

New development of internally contracted MRCI

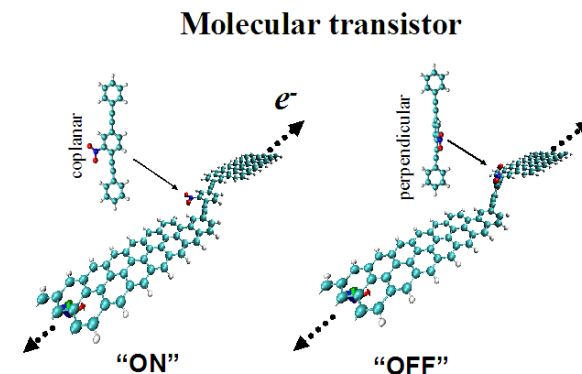
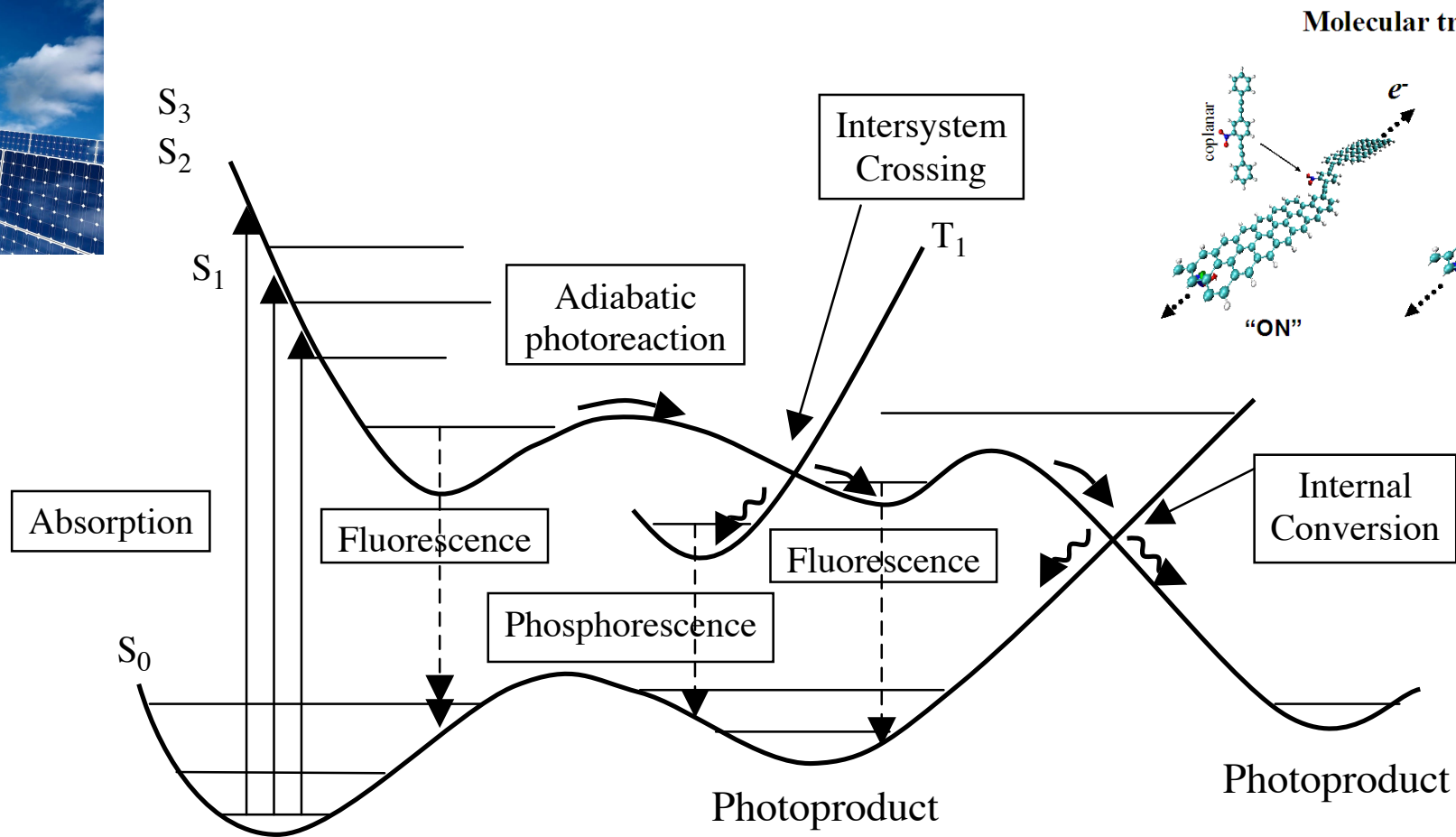
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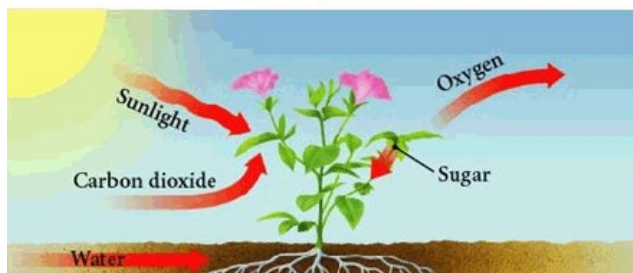
Contains

