TECHNISCHE UNIVERSITÄT DRESDEN

Emergent oscillations in interacting driven many-particle flows

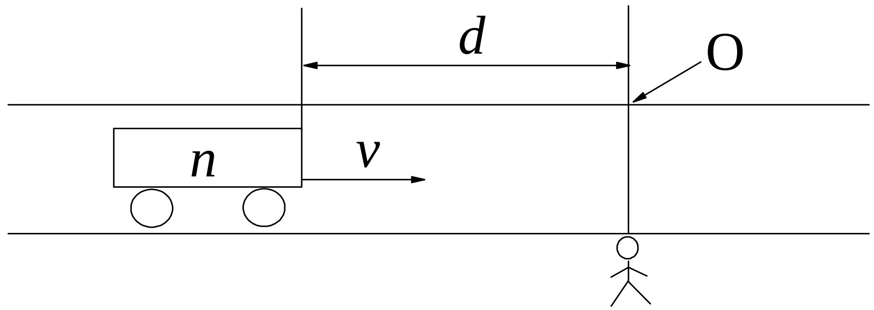
Dirk Helbing, Rui Jiang, and Martin Treiber

helbing@trafficforum.org

Institute for Transport & Economics

TU Dresden

Situation of Intersecting Vehicle and Pedestrian Flows



▸ Pedestrian behavior:

We will assume that pedestrians enter the sidewalk of the street at the crossing point O with probability $p = \lambda dt$ per time step dt (λ denotes the arrival rate of pedestrians).

Pedestrian Behavior

If there is no sufficient gap in the vehicle stream to cross the road, pedestrians accumulate around point O, but they start immediately to enter the road at time t, if (i.e. if the vehicle velocity is zero) or if $v(t) = 0$

$$d(t) > d_0 \quad \text{and} \quad \Delta t(t) := \frac{d(t)}{v(t)} \geq \sigma\tau$$

- Δt time to collision of the nearest approaching vehicle
- σ safety factor of pedestrians
- τ time period required for a pedestrian to cross (one lane of) the road.

Vehicle Behavior

The vehicle dynamics is given by a simple variant of the intelligent driver model (IDM)

$$f(\Delta x, v, v_n) = b\left[1 - \left(\frac{v_n}{v_0}\right)^4 - \left(\frac{s^*}{\Delta x}\right)^3\right]$$

$$s^* = s_0 + T v_n + \frac{v_n(v_n - v_{n-1})}{2b}$$

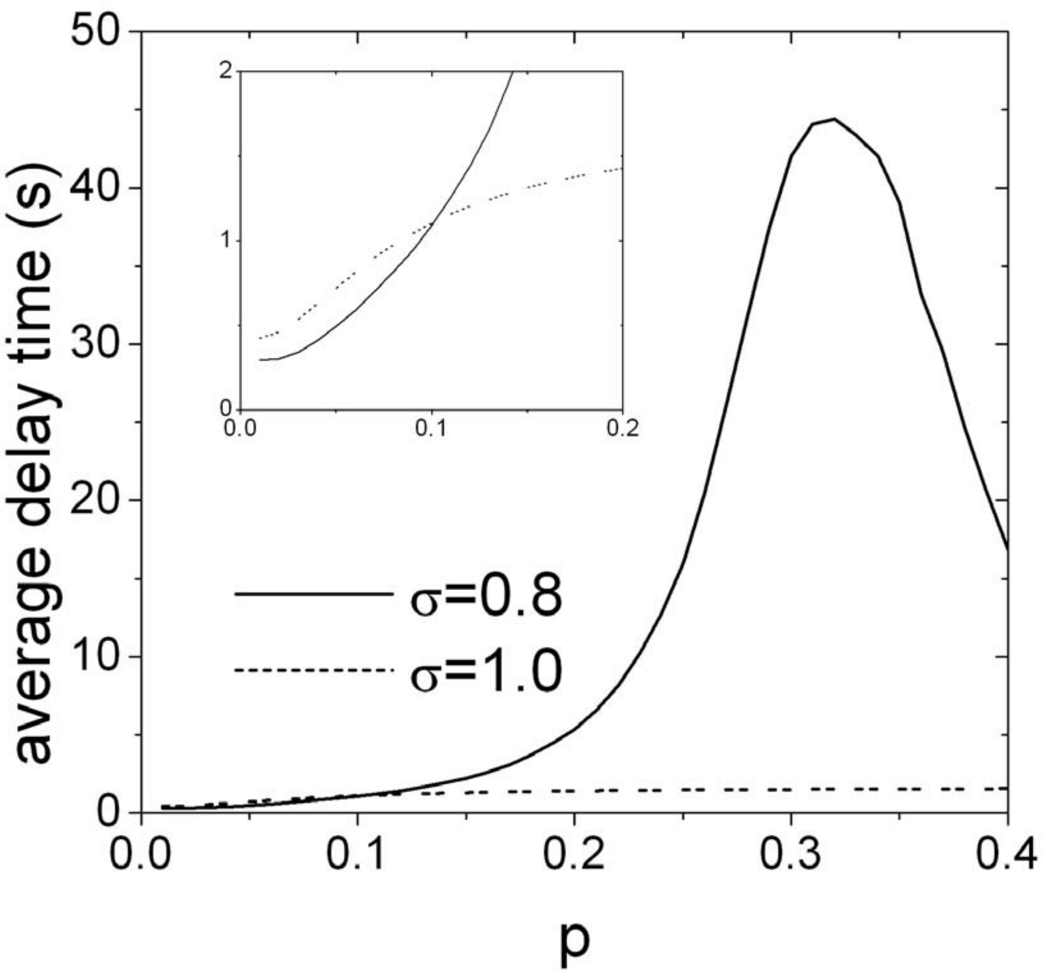
For the nearest vehicle n upstream of the crossing point O,

if a pedestrian is on the street, we have

$$\Delta x(t) = d(t) = x_O - x_n(t)$$

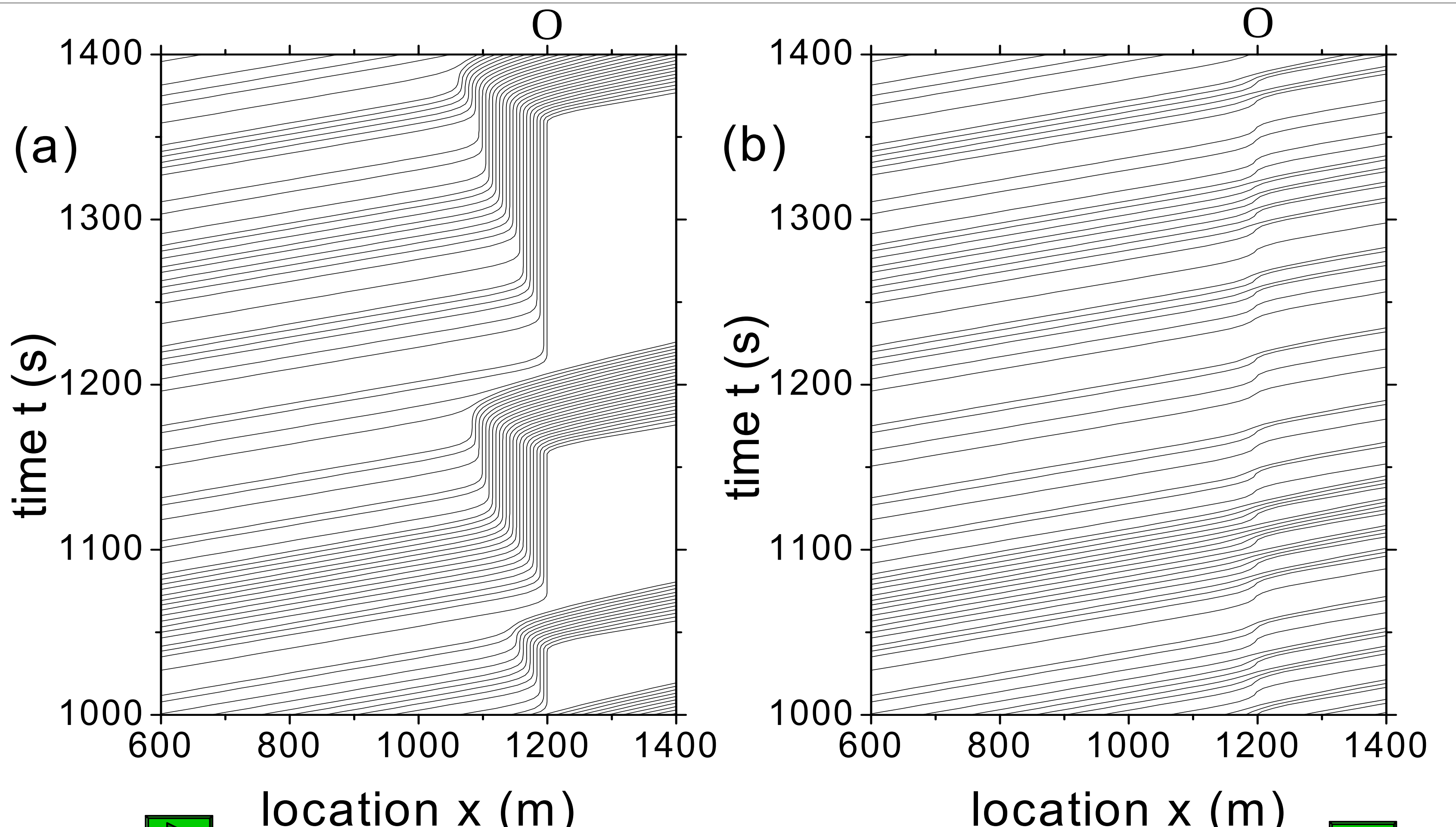
and $v_{n-1} = 0$

Simulation Result: “Faster-is-Slower Effect”



