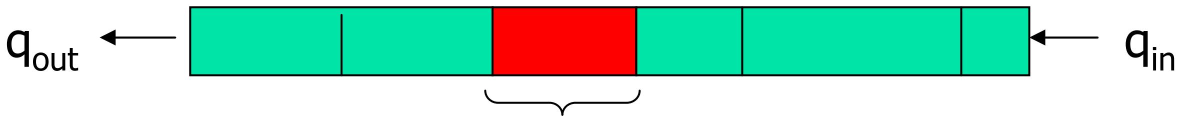
Multicommodity flow: intersections

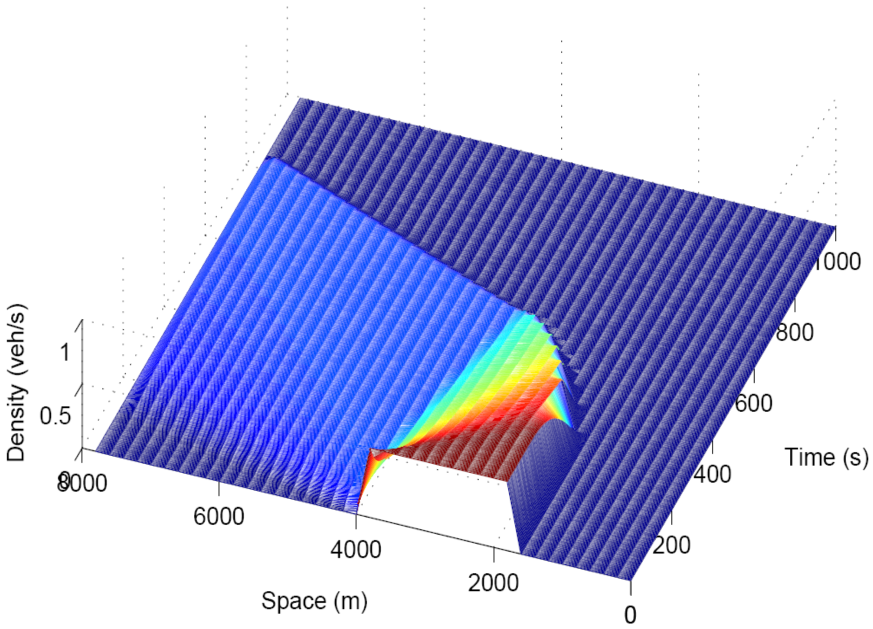
- No change, except…
- Turning movements result from assignment coefficients (behavioral: VMS, user choice…)

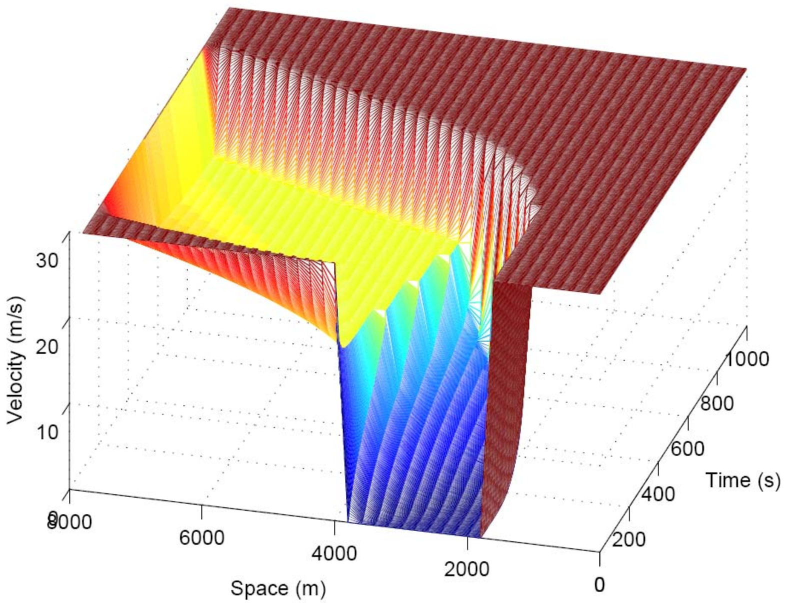
$$\gamma_{ij}(t) = \sum_d \beta_{ij}^d(t)\chi^d(n,t;i)$$