- *Excited state and none-adiabatic problem*
- **EN-GMFCI**
- icMRCI based on GUGA

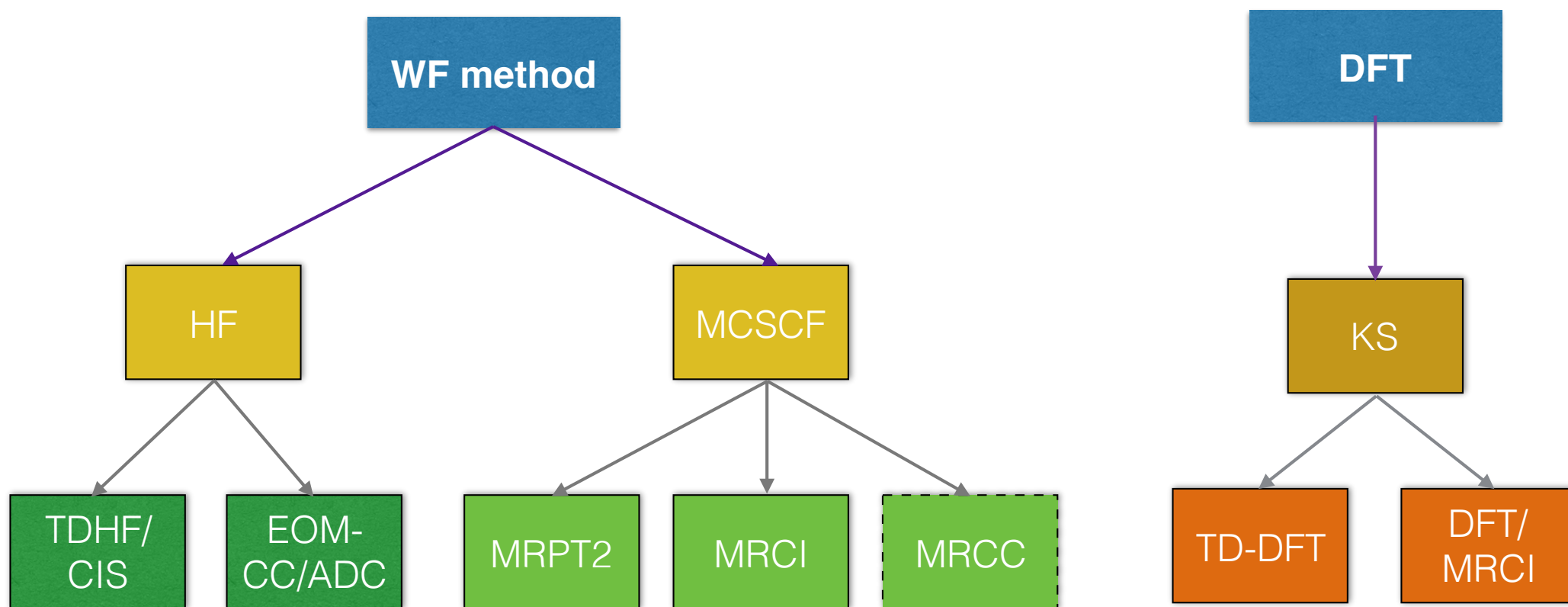
Photo-physics and photo-chemistry procedures