Average delay to pedestrians

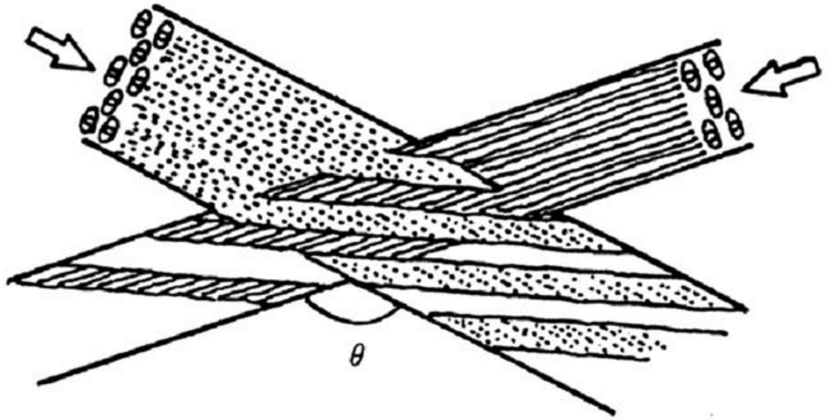
Simulation Results: Underlying Dynamics

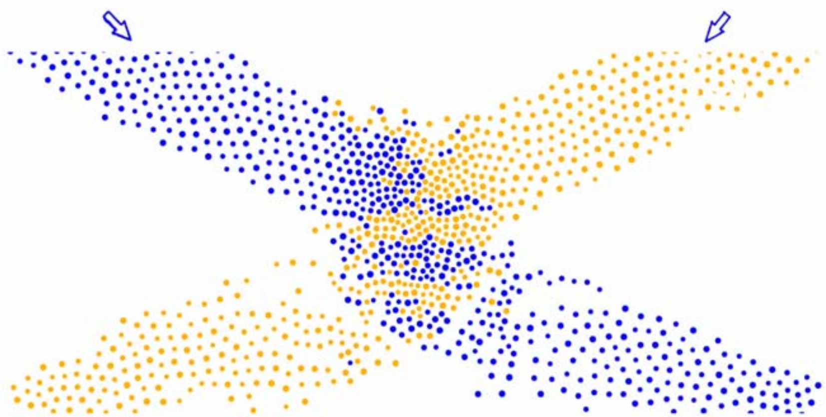


$\sigma = 0.8$ $\sigma = 1.0$

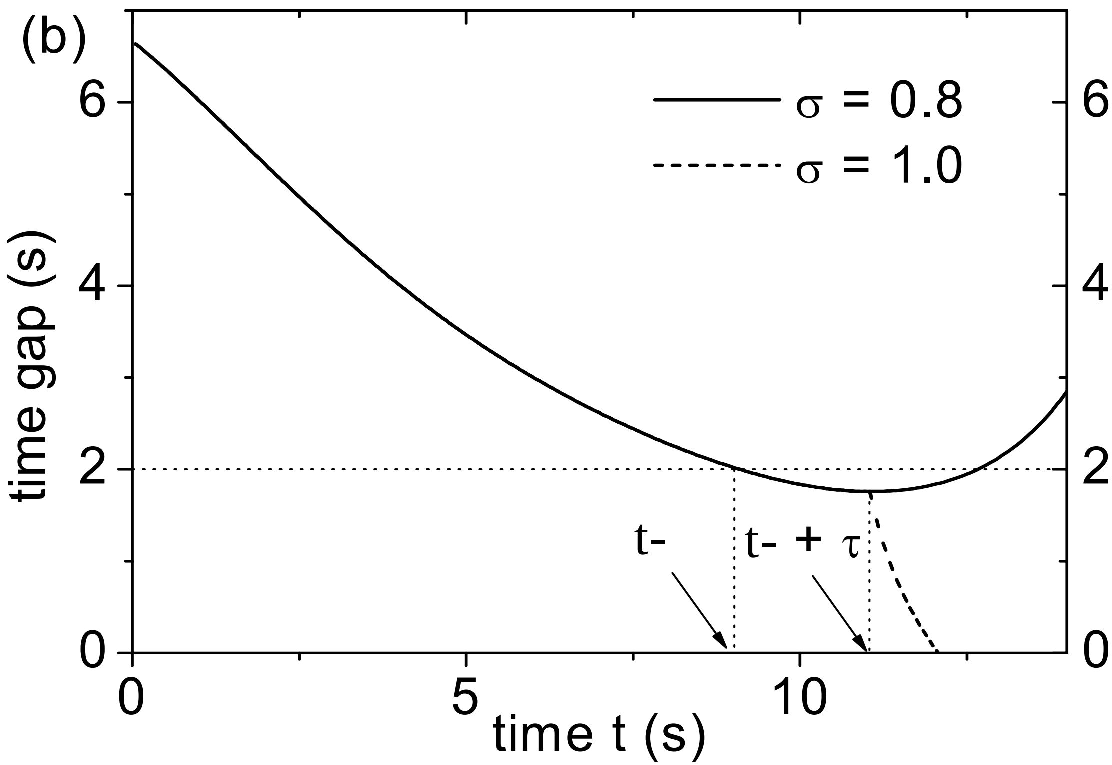
Representative space-over-time plots of vehicle trajectories

Self-Organization of Intersecting Pedestrian Flows





Critical Time Gap



Dynamically changing time gap $d(t)/v(t)$ of a vehicle, if pedestrians enter the street 100 meters ahead. If $\sigma\tau \lesssim 1.83$, pedestrians continue entering the street, which may stop the vehicle (solid growing curve). Otherwise, the crossing criterion is violated after some time and the vehicle can accelerate (dashed falling curve).

Analytical investigation

- **Pedestrian behavior is specified in the same way**

- **Vehicle behavior must be simplified:**

A vehicle with the speed v following a leading vehicle with speed v_* is assumed to decelerate with $dv/dt = -a$,

if $v > 0$ and

$$\Delta x < l_0 + d_0 + \frac{v^2}{2a} - \frac{{v_*}^2}{2a}$$

For a “>” sign, the vehicle accelerates with $dv/dt = a$, delayed by the reaction time T, until the maximum (free) speed is reached. For an “=“ sign, the velocity is not changed.

Driver Behavior

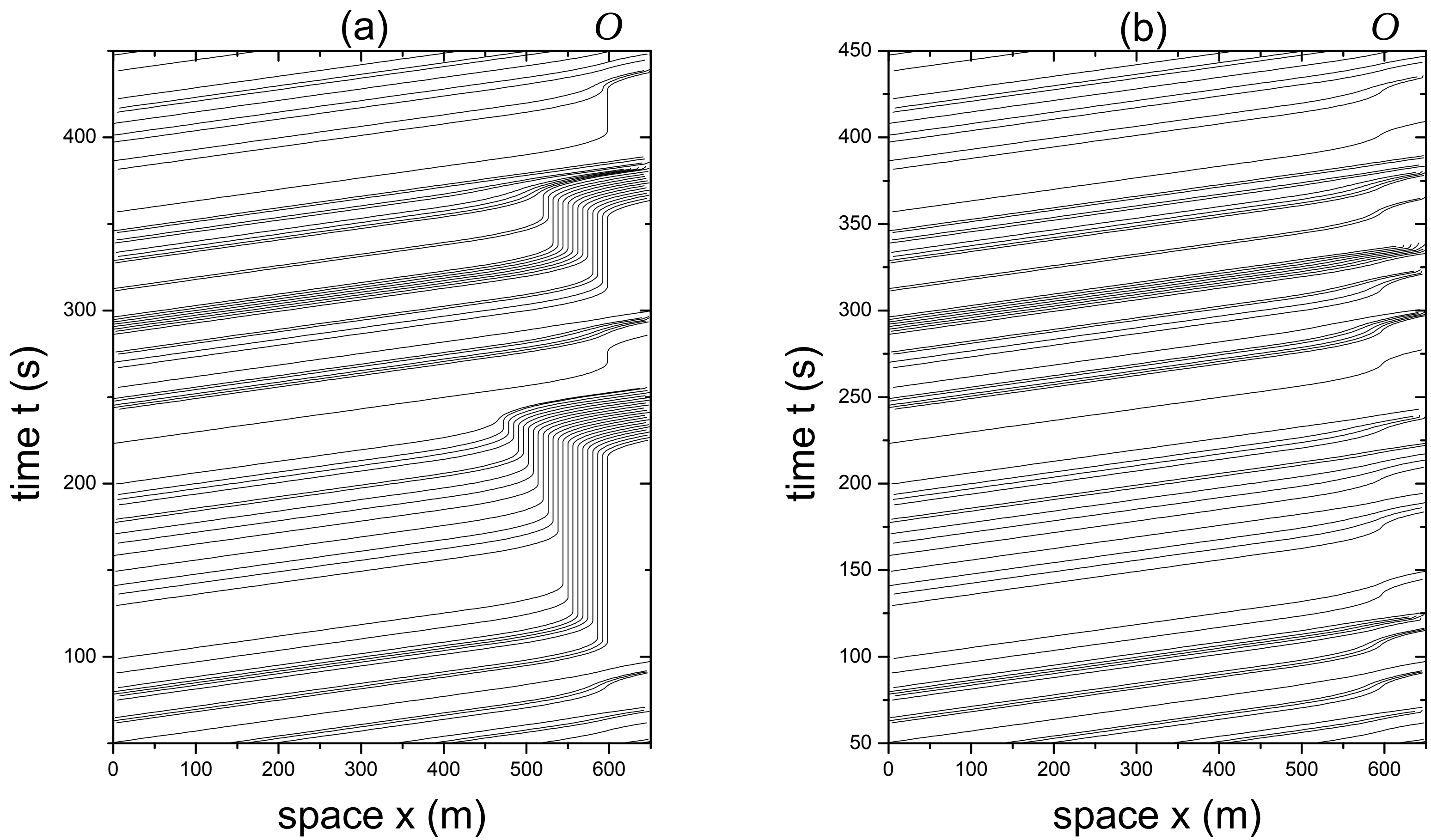
Careful drivers: The closest car to a pedestrian on the street decelerates, if the distance to the crossing point O is within the range

$$0 \le d(t) \le d_0 + \frac{v^2}{2a}$$

Aggressive drivers: The closest car starts to decelerate at the time t_0 determined so that the distance to the pedestrian corresponds to the safety distance

$d(t_n + \tau) = d_0$ at the time $t_n + \tau$ when the last pedestrian on the street (entering at time t_n) leaves the road after the crossing time τ

Simulation Results for Careful Drivers



$\sigma = 1.05$ $\sigma = 1.25$

Idealized Vehicle Distance Distribution: M/D/1 Queue

In our vehicle simulations, we have generated vehicles with initial velocity v_0 at the upstream boundary of the simulation stretch according to the exponential time gap distribution $Q_{\rm arr} {\rm e}^{-Q_{\rm arr} T'}$, where T' denotes the actual time gap. However, according to our car-following model, vehicles have gained at least their preferred distance, $D = l_0 + d_0 + v_0 T$ when they reach the maximum speed. According to theoretical considerations, this changes the effective time-gap distribution at the crossing point to

$$
\begin{aligned}
P(T') &= Q_{\rm arr} T_0 \, \delta(T' - T_0) \\
&\quad + (1 - Q_{\rm arr} T_0) Q_{\rm arr} {\rm e}^{-Q_{\rm arr}(T' - T_0)} \Theta(T' - T_0)
\end{aligned}
$$

with $T_0 = D/v_0$ That is, a fraction $Q_{\rm arr} T_0$ of vehicles will follow with the desired time gap T_0, while the rest has an exponentially distributed, larger time gap $T' > T_0$.

Reaction of Vehicles to Pedestrians

The time to collision evolves in time according to

$$\Delta t(t) = \frac{d(t)}{v(t)} = \frac{d(0) - v_0 t}{v_0} = \frac{d(0)}{v_0} - t \quad \text{if} \quad t > t_0$$

t_0: time point when the car starts to decelerate as response to a crossing pedestrian

<u>Careful drivers:</u>

$$t_0 = \frac{d(0) - d_0}{v_0} - \frac{v_0}{2a}$$

<u>Aggressive drivers:</u>

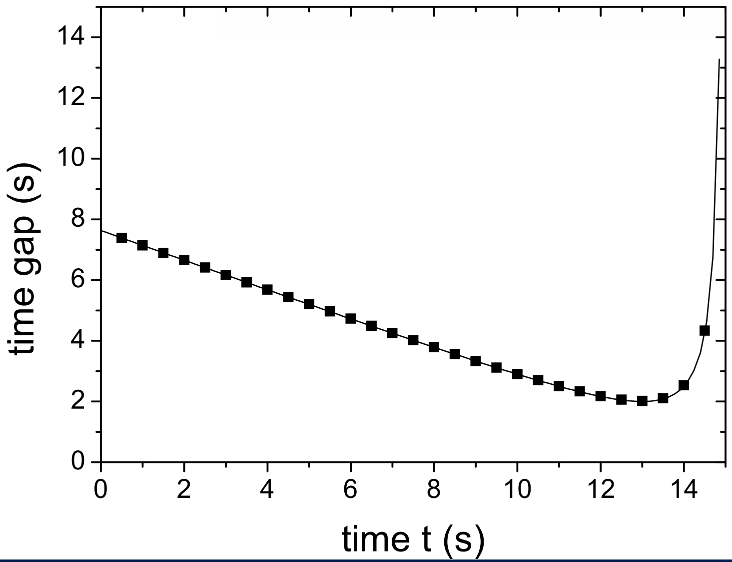
$$t_0 = \tau - \sqrt{\frac{2v_0\tau}{a} - 2\,\frac{d(0) - d_0}{a}}$$

Careful Drivers

$$\Delta t(t) = \frac{d_0 + {v_0}^2/(2a) - v_0(t-t_0) + a(t-t_0)^2/2}{v_0 - a(t-t_0)}$$

$$= \frac{v_0}{2a} - \frac{t-t_0}{2} + \frac{d_0}{v_0 - a(t-t_0)} \text{ if } t \geq t_0$$

$$v(t_0+\tau) = v_0 - a(\tau - t_0) = \frac{v_0}{2} - a\tau + a\,\frac{d(0)-d_0}{v_0}$$



Time-dependent time to collision for careful drivers, when pedestrians enter the road with probability

$$p = 1 \text{ and } \sigma = 1.05$$

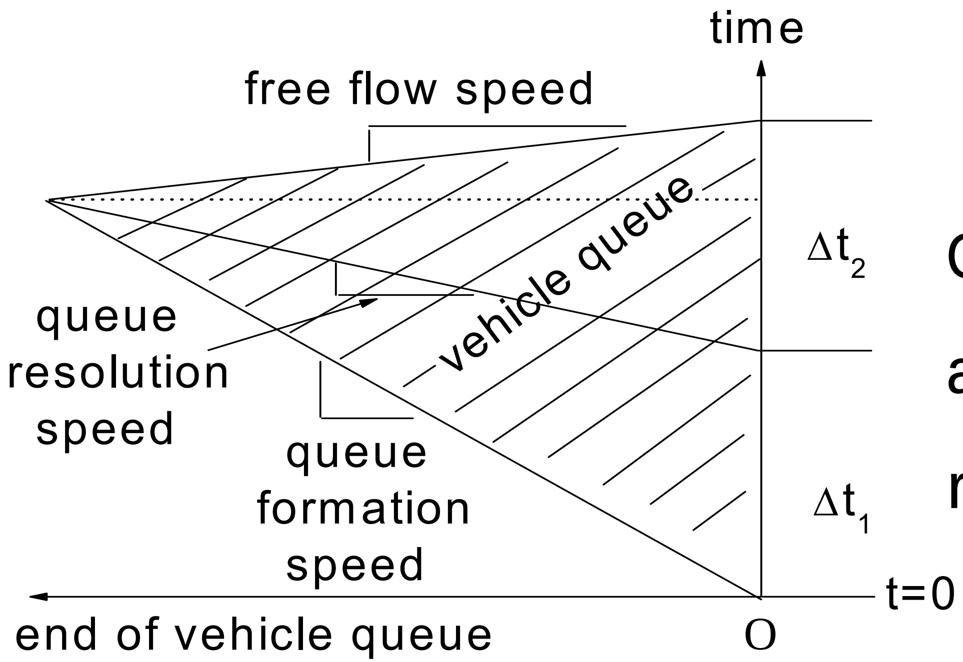
(symbols = numerically determined values, solid line = analytical formula)

Average Delay to Vehicles

Let us denote by $v_{\min}$ the minimum velocity before the car accelerates again. If only one pedestrian obstructs the car, we have $v_{\min} = v(t_0+\tau)$. The time delay to the car compared to a movement with the free velocity can be calculated as the distance $2(v_0 - v_{\min})^2/(2a)$ travelled less, divided by the desired velocity, which results in

$$\Delta t_{\rm br} = \frac{(v_0 - v_{\min})^2}{a v_0} \qquad (**)$$

Average Delay to Vehicles



Queue formation and queue resolution mechanism

Δt_1 : waiting time of the first stopped vehicle

$$\Delta t_2 = C\,\Delta t_1 \frac{1+|c|/v_0}{|c|-C}$$ jam resolution time with

$$C = \left(\frac{\rho_{\text{jam}}}{Q_{\text{arr}}} - \frac{1}{v_0}\right)^{-1}$$ and $$c = -1/(\rho_{\text{jam}} T) \approx -15 \text{ km/h}$$

Average Delay to Vehicles

If the vehicle is stopped, the time lost by the acceleration and deceleration process amounts to v_0/a. On top of this, we have to add the average waiting time t_w.