Example: Godunov discretization scheme. Shockwave

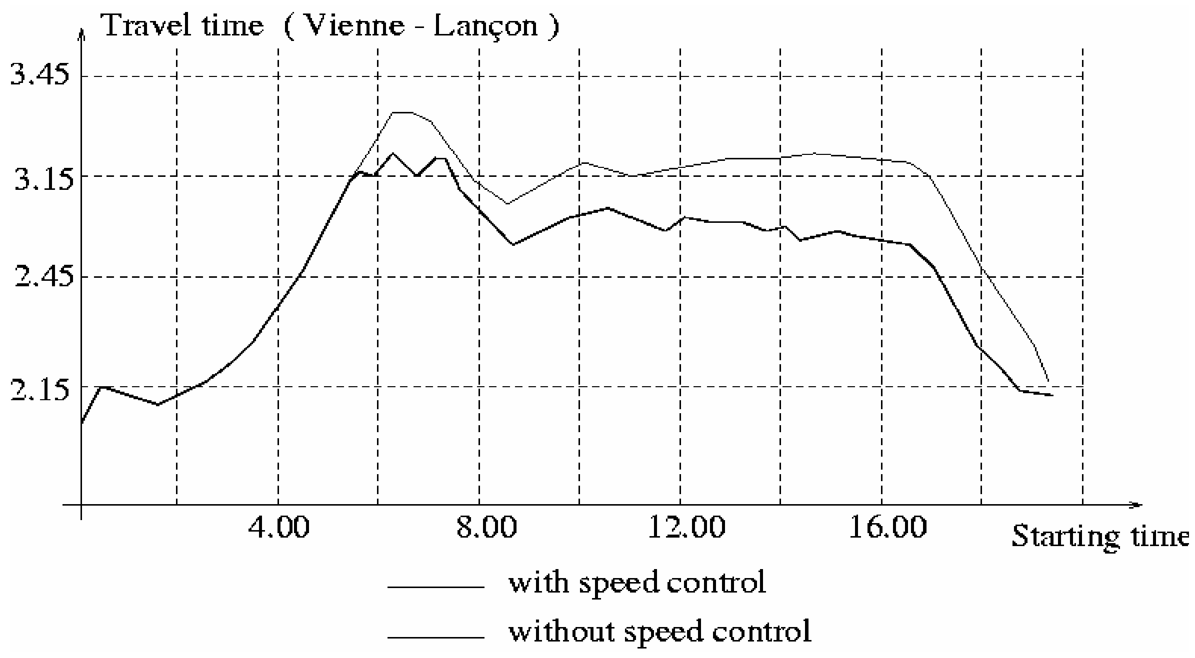






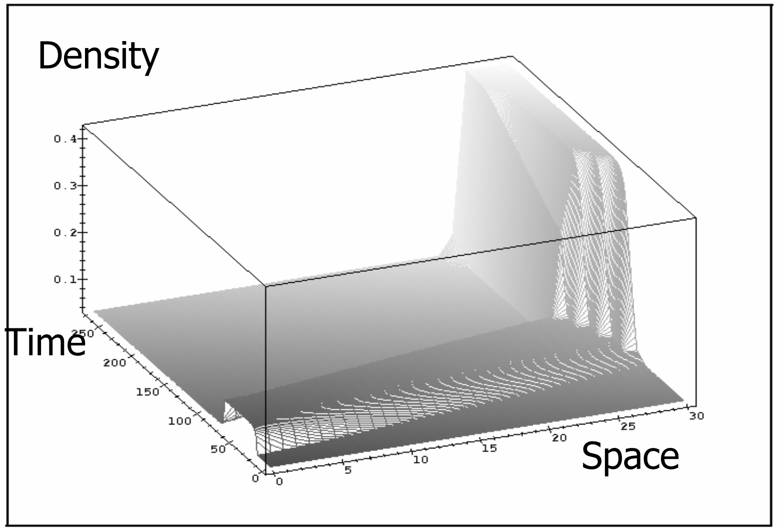
Example: speed control does work