Photosynthesis



Methods for the excited state



Beijing *D*ensity *F*unctional *p*ackage- *BDF*

- SCF level

RHF, UHF, ROHF, RKS, UKS,ROKS - Direct and none-direct, point group symmetry, CD/RI approximation.

DFT functional: LDA,GGA,Hybrid, range-sep Hybrid etc.

- MCSCF.
- Post SCF and excited states

MP2, Frag-LMP2

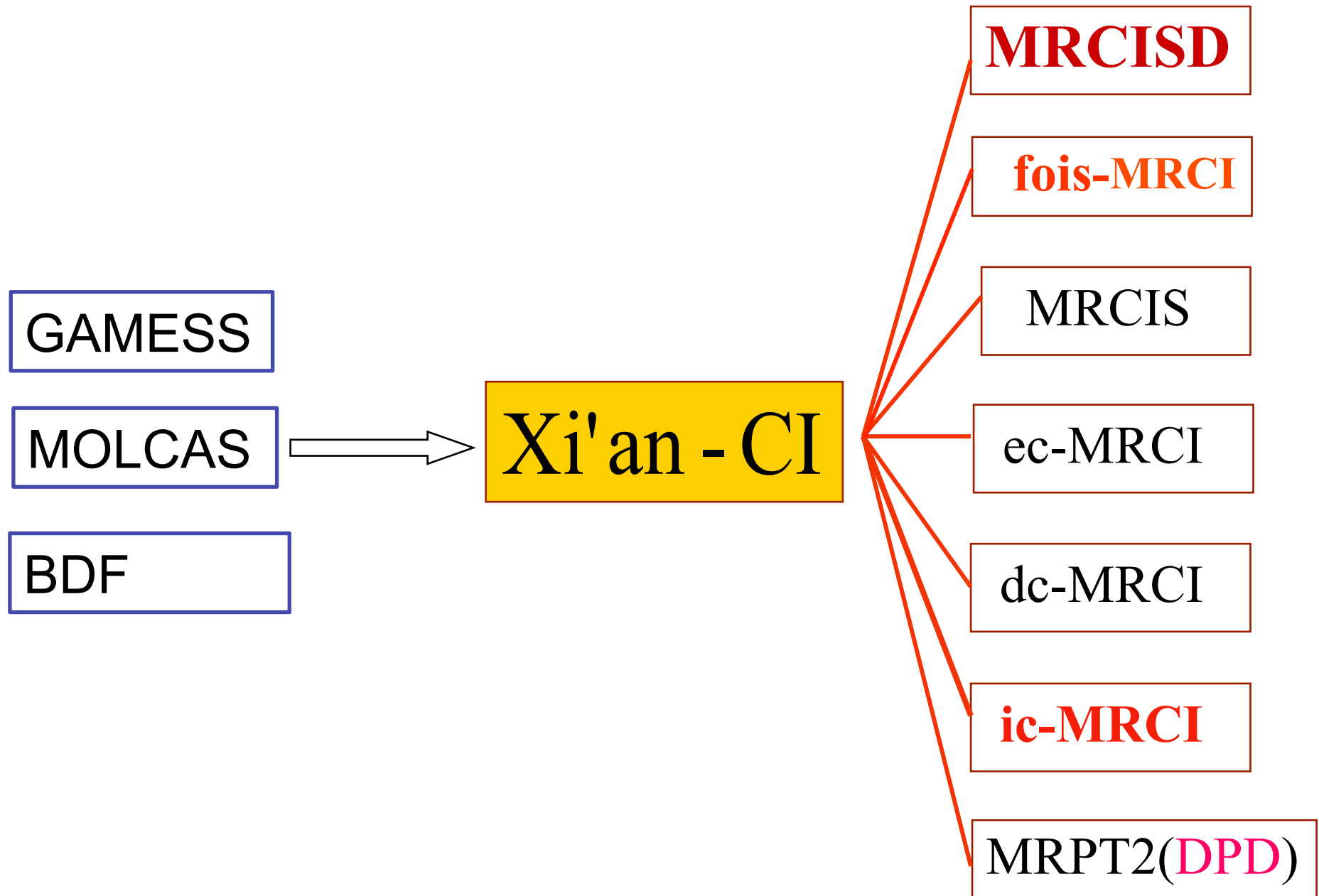
TDDFT/TDHF, TDDFT+SOC, TD Gradient and NAC