This can be obtained as follows: If Δt_1 denotes the waiting time of the first stopped vehicle, the number of vehicles queuing up behind it until the first car in the queue starts to accelerate is given by $\rho_\mathrm{jam} C\,\Delta t_1$. The delay of the last vehicle in the queue is the queue length $l = C\,\Delta t_1$, divided by the queue resolution speed c. As the waiting time between the first and the last vehicle in the queue progresses approximately linearly, their cumulative waiting time is

$$\frac{\rho_\mathrm{jam} C\,\Delta t_1}{2}\left(\Delta t_1 + \frac{C\,\Delta t_1}{c}\right) = \frac{\rho_\mathrm{jam} C(\Delta t_1)^2}{2}\left(1 + \frac{C}{c}\right)$$

Average Delay to Vehicles

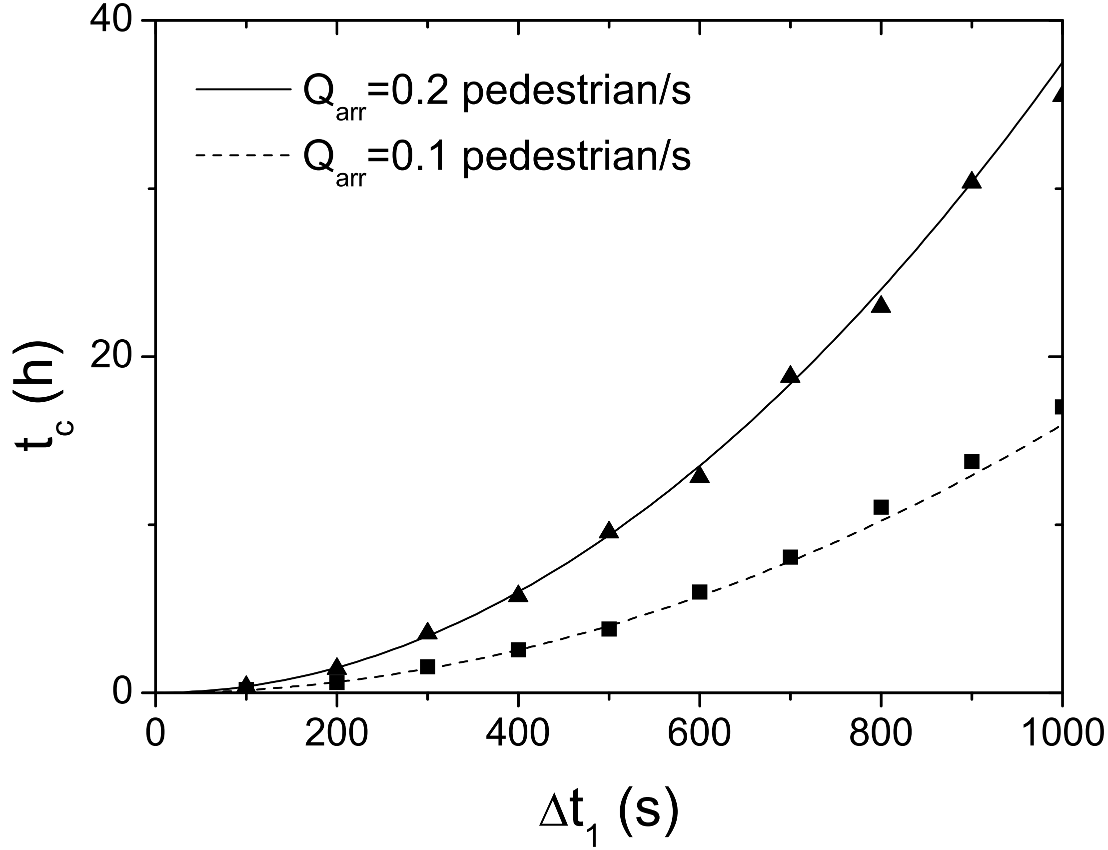
Moreover, up to the time point when the queue formed within the stopping time Δt_1 has resolved, another $\rho_{\text{jam}} l C/(c - C)$ vehicles have joined the queue. While the waiting time of the first of these additional vehicles is approximately $l/c = C\,\Delta t_1/c$ (as the one of the last vehicle in the first part of the queue), the waiting time of the last vehicle is basically zero, which implies a cumulative waiting time of

$$\frac{\rho_{\text{jam}} C\,\Delta t_1}{2}\left(\frac{C}{c-C}\,\frac{C\,\Delta t_1}{c} + 0\right) = \frac{\rho_{\text{jam}} C(\Delta t_1)^2}{2}\,\frac{C^2}{c^2 - cC}$$

Adding this to the average delay shown in previous slide gives the cumulative waiting time

$$t_c = \frac{\rho_{\text{jam}}}{2}(\Delta t_1)^2\,\frac{cC}{c-C}$$

Average Delay to Vehicles



Average of the cumulative waiting times t_c of vehicles as a function of the time period Δt_1 the first vehicle in the queue has to wait, for different values of the vehicle arrival rate $Q_{\rm arr}$.

Transition Point to Alternating Flows

For small values of σ, there exists a time point t_-,after which the safety criterion prohibits a further entering of pedestrians into the road. This time point is given by the earlier time fulfilling the critical safety condition $\Delta t(t_\mp) = \sigma\tau$. Together with the expressions for the times to collision, this eventually implies

$$t_\mp - t_0 = \frac{v_0}{a} - \sigma\tau \mp \sqrt{(\sigma\tau)^2 - \frac{2d_0}{a}}$$

for careful drivers. t_+ is the first time point at which pedestrians may re-enter the road again, as the time to collision $\Delta t(t)$ increases close to the crossing point.

Transition Point to Alternating Flows

The car reaches its minimum possible velocity a time period τ after t_-, i.e. after the latest entering pedestrian has left the road at time $t_- + \tau$. This implies

$$v(t_- + \tau) = a\tau(\sigma - 1) + \sqrt{(a\sigma\tau)^2 - 2ad_0}$$

for careful drivers. To exclude stopped vehicles, on the one hand, this minimum velocity should be positive, i.e.

$$\left(\sigma - \frac{1}{2}\right) a\tau^2 > d_0 \qquad (*)$$

Transition Point to Alternating Flows

In order to avoid the stopping of vehicles by multiple crossing pedestrians, we have to demand

$$t_+ - t_- = 2\sqrt{(\sigma\tau)^2 - \frac{2d_0}{a}} > \tau$$

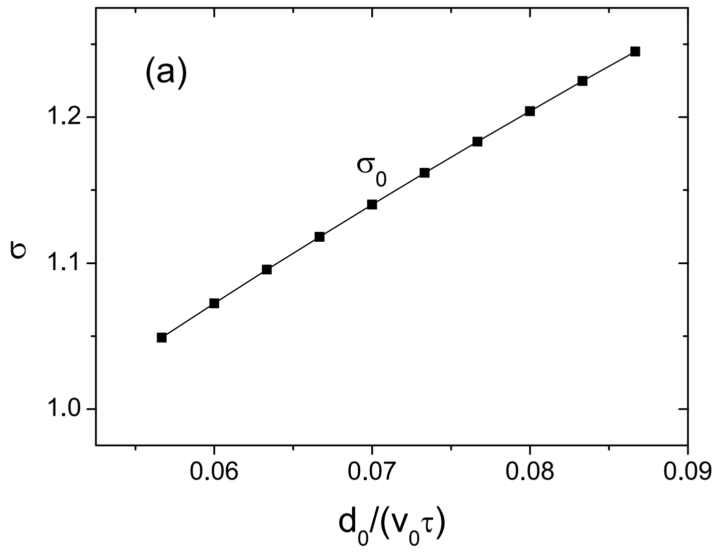
which results in

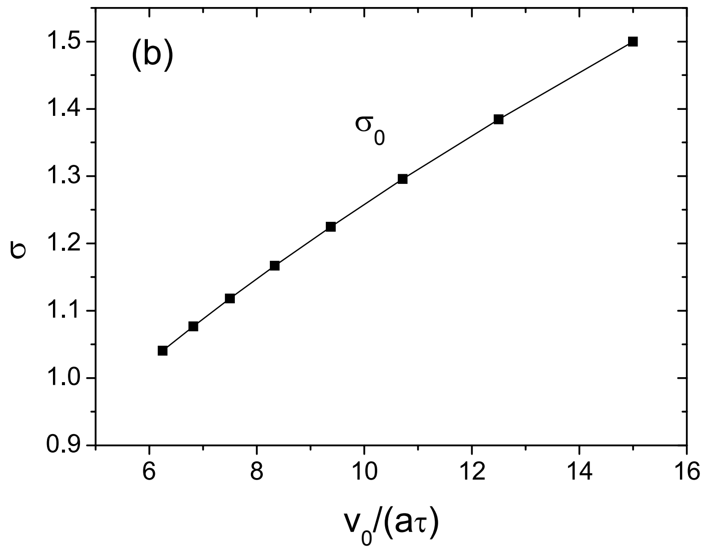
$$\sigma > \sqrt{\frac{2d_0}{a\tau^2} + \frac{1}{4}}$$

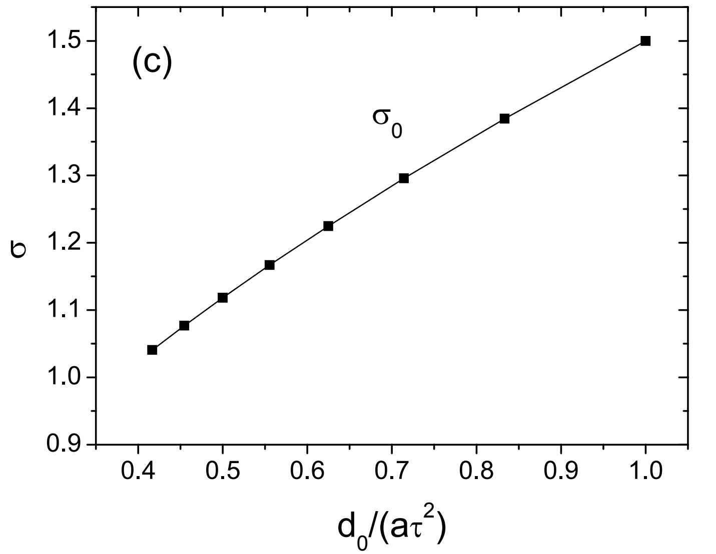
Together with condition (*) on the previous slide, we find that a careful driver cannot be stopped completely under the condition

$$\sigma > \sigma_0 = \max\left(\frac{d_0}{a\tau^2} + \frac{1}{2}, \sqrt{\frac{2d_0}{a\tau^2} + \frac{1}{4}}\right)$$

Transition Point to Alternating Flows







Transition point σ_0 to alternating vehicle and pedestrian flows as a function of different dimensionless parameters.

Calculation of Earlier Acceleration

Due to the statistical arrival of pedestrians, it is likely that the time point $t_n \le t_-$ of the last (*n*th) pedestrian entering the road is smaller than the latest *possible* entering time t_-. We are, therefore, interested in calculating the mean value of the time gap $\langle t_- - t_n \rangle$.