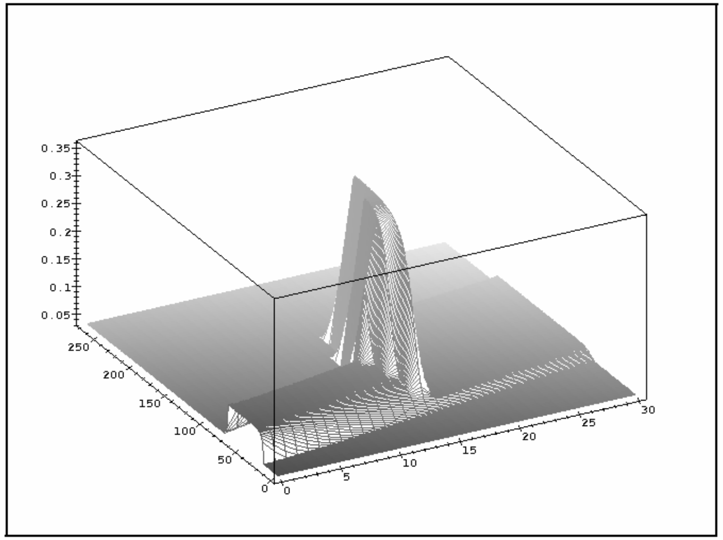
- The facts



No Control vs Speed Control

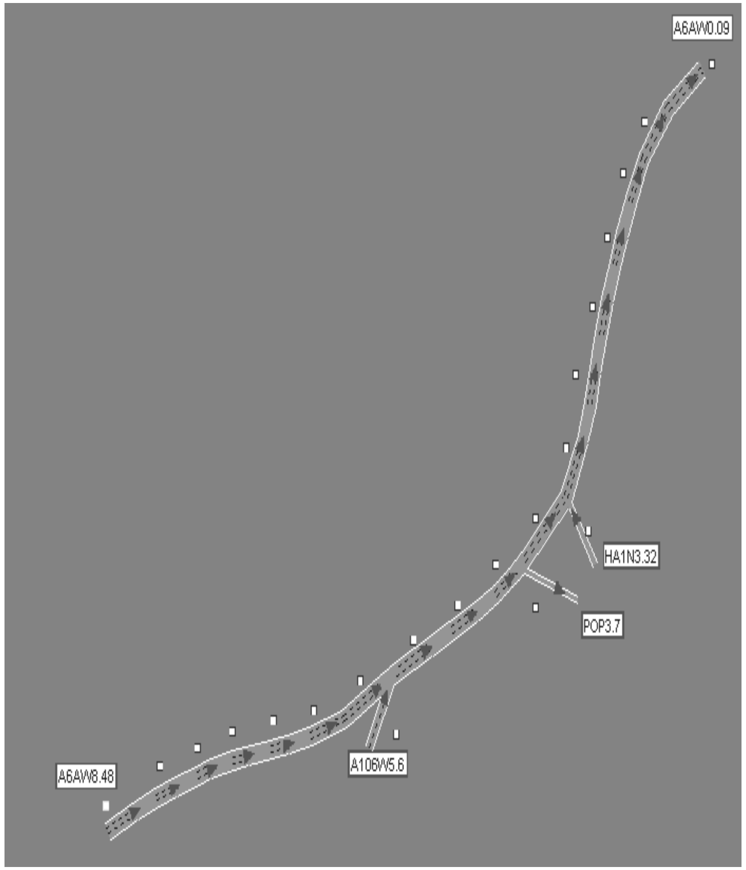
- Bounded acceleration node model for the exit of motorway