CCSD,CCSD(T), ***EOM-IP/EA/EE,CCSD+SOC***

MRCI,MRPT2, MRCI-SOC,MCCEPA

EN-GMFCI

Xi'an-Cl package



Contains

- Excited state and non-adiabatic problem
- ***EN-GMFCI***
- icMRCI based on GUGA

Electron-Nuclei General mean field configuration interaction

$$\hat{H}(\vec{R}^e, \vec{Q}) = \hat{H}(\vec{R}^e) + \hat{H}(\vec{Q}) + \hat{H}(\vec{R}^e, \vec{Q})$$

$$\hat{H}(\vec{R}^e) = -\frac{1}{2} \sum_{i=1}^p \nabla_{\vec{r}_i^e}^2 + \sum_{1 \leq i < j \leq p} \frac{1}{\|\vec{r}_i^e - \vec{r}_j^e\|} \quad \text{Electron}$$

$$\hat{H}(\vec{Q}) = -\frac{1}{2} \sum_{i=1}^q \nabla_{\vec{Q}_i}^2 + \sum_{1 \leq a < b \leq N} \frac{Z_a Z_b}{\|\vec{r}_i^e - \vec{r}_a^0 - \hat{G}_a^{-1} \hat{L}^T \vec{Q} - \hat{G}_b^{-1} \hat{L}^T \vec{Q}\|} \quad \text{Nuclei}$$

$$\hat{H}(\vec{Q}, \vec{R}^e) = - \sum_i^p \sum_a^N \frac{Z_a Z_b}{\|\vec{r}_i^e - \vec{r}_a^0 - \hat{G}_a^{-1} \hat{L}^T \vec{Q}\|} \quad \text{Coupling term}$$