For this, let $K = t_-/dt$ be the number of time steps between the last entering pedestrian and t_-. As the probability that no pedestrian enters in a time step is given by $r = (1-p)$, $(1-p)^K$ is the probability that nobody enters between $t = 0$ and $t = t_-$, and $p(1-p)^{K-k}$ the probability that the last pedestrian enters at time

$t_- - (K-k)\,dt = k\,dt$.

Calculation of Earlier Acceleration

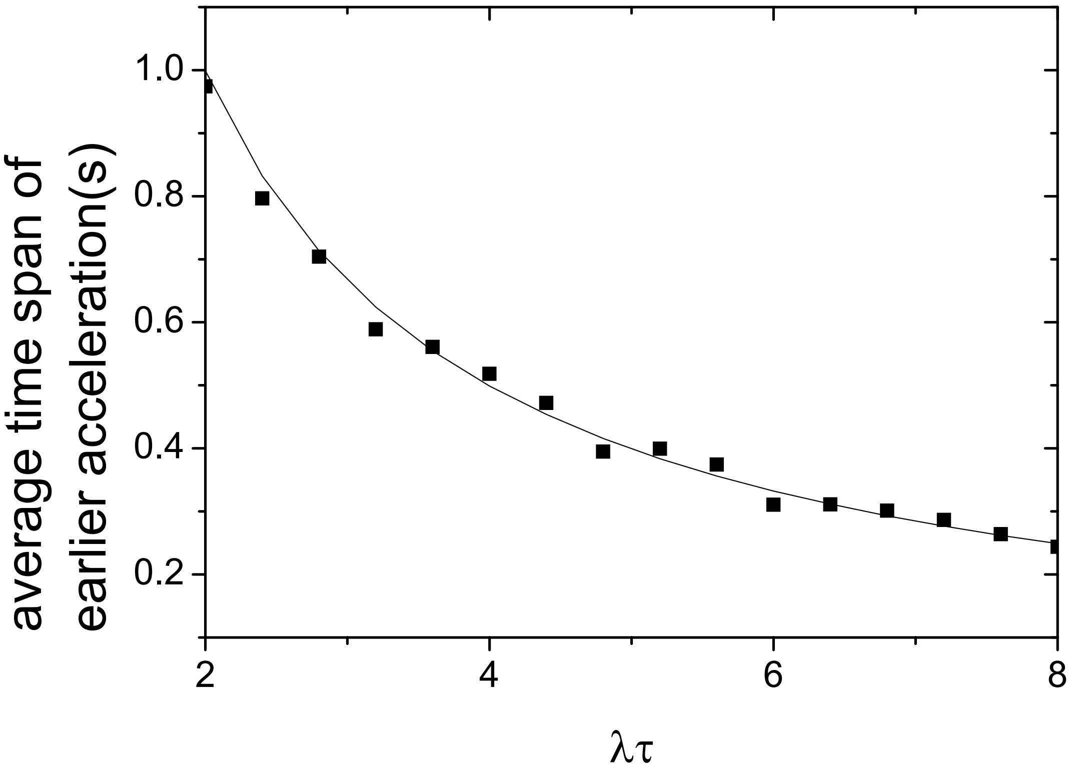
The expected value of $t_- - t_n$ is

$$\frac{\langle t_- - t_n \rangle}{dt} = K(1-p)^N + p\sum_{k=1}^{K}(K-k)(1-p)^{K-k}$$

$$= Kr^K + (1-r)r\frac{d}{dr}\sum_{k=1}^{K} r^{K-k} = \frac{r(1-r^K)}{1-r}$$

$$= (1-p)\frac{1-(1-p)^K}{p}$$

Calculation of Earlier Acceleration



Average time span $t_- - \langle t_n \rangle$ between the latest *possible* entering of the street by a pedestrian and the time point when the last pedestrian *actually* enters the street as a function of the scaled pedestrian arrival rate $\lambda\tau$.

Waiting Time of the First Vehicle

If a vehicle is stopped by crossing pedestrians, it will have to wait until a time gap of duration τ in the pedestrian flow occurs. A gap of length $\tau = N\,dt$ or greater occurs with probability

$$(1-p)^N = (1-p)^{\tau/dt} = [\underbrace{(1-p)^{1/dt}}_{=\mathrm{e}^{-\lambda}}]^\tau = \mathrm{e}^{-\lambda\tau}$$

if pedestrian gap sizes are exponentially distributed, as expected. Here, we have assumed $\ln(1-p) \approx -p$, but the required small values of $p = \lambda\,dt$ can be reached by sufficiently small choice of the time steps dt. In fact, in the following considerations, we will study the limit $dt \to 0$. Therefore, we have used the value dt =0.001 s in our computer simulations.

Waiting Time of First Vehicle

Now, let k_i denote the size of the ith gap $T_i = t_i - t_{i-1}$ (i.e. the number of time steps dt with no pedestrian arrival). Then, the expected value for the time period until a time gap of length $\tau = N\,dt$ or greater starts is given by

$$\sum_{n=0}^{\infty} \sum_{k_1=0}^{N} \cdots \sum_{k_n=0}^{N} (k_1 + 1 + \cdots + k_n + 1)$$

$$\times (1-p)^{k_1} p \cdot \cdots \cdot (1-p)^{k_n} p \cdot (1-p)^N$$

$$\approx \sum_{n=0}^{\infty} \lambda^n \int_0^{\tau} dT_1 \ldots \int_0^{\tau} dT_n$$

$$\times (T_1 + \cdots + T_n) \mathrm{e}^{-\lambda(T_1+\cdots+T_n)} \mathrm{e}^{-\lambda\tau}$$

$$= -\mathrm{e}^{-\lambda\tau} \sum_{n=0}^{\infty} \lambda^n \frac{d}{d\lambda} \prod_{i=1}^{n} \left(\int_0^{\tau} dT_i \, \mathrm{e}^{-\lambda T_i} \right)$$

Waiting Time of First Vehicle

$$
\begin{aligned}
&= -\mathrm{e}^{-\lambda\tau} \sum_{n=0}^{\infty} \lambda^n \frac{d}{d\lambda} \left[\frac{1}{\lambda^n} \left(1 - \mathrm{e}^{-\lambda\tau}\right)^n \right] \\
&= -\mathrm{e}^{-\lambda\tau} \sum_{n=0}^{\infty} n \left(1 - \mathrm{e}^{-\lambda\tau}\right)^n \left(\frac{\tau \mathrm{e}^{-\lambda\tau}}{1 - \mathrm{e}^{-\lambda\tau}} - \frac{1}{\lambda} \right) \\
&= \left(\frac{1}{\lambda} - \frac{\tau \mathrm{e}^{-\lambda\tau}}{1 - \mathrm{e}^{-\lambda\tau}} \right) \mathrm{e}^{-\lambda\tau} s \frac{d}{ds} \sum_{n=0}^{\infty} s^n \text{ with } s = 1 - \mathrm{e}^{-\lambda\tau} \\
&= \frac{1}{\lambda} [\mathrm{e}^{\lambda\tau} - (1 + \lambda\tau)] \approx \frac{\lambda\tau^2}{2} + \dots \qquad (38)
\end{aligned}
$$

Note, however, that the waiting time is reduced by the gap between the time $M\,dt := v_0/a$ when the vehicle is stopped and the time $t_n \leq t_-$ at which the last pedestrian has entered the street before.

Waiting Time of First Vehicle

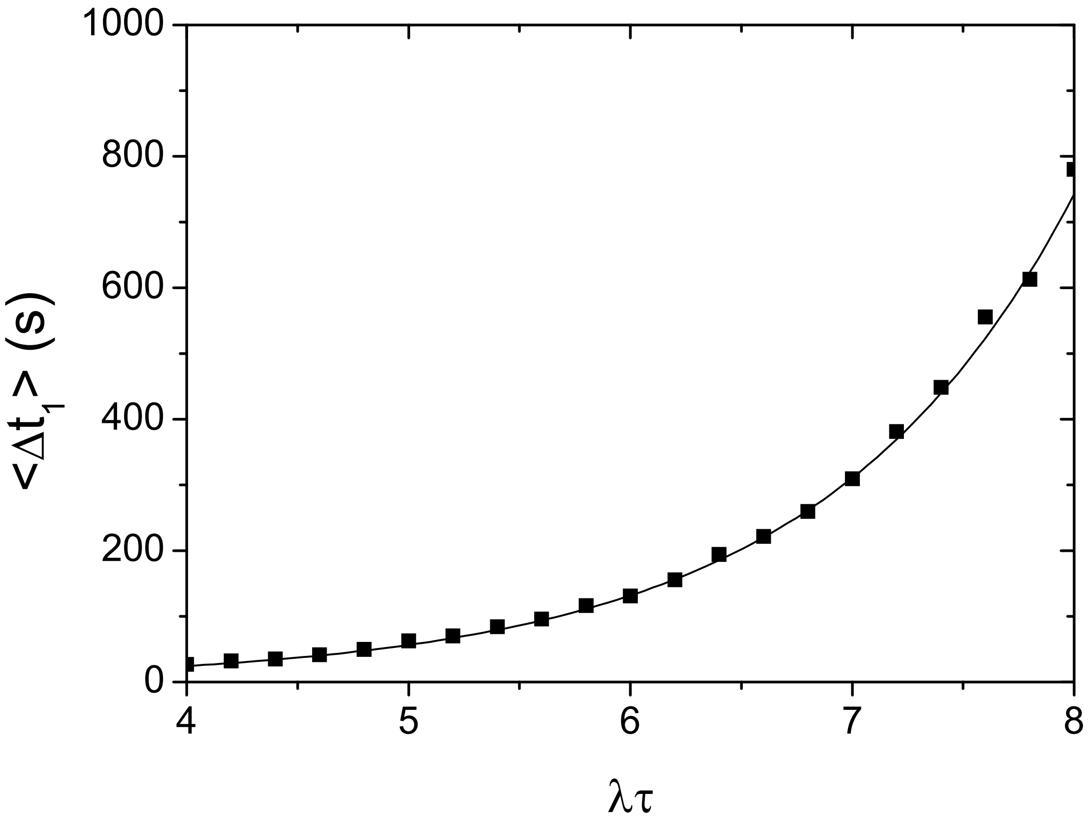
We can calculate the expected value of this time gap as

$$\langle v_0/a - t_n \rangle = (1-p)\frac{1-(1-p)^M}{p/dt} = \frac{1-\mathrm{e}^{-\lambda v_0/a}}{\lambda}$$

since we have $(1 - p) \to 1$ in the limit $dt \to 0$. As a consequence, the expected value $\langle \Delta t_1 \rangle$ that the first vehicle in the queue has to wait can be estimated as

$$\begin{aligned}\langle \Delta t_1 \rangle &= \frac{1}{\lambda}[\mathrm{e}^{\lambda\tau} - (1+\lambda\tau)] - \frac{1-\mathrm{e}^{-\lambda v_0/a}}{\lambda} \\ &= \frac{1}{\lambda}\left(\mathrm{e}^{\lambda\tau} + \mathrm{e}^{-\lambda v_0/a} - 2 - \lambda\tau\right)\end{aligned}$$

Waiting Time of First Vehicle



Average waiting time of the first vehicle in the queue as a function of the scaled pedestrian arrival rate $\lambda\tau$.

Average Delay to Pedestrians

After a time interval Δt_1, i.e. a time period τ after the last pedestrian has entered the road, the first vehicle in the queue can accelerate again. When the vehicle has started to move again, no pedestrian will be able to cross the road until the last vehicle of the queue has passed point O.