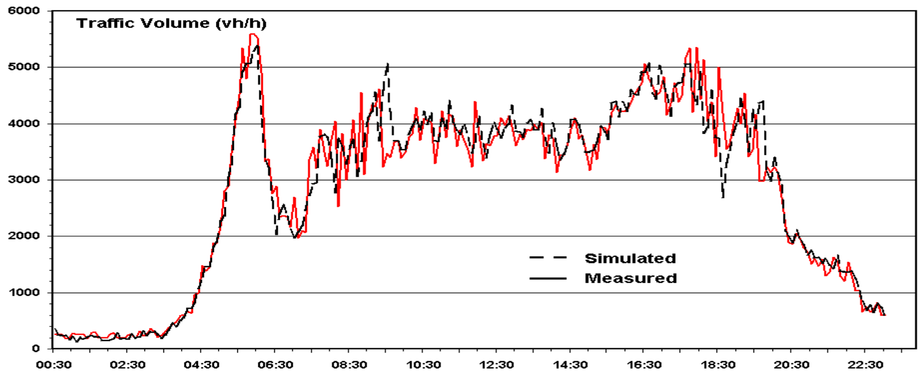
Data reconstruction works too

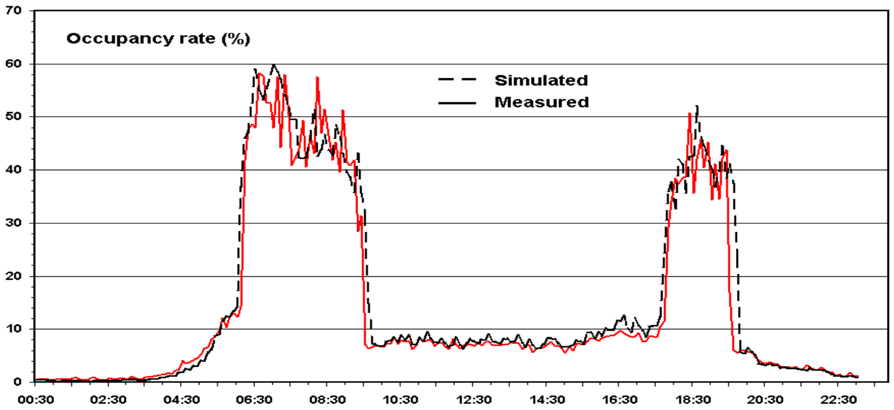
- Problem: reconstruct loop data (usually 10% out of order at any time)
- Solution: LWR model, fed by loop data

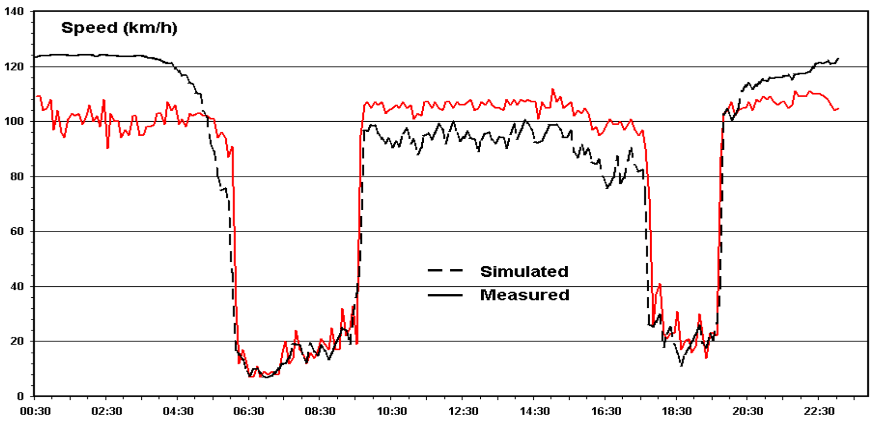


Data reconstruction

- Reconstructed vs actual loop data
- Volume (flow),
- Occupancy (density)
- Speed
- Haj-Salem, Lebacque 2002



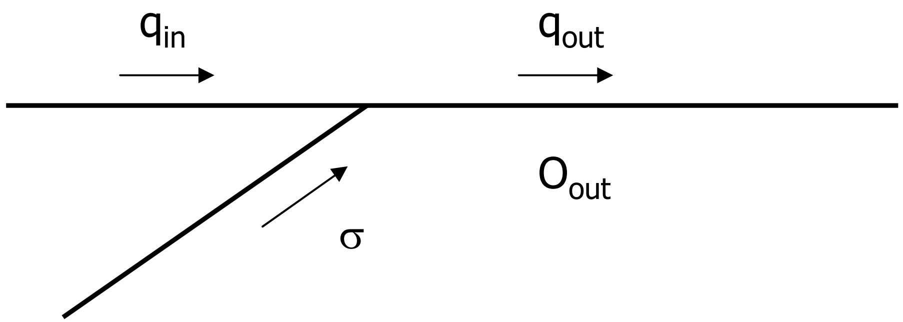




Ramp metering

- Principle: limit access to motorway ⇔ limit conflicts and reacceleration
- Nominal capacity reduction ⇔ effective capacity increase (Braess-like paradox)
- Uses bounded acceleration node + ALINEA (linear feedback)