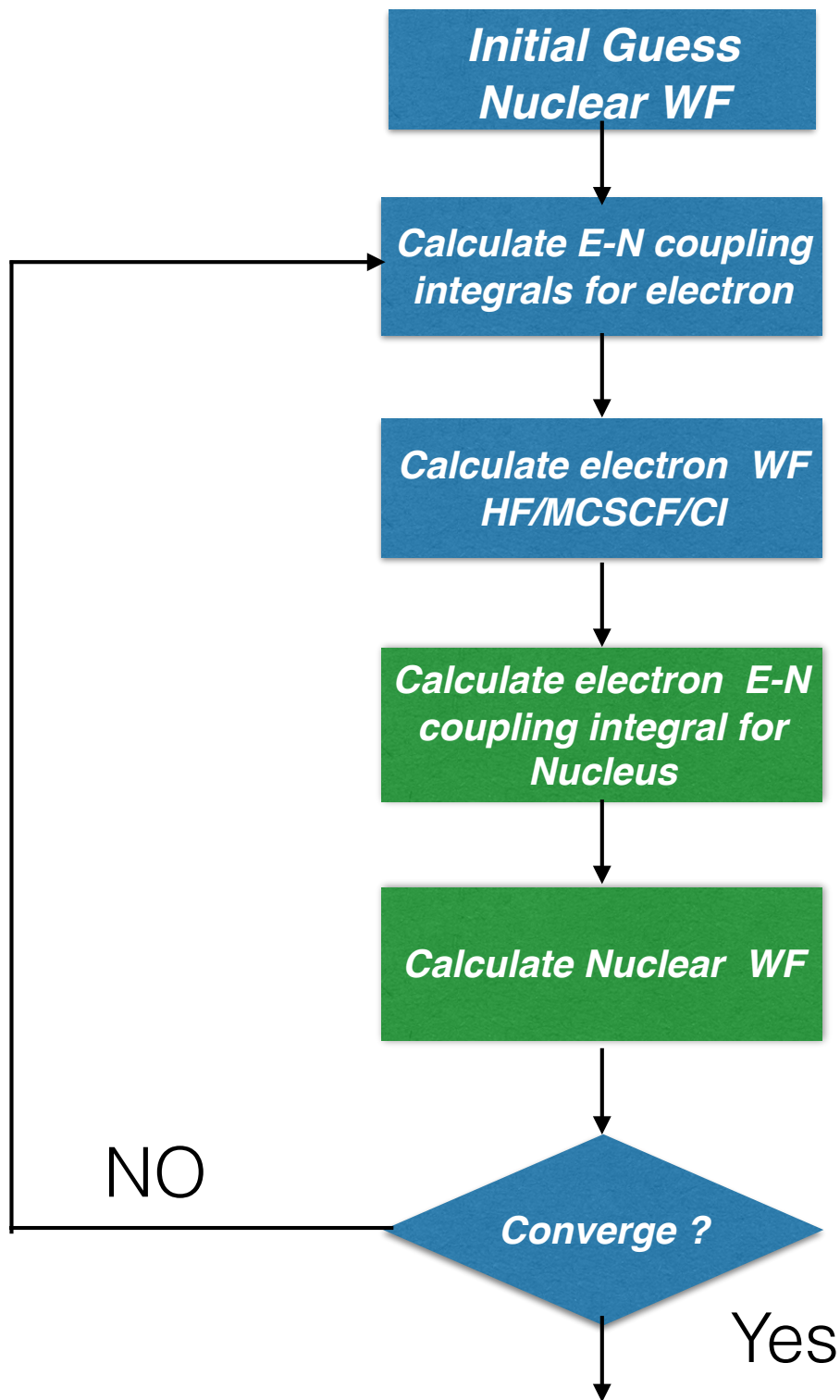
General Mean field approach

Electrons are in a mean field generated by nucleus

$$\hat{H}^{eff}(\vec{R}^e) = \hat{H}(\vec{R}^e) + \langle \phi_{\vec{0}}^{(0)}(\vec{Q}) | \hat{H}(\vec{Q}) + \hat{H}(\vec{R}^e, \vec{Q}) | \phi_{\vec{0}}^{(0)}(\vec{Q}) \rangle_{\vec{Q}}$$

Nucleus are in a mean field generated by electrons

$$\hat{H}^{eff}(\vec{Q}) = \hat{H}(\vec{Q}) + \langle \phi_{\vec{0}}^{(1)}(\vec{R}^e) | \hat{H}(\vec{R}^e) + \hat{H}(\vec{R}^e, \vec{Q}) | \phi_{\vec{0}}^{(1)}(\vec{R}^e) \rangle_{\vec{R}^e}$$

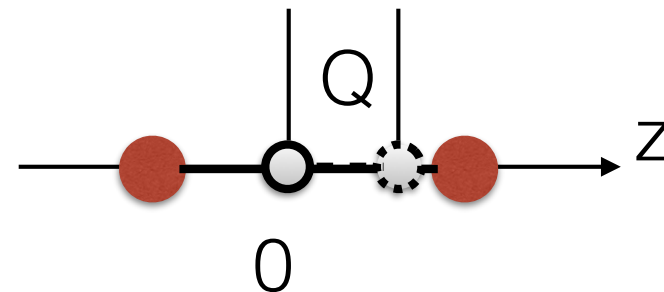


Self-consistent solving EN general mean field problem is solved.