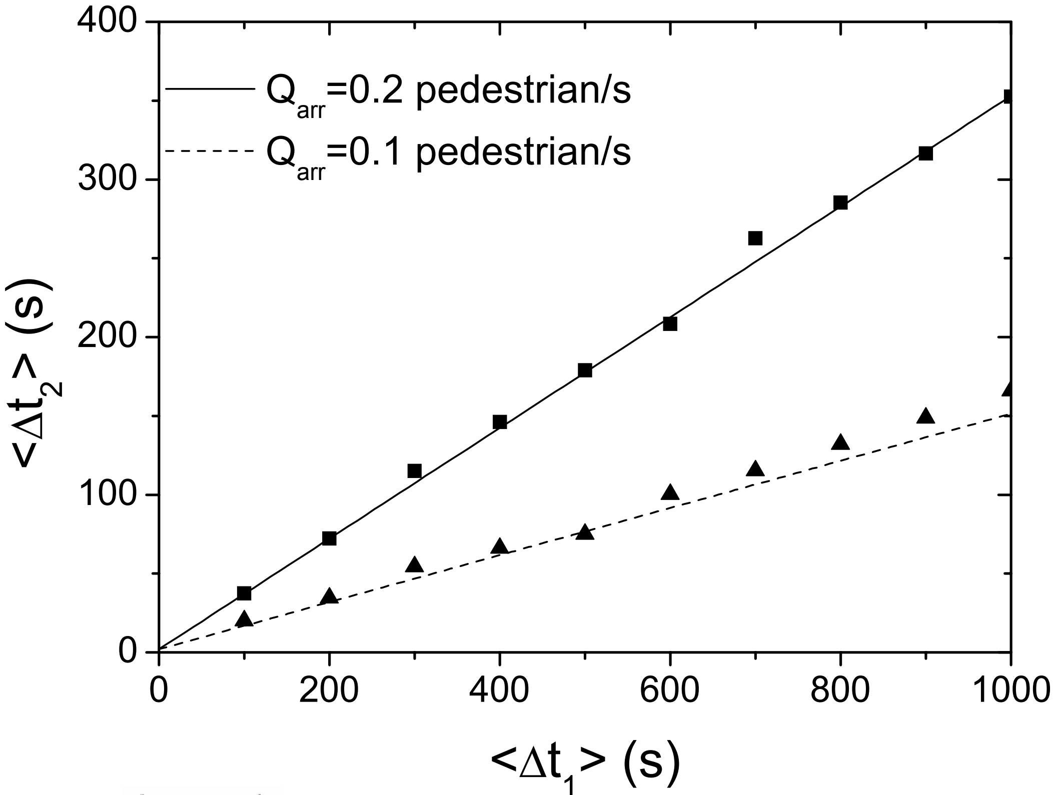
This time period can be calculated as $\Delta t_2 = C\,\Delta t_1 \frac{1 + c/v_0}{c - C}$

The expected value of Δt_2 is

$$\langle \Delta t_2 \rangle = C\,\langle \Delta t_1 \rangle \frac{1 + c/v_0}{c - C} + \sqrt{\frac{2d_0}{a}}$$

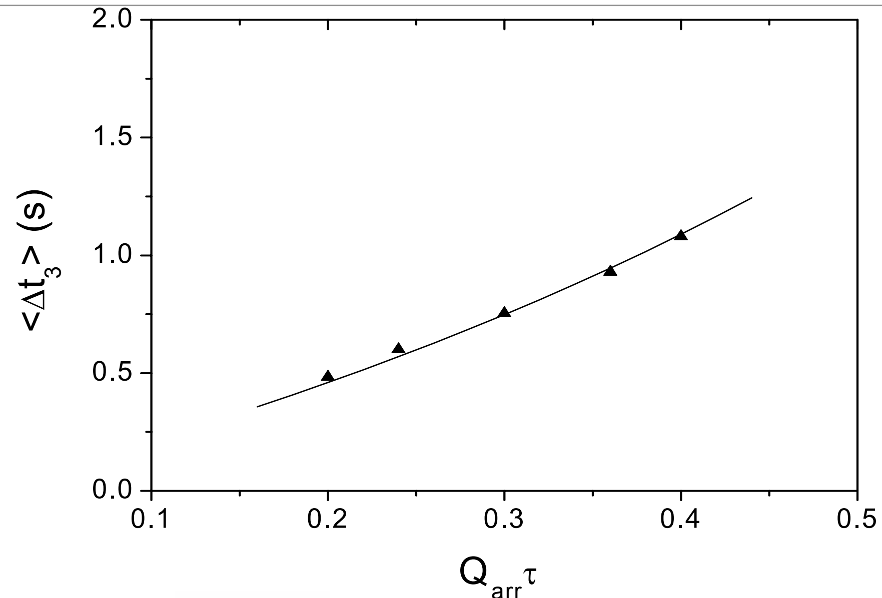
where we have also taken into account the additional amount $\sqrt{2d_0/a}$ required by a vehicle to get from $x = -d_0$ to point O.

Average Delay to Pedestrians



Average time $\langle \Delta t_2 \rangle$ needed to dissolve a vehicle queue as a function of the average time $\langle \Delta t_1 \rangle$ for which the first vehicle has been waiting, for various values of the vehicle arrival rate $Q_{\rm arr}$.

Average Delay to Pedestrians



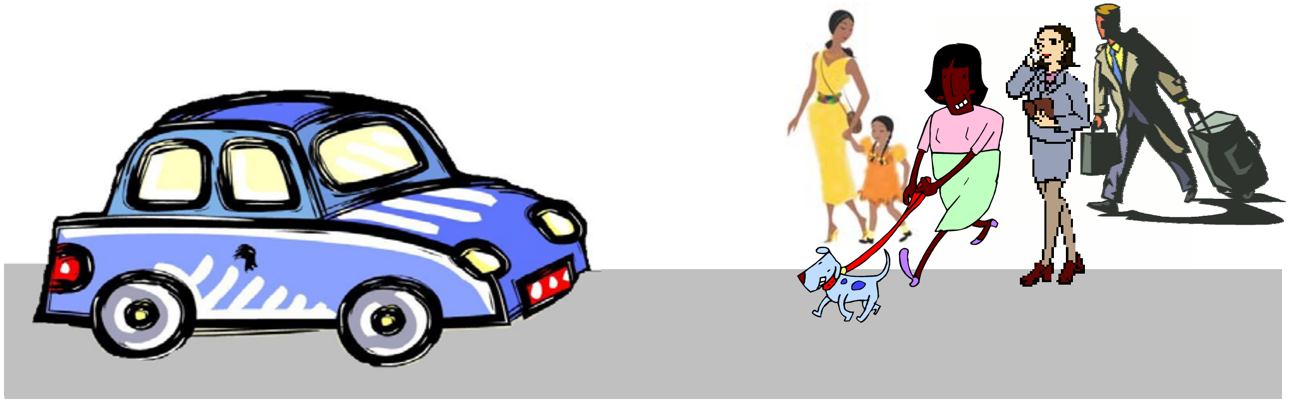
Average waiting time $\langle \Delta t_3 \rangle$ until a pedestrian enters the road after a vehicle queue has completely dissolved, as a function of the scaled vehicle arrival rate $Q_{\mathrm{arr}}\tau$ for $\sigma = 1.05$. Our numerical simulation assumes the special case that a pedestrian arrives just at time when a vehicle of the queue passes the crossing point and vehicle time gaps are not smaller than T_0 .

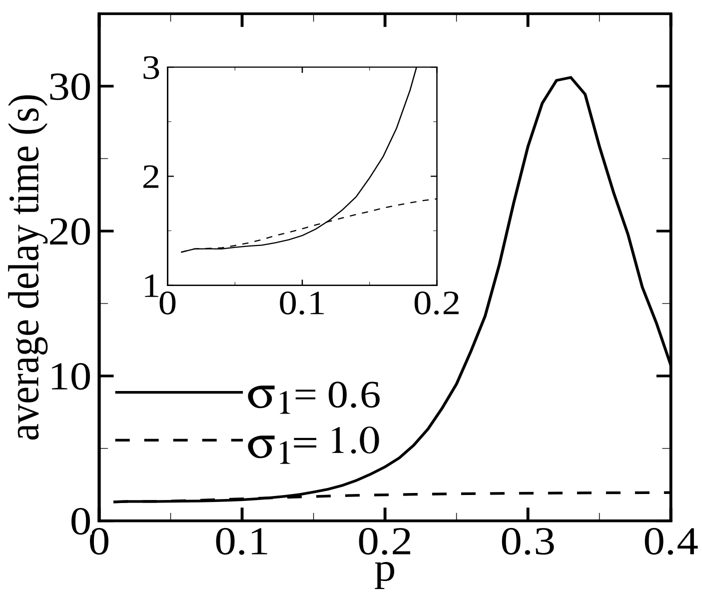
Discussion

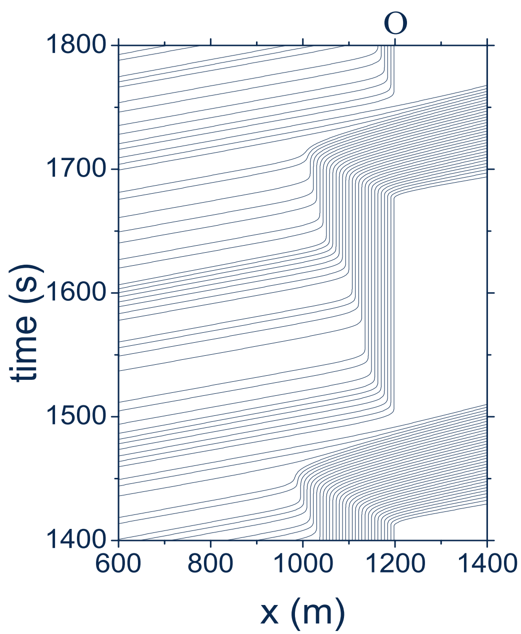
- Emergent oscillations have been discovered in so different systems as the saline (i.e. saltwater-water) oscillator, ticking hour glass, RNA Polymerase traffic on DNA, pedestrians passing a bottleneck, and ants.

- In this contribution, we have studied the example of intersecting pedestrian and vehicle flows. This system is found to show a transition to emergent oscillations. However, contrary to our expectations, the oscillations are not an efficient pattern of motion. Instead, they are related with a considerable reduction of the throughput and increased waiting times ("faster-is-slower effect").

Intersecting Vehicle and Pedestrian Streams

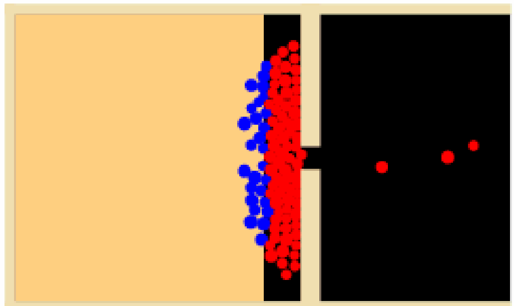




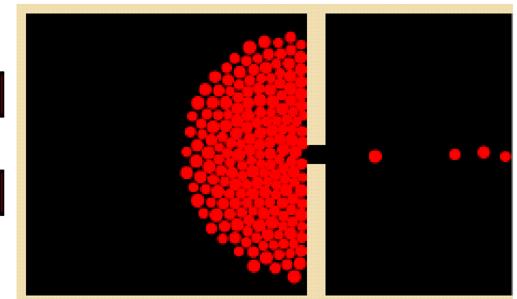


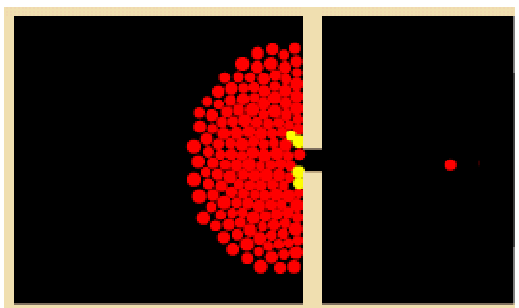
Clogging at Bottlenecks and "Faster-is-Slower Effect"

Physical Interactions and Friction Effects due to Uncontrolled Rush and Pushy Behavior