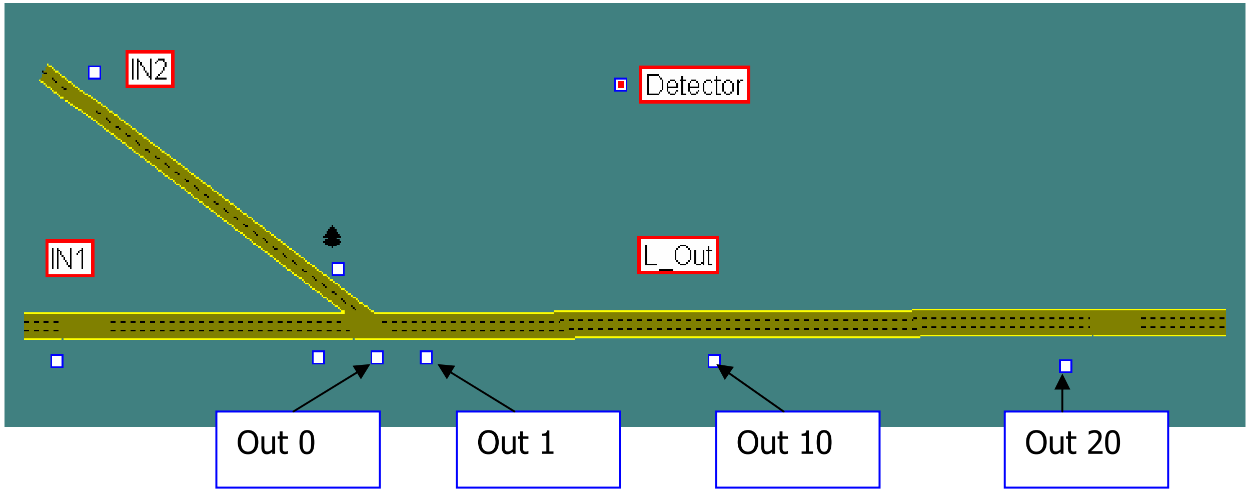
Ramp metering (ALINEA)



- $\sigma(t) = q(t-1) + c\,(O^* - O_{out}(t))$

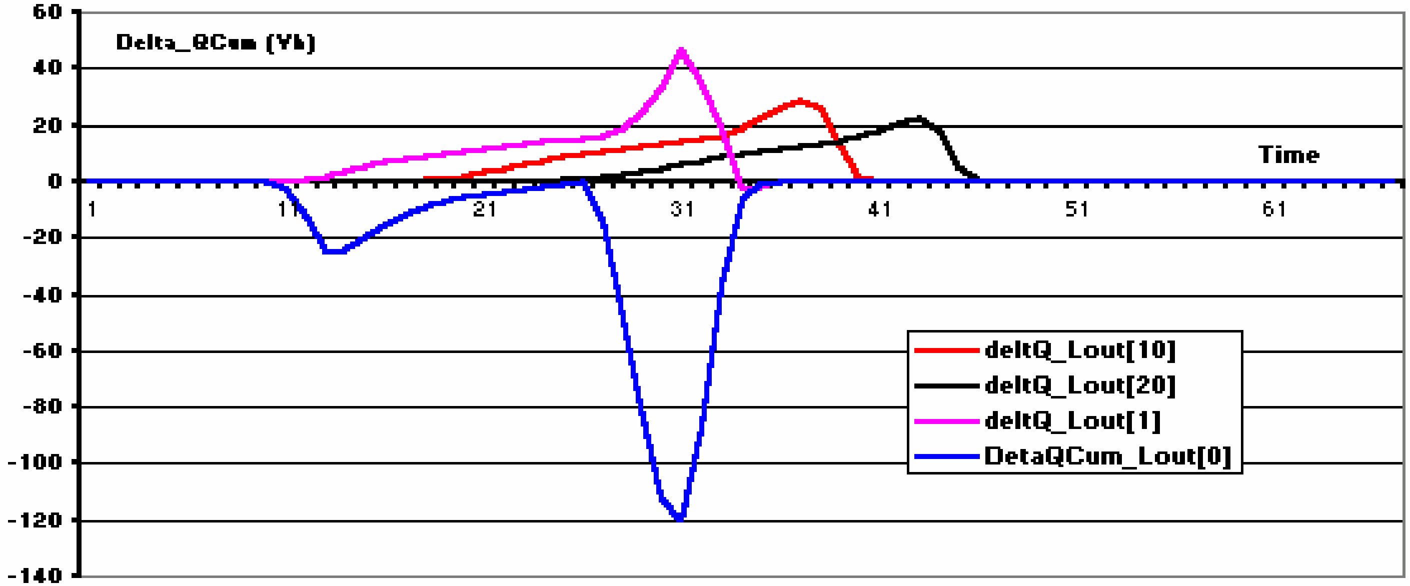
- $q(t) = \mathrm{Min}\,\{\sigma(t), \delta(t)\}$