Configuration interaction can be evoked based on mean field result.

$$|\Phi_{ni}(\vec{Q}, \vec{R})\rangle = |\phi_n(\vec{Q})\rangle |\phi_i(\vec{R}^e)\rangle$$

For diatomic molecule



$$Q = \sqrt{\mu_{ab}}(r_{a_z} - r_{a_z}^0 - r_b + r_{b_z}^0) \quad \mu_{ab} = \frac{m_a m_b}{m_a + m_b}$$

Nucleus

$$\hat{H}^{eff}(Q) = -\frac{1}{2} \sum_{i=1}^q \nabla_{Q_i}^2 + \frac{\sqrt{\mu_{ab}} Z_a Z_b}{|\xi_{ab}^0 + Q|} + \langle \phi_{\vec{0}}^{(1)}(\vec{R}^e) | -\frac{1}{2} \sum_{i=1}^p \nabla_{\vec{r}_i^e}^2 + \sum_{1 \leq i < j \leq p} \frac{1}{\|\vec{r}_i^e - \vec{r}_j^e\|} - \sum_{i=1}^p \frac{Z_a}{\sqrt{(r_{i_x}^e)^2 + (r_{i_y}^e)^2 + (r_{i_z}^e - r_{a_z}^0 - \frac{\sqrt{\mu_{ab}} Q}{m_a})^2}} + \frac{Z_b}{\sqrt{(r_{i_x}^e)^2 + (r_{i_y}^e)^2 + (r_{i_z}^e - r_{b_z}^0 + \frac{\sqrt{\mu_{ab}} Q}{m_b})^2}} | \phi_{\vec{0}}^{(1)}(\vec{R}^e) \rangle_{\vec{R}^e}$$

Electron

$$\hat{H}^{eff}(\vec{R}^e) = -\frac{1}{2} \sum_{i=1}^p \nabla_{\vec{r}_i^e}^2 + \sum_{1 \leq i < j \leq p} \frac{1}{\|\vec{r}_i^e - \vec{r}_j^e\|} + \langle \phi_0^{(0)}(Q) | -\frac{1}{2} \nabla_Q^2 + \frac{\sqrt{\mu_{ab}} Z_a Z_b}{|\xi_{ab}^0 + Q|} - \sum_{i=1}^p \frac{Z_a}{\sqrt{(r_{i_x}^e)^2 + (r_{i_y}^e)^2 + (r_{i_z}^e - r_{a_z}^0 - \frac{\sqrt{\mu_{ab}} Q}{m_a})^2}} + \frac{Z_b}{\sqrt{(r_{i_x}^e)^2 + (r_{i_y}^e)^2 + (r_{i_z}^e - r_{b_z}^0 + \frac{\sqrt{\mu_{ab}} Q}{m_b})^2}} | \phi_0^{(0)}(Q) \rangle_Q$$

Nuclear wave function

Kratzer potential

$$\left[\frac{\hbar^2}{2\mu} \left(-\frac{\partial^2}{\partial r^2} + \frac{J(J+1)}{r^2} \right) + \frac{A}{r^2} - \frac{B}{r} \right] \psi_n^j(r) = E_{nj} \psi_n^j(r)$$

Reduced formula from Kratzer $x = \frac{r}{r_e}$

$$\left\{ -\frac{\partial^2}{\partial x^2} + K \left(\frac{1}{x^2} - \frac{2}{x} \right) \right\} \psi_n^j(x) = \epsilon_{nj} \psi_n^j(x)$$

$$K = J(J+1) + A \frac{2\mu}{\hbar^2}$$

EN coupling integrals

$$I_{e-n}[Z_I, r_{I_z}^0, \eta] = \langle \phi_i^{kra}(Q) \chi_1(\vec{r}^e) | \frac{Z_I}{\sqrt{(r_x^e)^2 + (r_y^e)^2 + (r_z^e - r_{I_z}^0 + \eta Q)^2}} | \phi_j^{kra}(Q) \chi_2(\vec{r}^e) \rangle$$