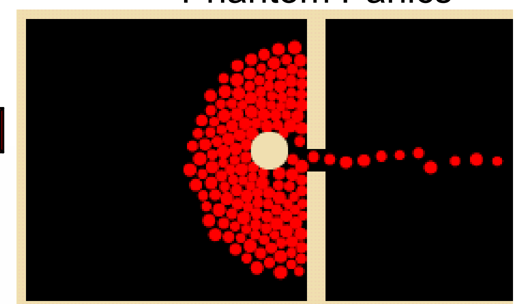


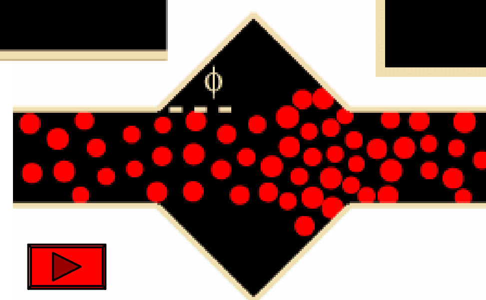
Faster-is-Slower Effect





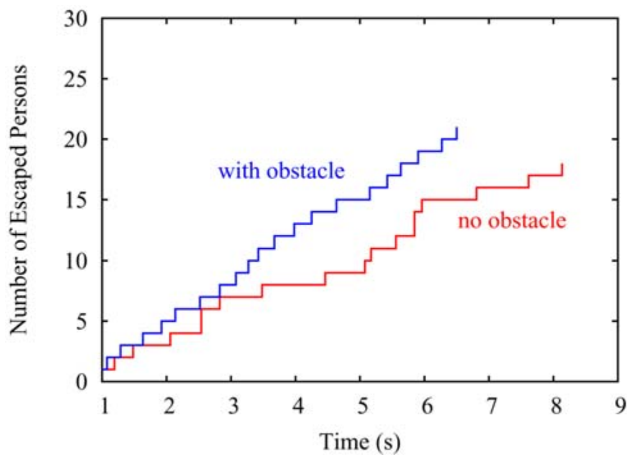
Phantom Panics

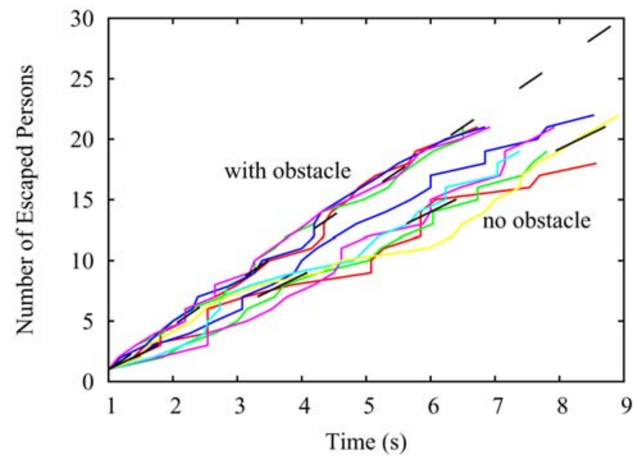




Application to the Simulation of Evacuation Scenarios

Practical Implications and Design Solutions

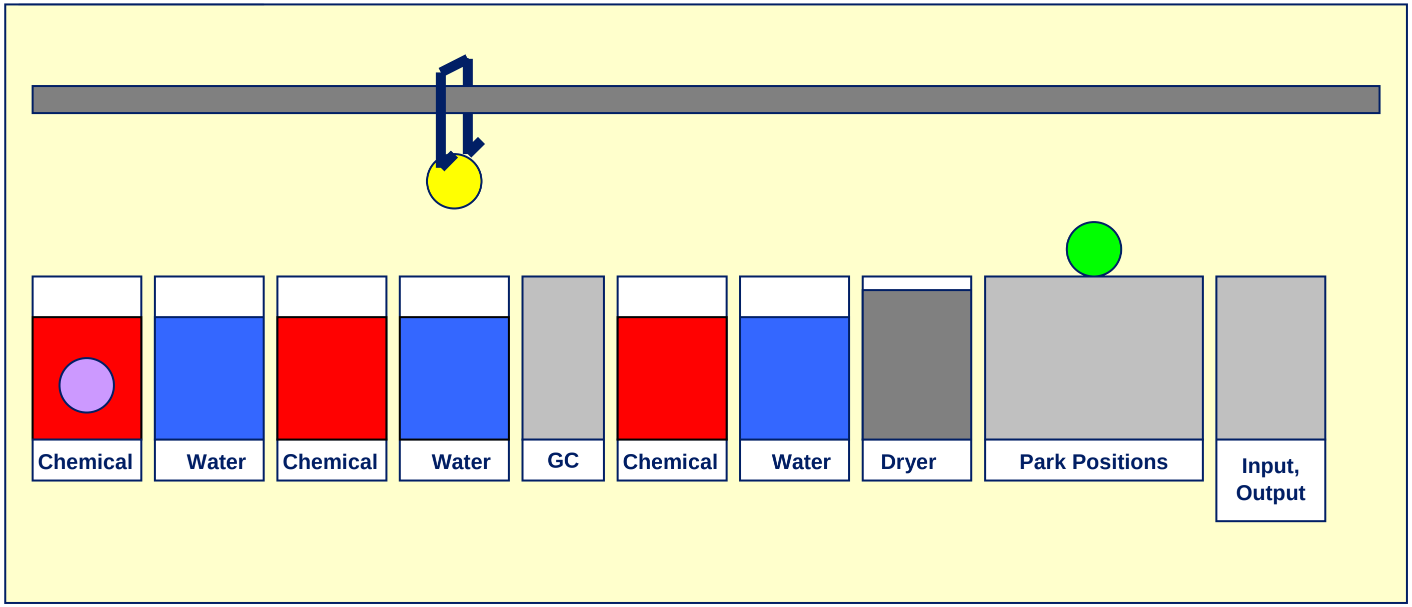




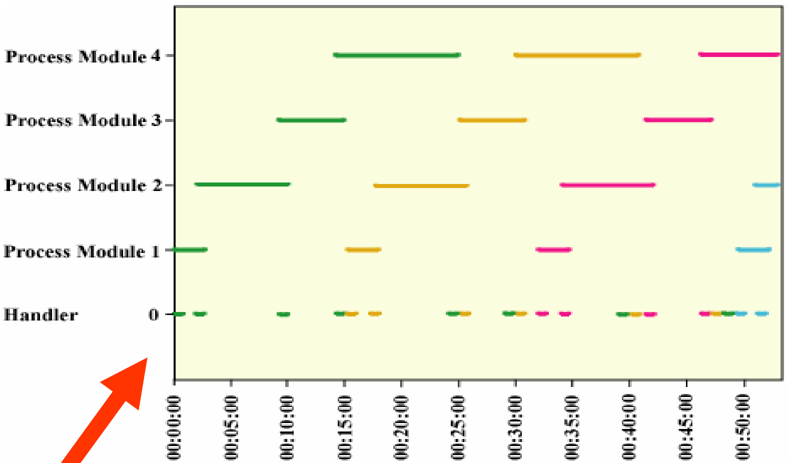
Without an obstacle one can observe clogging effects and a tendency of people to fall in panic situations (left).

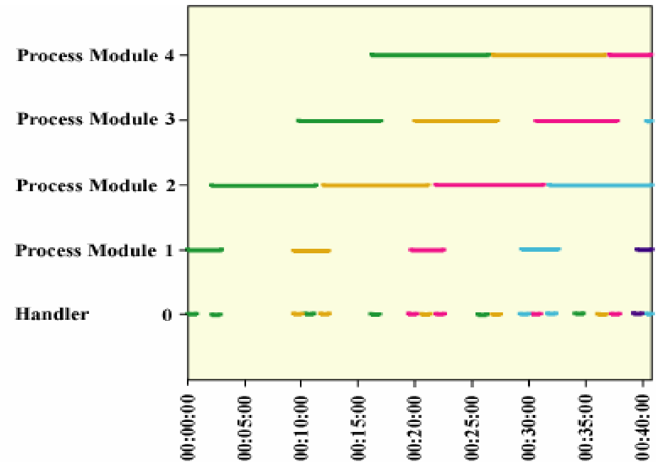
The clogging effect can be significantly reduced by a suitable obstacle, which increases the efficiency of escape and diminishes the tendency of falling (right).

Illustration of a Wet-Bench



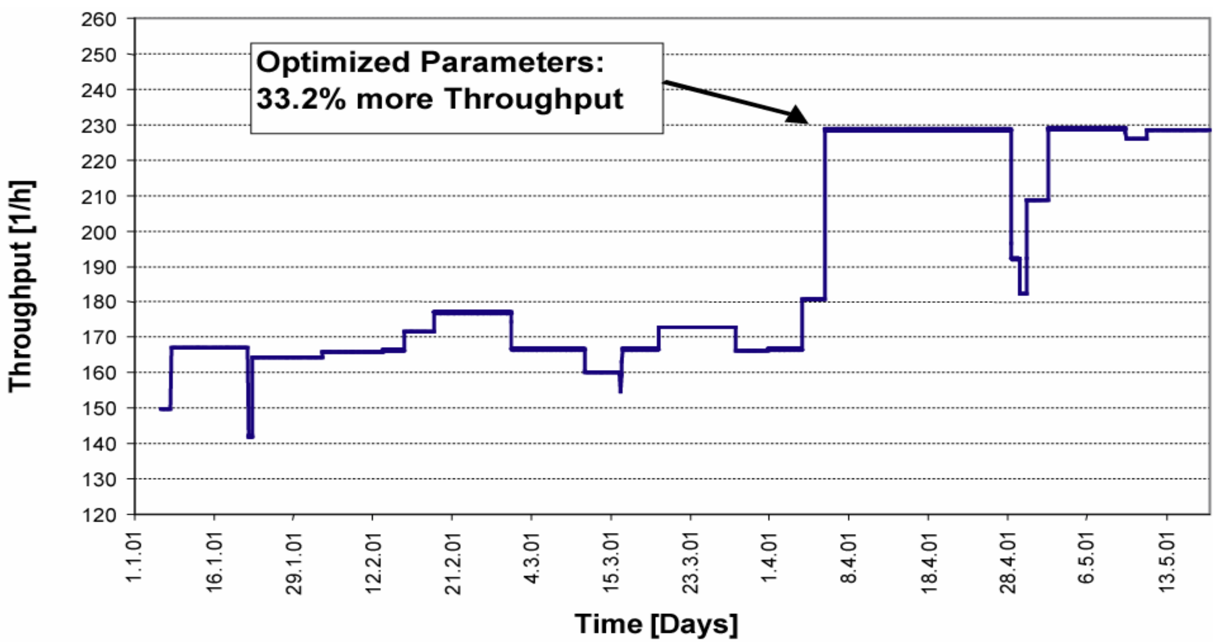
Slower-is-Faster Effect in Semiconductor Production





Slower-is-faster effect

Old recipe: ca. 170/h



New recipe: ca. 230/h

Modeling Macroeconomic Commodity Flows

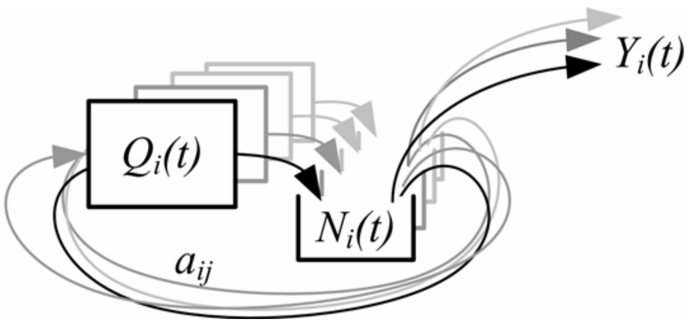
Conservation of resources

$$\dot{N}_i(t) = \underbrace{Q_i(t)}_{\text{supply}} - \underbrace{\sum_{j=1}^{m} a_{ij} Q_j(t)}_{\text{re-entrant}} - \underbrace{Y_i(t)}_{\text{outflow}}$$

(a_{ij})	input matrix **A**
$N_i(t)$	inventory level
$Q_i(t)$	delivery rate
$Y_i(t)$	consumption rate
$P_i(t)$	price level

Adaptation of delivery rates

$$\frac{\dot{Q}_i(t)}{Q_i(t)} = \hat{\nu} \underbrace{\left(\frac{N_i^0}{N_i(t)} - 1\right)}_{\substack{\text{deviations from} \\ \text{desired level}}} - \hat{\mu} \underbrace{\frac{\dot{N}_i(t)}{N_i(t)}}_{\substack{\text{temporal} \\ \text{changes}}}$$

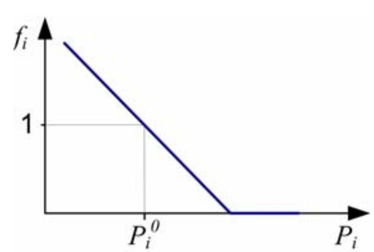


Adaptation of prices

$$\frac{\dot{P}_i(t)}{P_i(t)} = \nu \underbrace{\left(\frac{N_i^0}{N_i(t)} - 1\right)}_{\substack{\text{deviations from} \\ \text{desired level}}} - \mu \underbrace{\frac{\dot{N}_i(t)}{N_i(t)}}_{\substack{\text{temporal} \\ \text{changes}}}$$

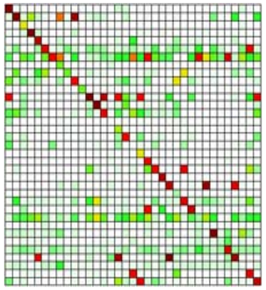
Consumption

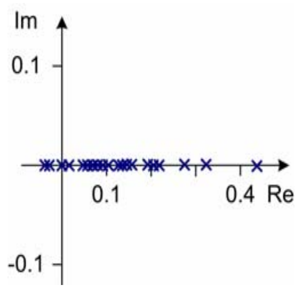
$$Y_i(t) = \left[Y_i^0 + \xi_i(t)\right] f_i\left(P_i(t)\right)$$

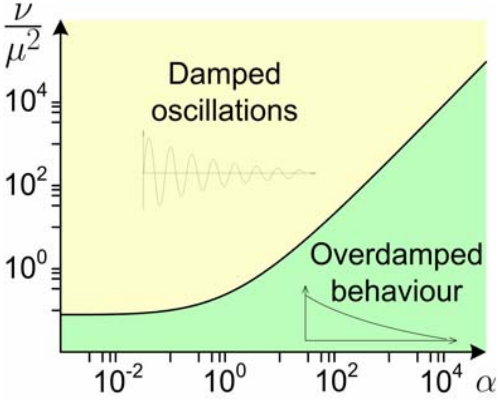


Network-Induced Oscillatory Behaviour

Input matrices with **real** eigenvalues only

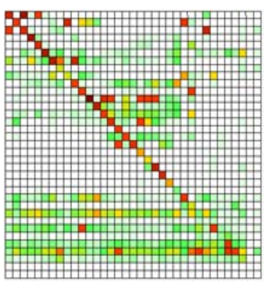


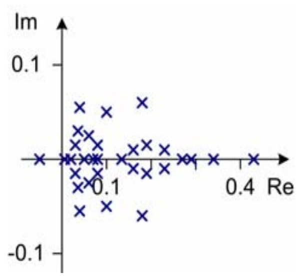


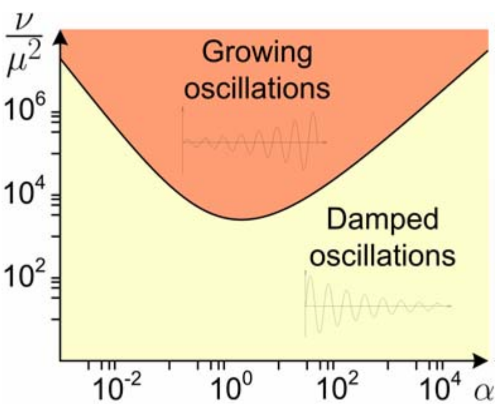


Overdamped behaviour possible.
Oscillations are **never growing**.