Ramp metering 2



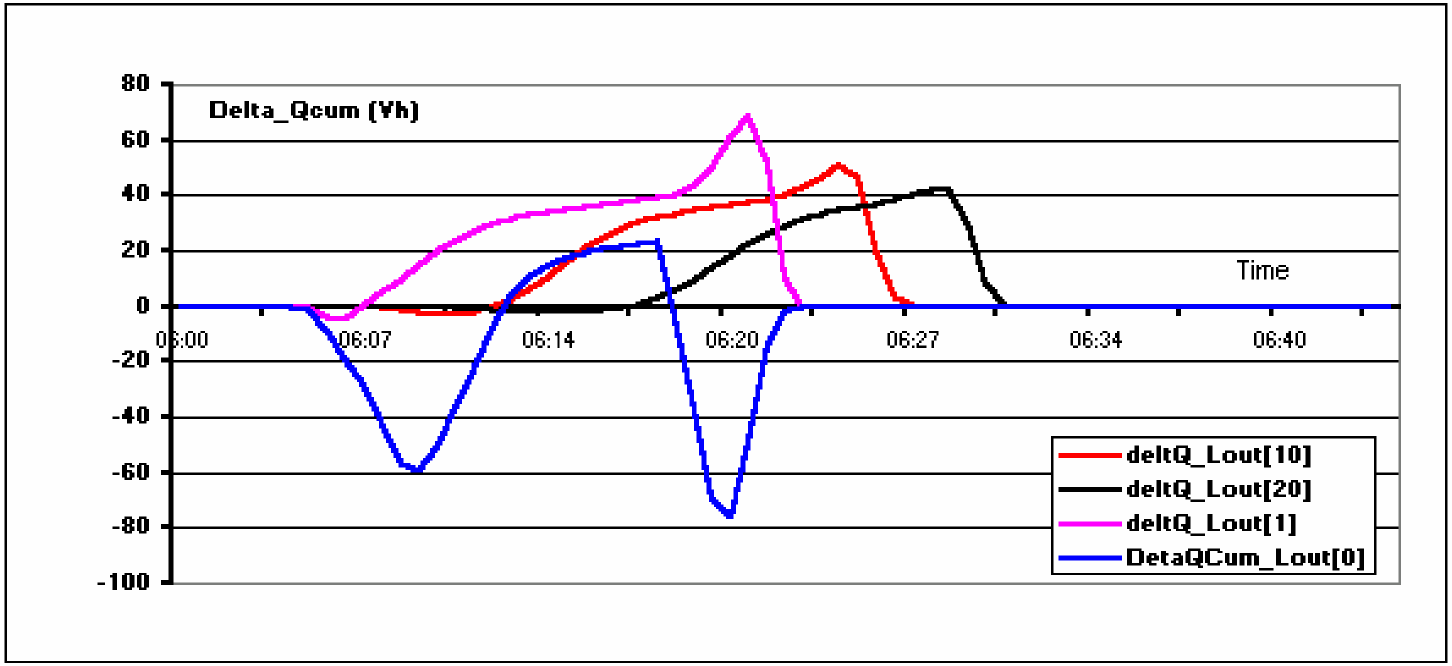
- Two strategies:
  - Ramp metering
  - Ramp metering + Speed control (highway)