Input matrices with **complex** eigenvalues





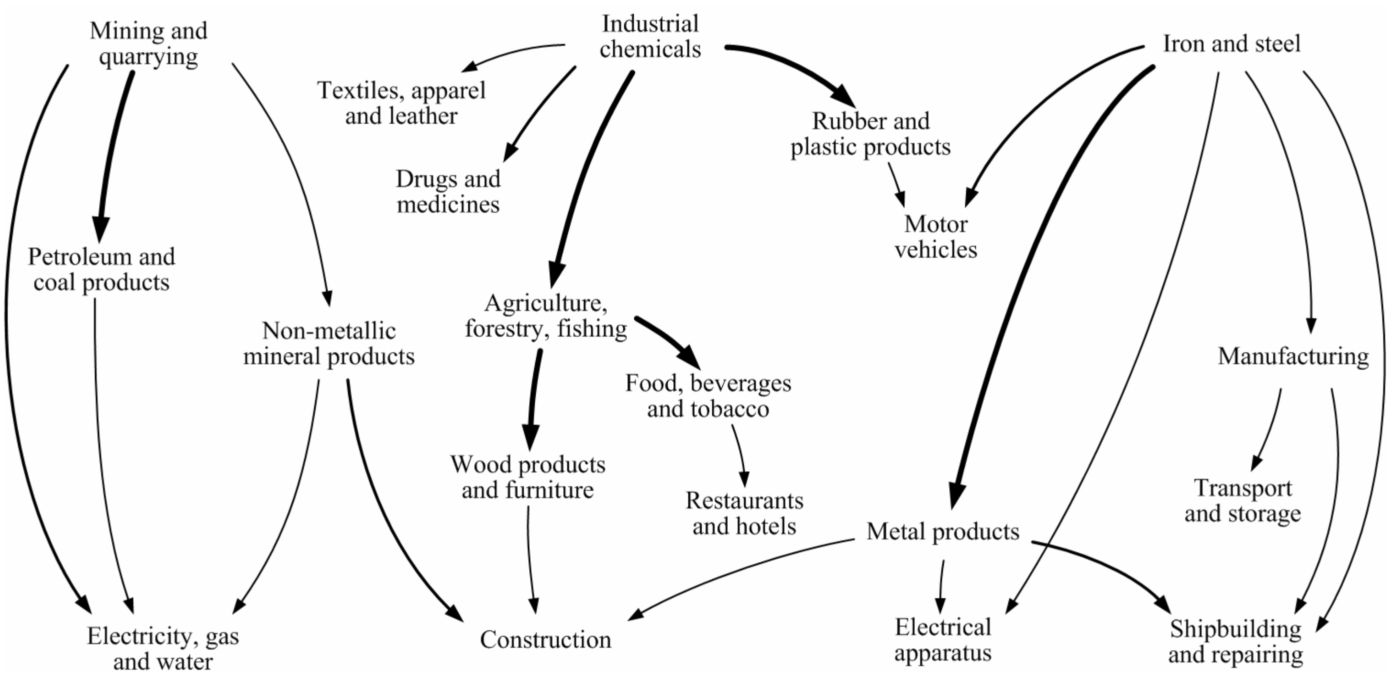


Always oscillating.
Growing oscillations are likely.

Empirical Supply Networks

Commodity flow (average of FRA, GER, JAP, UK, USA)

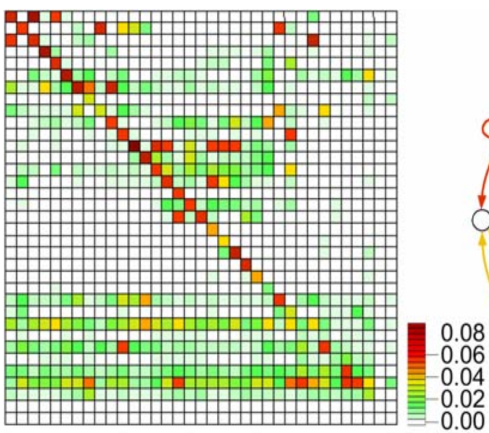
Network structure



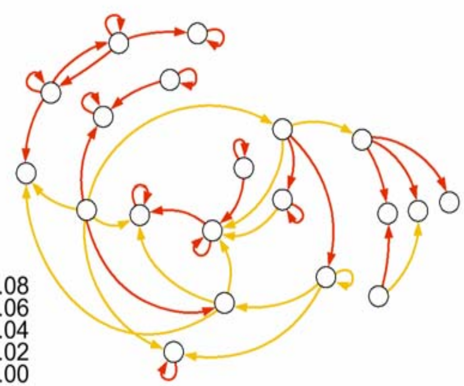
An Alternative Explanation of Business Cycles

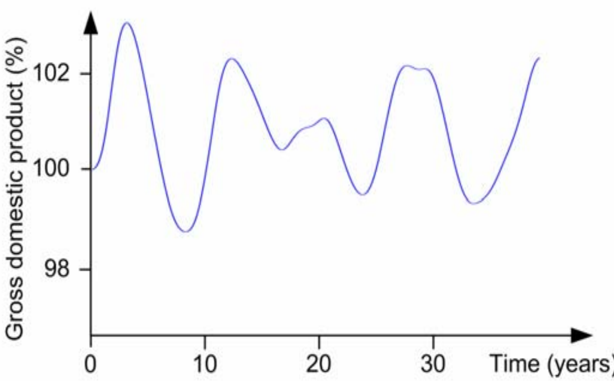
- Investigation of the network structure:
- **Positive** and **negative feedbacks** in production processes
- **Time lags** in the information flow and adaptation process

Business cycles because of the structure of production networks?



Input output matrix

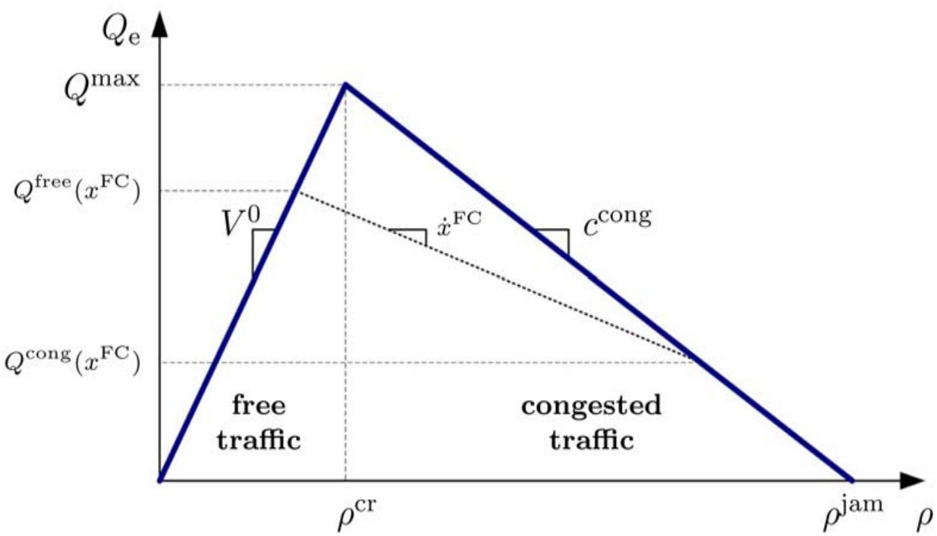

Related delivery network

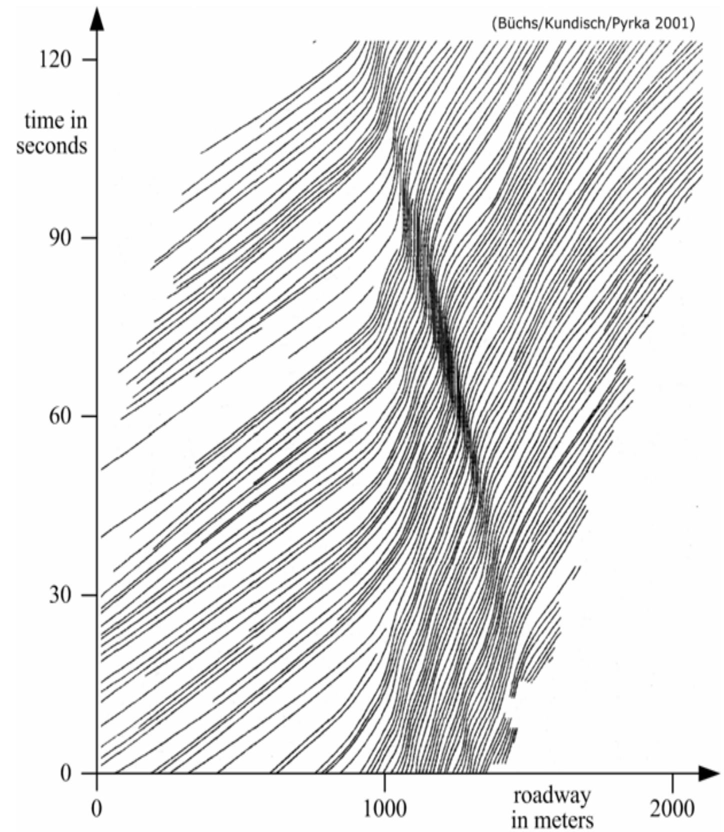


Resulting oscillations in the gross domestic product

Fluid-Dynamic Transport Model

Flow Q and density ρ are empirically correlated via the fundamental diagram $Q_e(\rho)$



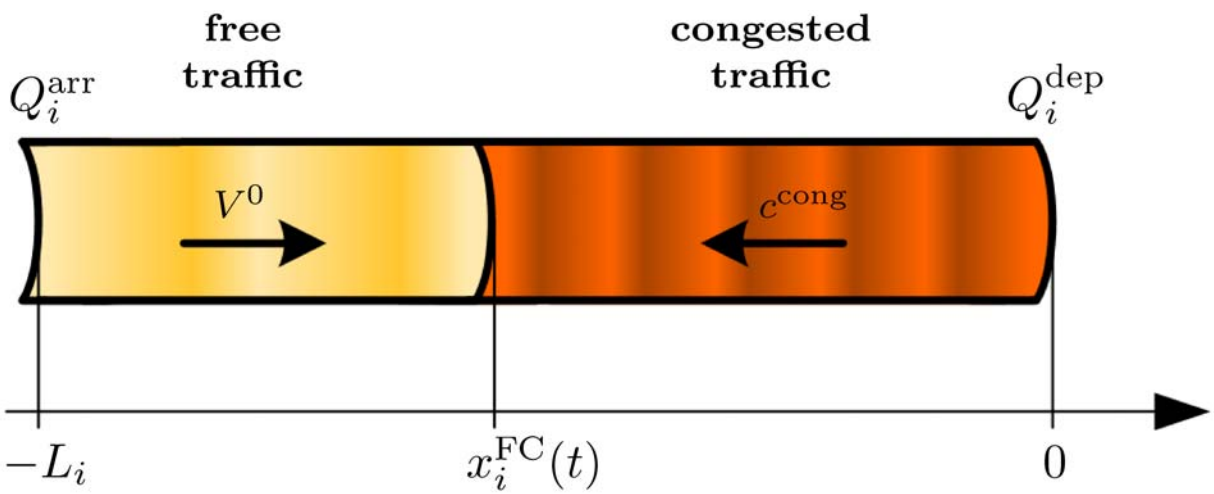


$$\frac{\partial \rho}{\partial t} + \underbrace{\frac{dQ_e(\rho)}{d\rho}}_{c(\rho)} \cdot \frac{\partial \rho}{\partial x} = 0$$

Derivative Q_e' plays an important role!

Network Links: Homogenous Road Sections

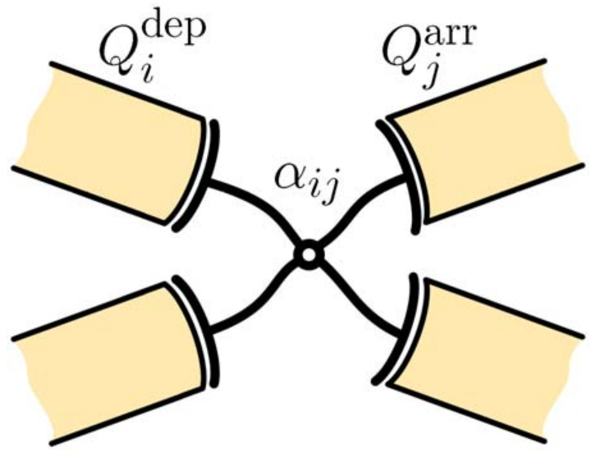
Integration of LWR model leads to efficient, section-based traffic model!



- Movement of congestion
$$\frac{\mathrm{d}}{\mathrm{d}t} x_i^{\mathrm{FC}} = \frac{\Delta Q\left(x_i^{\mathrm{FC}}\right)}{\Delta \rho\left(x_i^{\mathrm{FC}}\right)}$$
- Number of vehicles
$$\frac{\mathrm{d}}{\mathrm{d}t} N_i(t) = Q_i^{\mathrm{arr}}(t) - Q_i^{\mathrm{dep}}(t)$$
- Travel time
$$\frac{\mathrm{d}}{\mathrm{d}t} T_i(t) = 1 - \frac{Q_i^{\mathrm{dep}}(t)}{Q_i^{\mathrm{arr}}(t - T_i(t))}$$

Network Nodes: Connectors

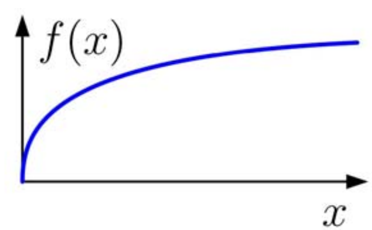
- Side Conditions
  - Conservation $$\sum_i Q_i^{\text{dep}} = \sum_j Q_j^{\text{arr}}$$
  - Non-negativity $$Q_i^{\text{dep}} \geq 0$$ $$Q_j^{\text{arr}} \geq 0$$
  - Upper boundary $$Q_i^{\text{dep}} \leq Q_i^{\text{dep,pot}}$$ $$Q_j^{\text{arr}} \leq Q_j^{\text{arr,pot}}$$
  - Branching $$\sum_i \alpha_{ij} Q_i^{\text{dep}} = Q_j^{\text{arr}}$$



- Goal function

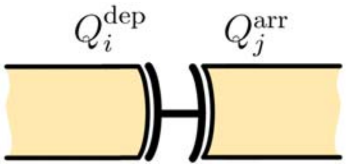
$$F = \sum_i f\left(Q_i^{\text{dep}}\right) \to \max$$

$$f(x) = x^p \quad \text{with } p \ll 1$$



Network Nodes: Special Cases

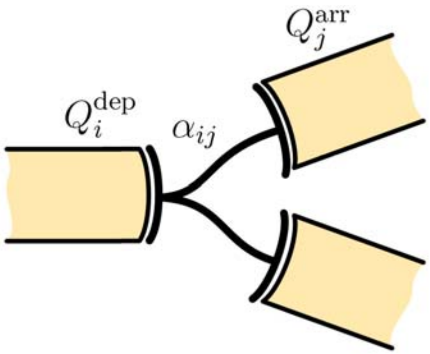
- **1 to 1:** $Q_i^{\text{dep}} = Q_j^{\text{arr}} = \min\left\{Q_i^{\text{dep,pot}}, Q_j^{\text{arr,pot}}\right\}$



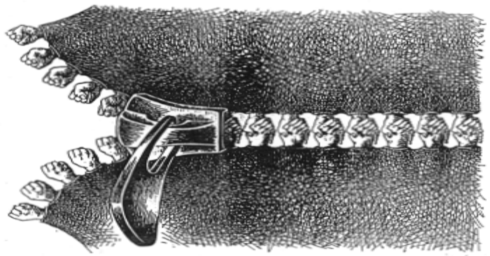
- **1 to n:** Diverging with branch weight α_{ij}

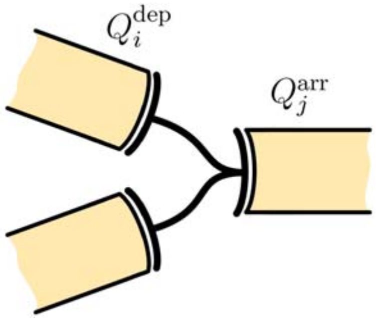
$$Q_i^{\text{dep}} = \min\left\{Q_i^{\text{dep,pot}}, \min_j \frac{Q_j^{\text{arr,pot}}}{\alpha_{ij}}\right\}$$

$$Q_j^{\text{arr}} = \alpha_{ij} Q_i^{\text{dep}}$$

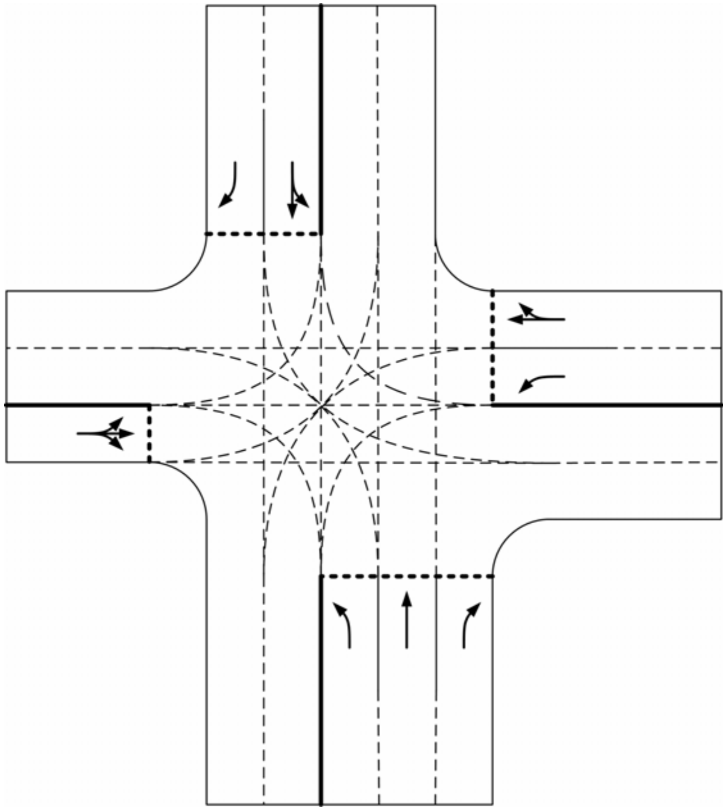


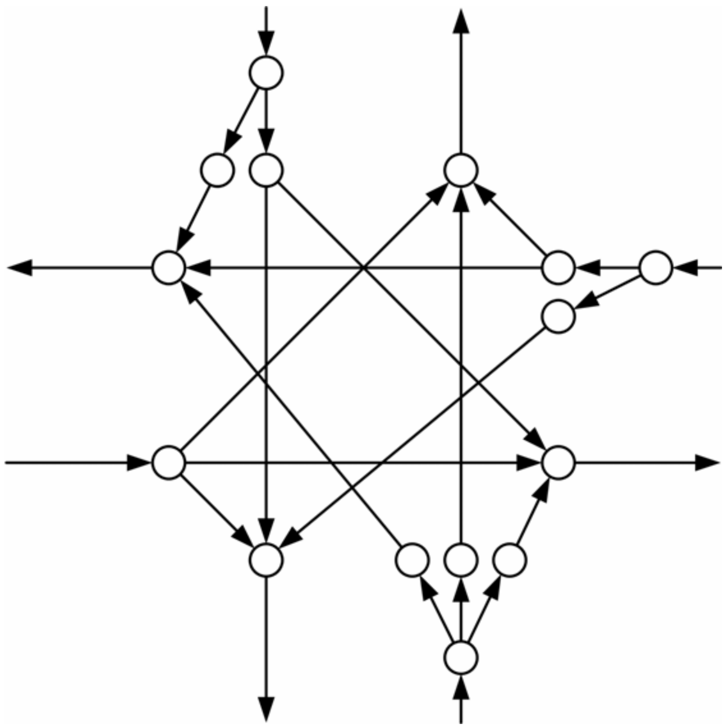
- **n to 1:** Merging



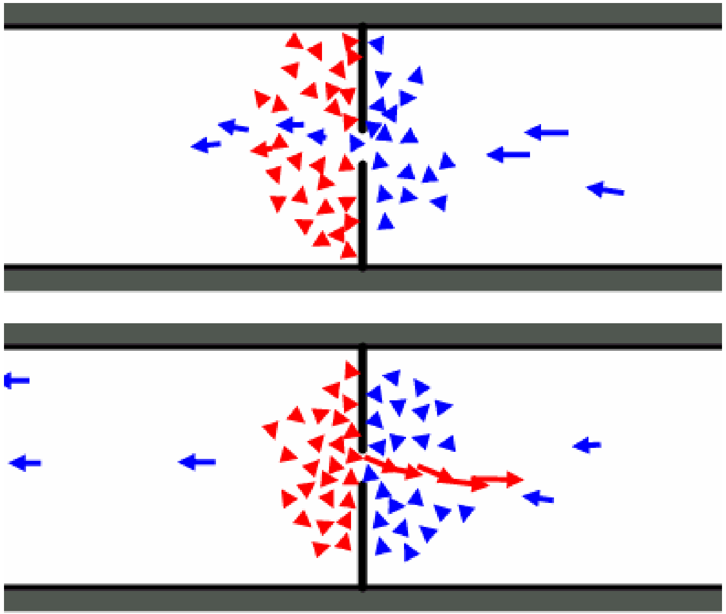


Network Representation of Intersections

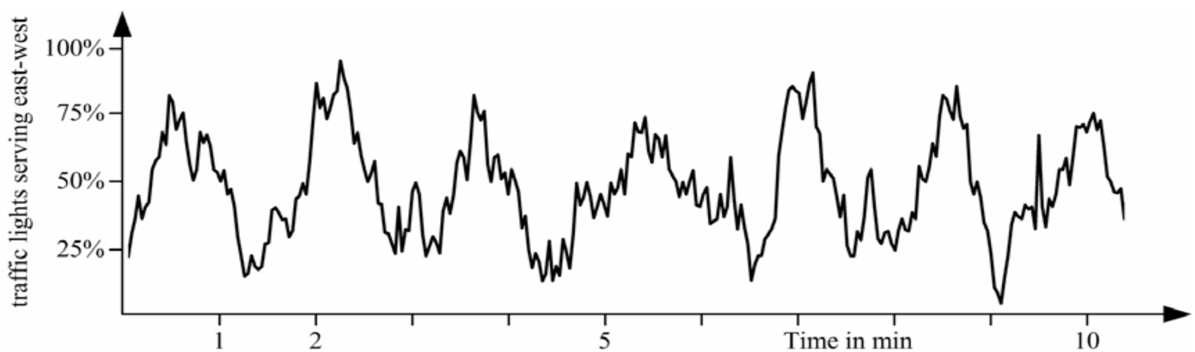


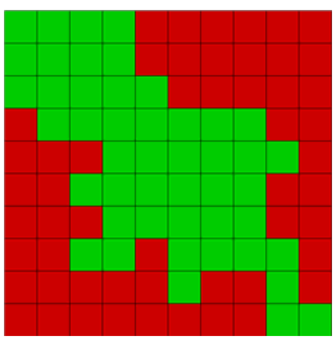


Self-Organized Oscillations at Bottlenecks and Synchronization



- Pressure-oriented, autonomous, distributed signal control:
  - Major serving direction alternates, as in pedestrian flows at intersections
  - Irregular oscillations, but ‘synchronized’
- In huge street networks:
  - ‘Synchronization’ of traffic lights due to vehicle streams spreads over large areas





Application Example: City Center of Dresden

- Simulation “Pirnaischer Platz” (City center of Dresden)

THANK YOU FOR YOUR ATTENTION!

Dirk Helbing

helbing@trafficforum.org

Institute for Transport & Economics

References:

• *Phys. Rev. E* **72**, 046130 (2005)

• *J. Phys. A: Math. Gen.* **36**, L593 (2003)

• *New J. Physics* **5**, 90.1-90.28 (2003)

• *Phys. Rev. E* **70**, 056118 (2004)

• *Phys. Rev. E* **70**, 066116 (2004)

• www.trafficforum.org/trafficlights