Ramp metering 3 (simulation results)



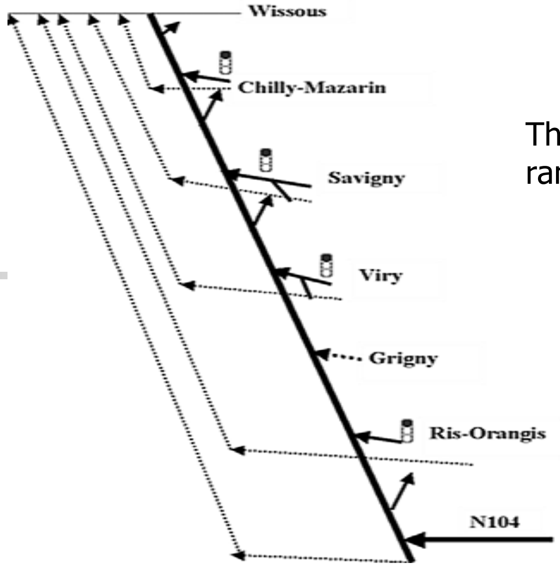
- Ramp metering alone
  - Gains downstream of node

Ramp metering 4 (simulation results)



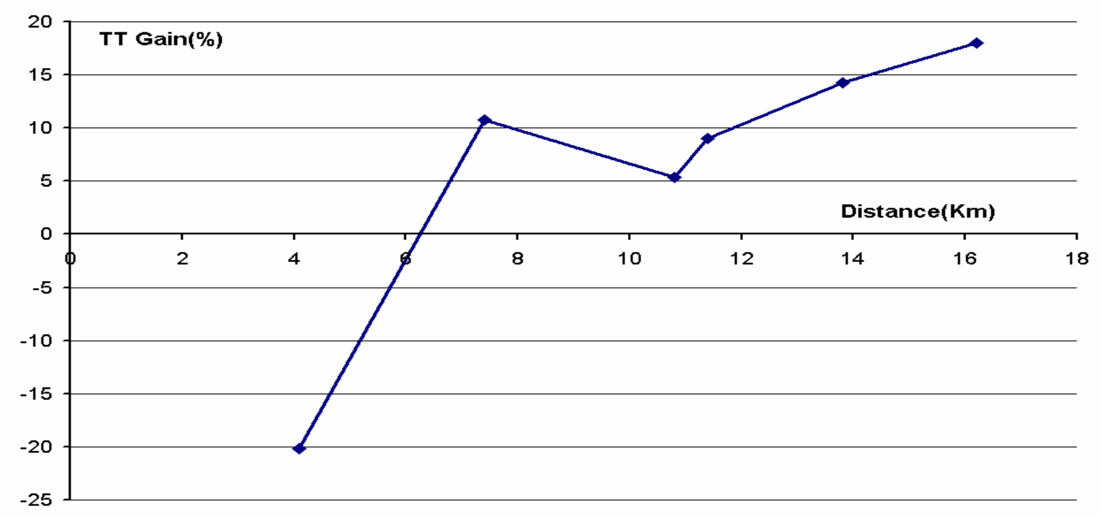
- Ramp metering + Speed control
  - Greater gains downstream of node

Ramp metering 4:



The on-ramps

- Persistence of gains with respect to the distance traveled
- Cause: fluidity, regularity, no hysteresis



Conclusion & next steps

- Systematic approach to intersection modelling
- Network models (FIFO)
- Satisfactory empirical evidence

Next steps:

- Multicommodity, non FIFO flows on networks
- New integration methods: Hamilton-Jacobi, cumulative flows
- More physical intersection models
- Development hybrid models (Micro +macroscopic modeling)
- Development of the MAESTRAU kernel