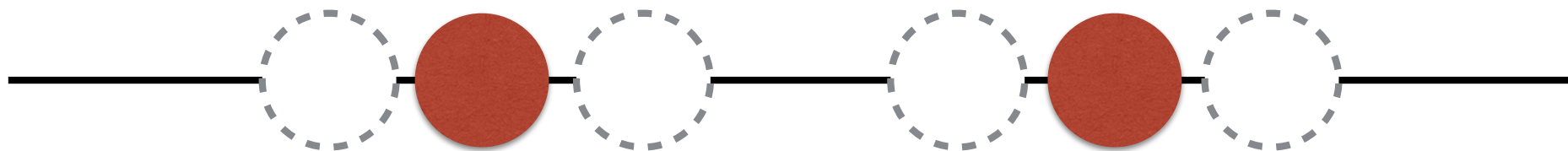
Here, the Kratzer functions are

$$\phi_i^{kra}(Q) = \frac{N_i^{kra}}{\sqrt{\xi_{ab}^0}} \left(1 + \frac{Q}{\xi_{ab}^0}\right)^\lambda \text{Exp} \left[\frac{\lambda(1-\lambda)}{\lambda+i} \left(1 + \frac{Q}{\xi_{ab}^0}\right) \right] {}_1F_1[-i, 2\lambda; \frac{2\lambda(\lambda-1)}{\lambda+i} \left(1 + \frac{Q}{\xi_{ab}^0}\right)]$$

1 Rys and generalized Laguerre double quadrature formulas

2 Multi-Precision based on library arprec

Off-center basis:



H₂, Convergence of total energy with number of Kratz and Off-center basis, Full-CI, cc-PVTZ

	K=4	K=8	K=12	K=16
n=4	-1.1621504	-1.1622377	-1.1622418	-1.1622419
n=8	-1.1623184	-1.1624301	-1.1624334	-1.1624341
n=12	-1.1623574	-1.1624676	-1.1624750	-1.1624754
n=16	-1.1623877	-1.1624996	-1.1625050	-1.1625060

Convergence of EN-GMFCI energy with basis sets and off-center basis

	cc-pVDZ	cc-pVTZ	cc-pVQZ	cc-pV5Z
n=4	-1.1576745 (18,0)	-1.1622419 (36,0)	-1.1632984 (68,0)	-1.1636441 (118,0)
n=8	-1.1579290 (26,5)	-1.1624380 (44,4)	-1.1634754 (76,2)	-1.1638262 (126,0)
n=12	-1.1581072 (34,10)	-1.1624953 (52,9)	-1.1636184 (84,5)	-1.1638365 (134,5)
n=16	-1.1581475 (42,16)	-1.1625060 (60,16)	-1.1635140 (92,10)	-1.1638438 (142,8)

Two numbers in parentheses are number of basis functions and number of basis functions should be eliminated due to basis set linear dependent.

Full CI v.s. Limited CI

	Full CI	Limited CI	
Numbers of CSF's	144720	6286	4.4%
E^0 (hartree)	-1.1638438	-1.1638413	
$\nu; 0 \rightarrow 1$ (cm ⁻¹)	4165.36	4165.86	

cc-pV5Z, n=16

Transition	TF-NOMO/CIS	TF-NOMO/FCI	This work	Exp.
H ₂				
v: 0 → 1	4655	4182	4165	4161
v: 0 → 2	9406	NA	8110	8087
Σ_g^+ : 0 → 1	106556	NA	91711	91700
D ₂				
v: 0 → 1	3549	3006	2994	2994
v: 0 → 2	7026	NA	5874	5869
Σ_g^+ : 0 → 1	107628	NA	92182	91697
T ₂				
v: 0 → 1	2929	2477	2465	2465
v: 0 → 2	5843	NA	4851	4849
Σ_g^+ : 0 → 1	108043	NA	92375	91696

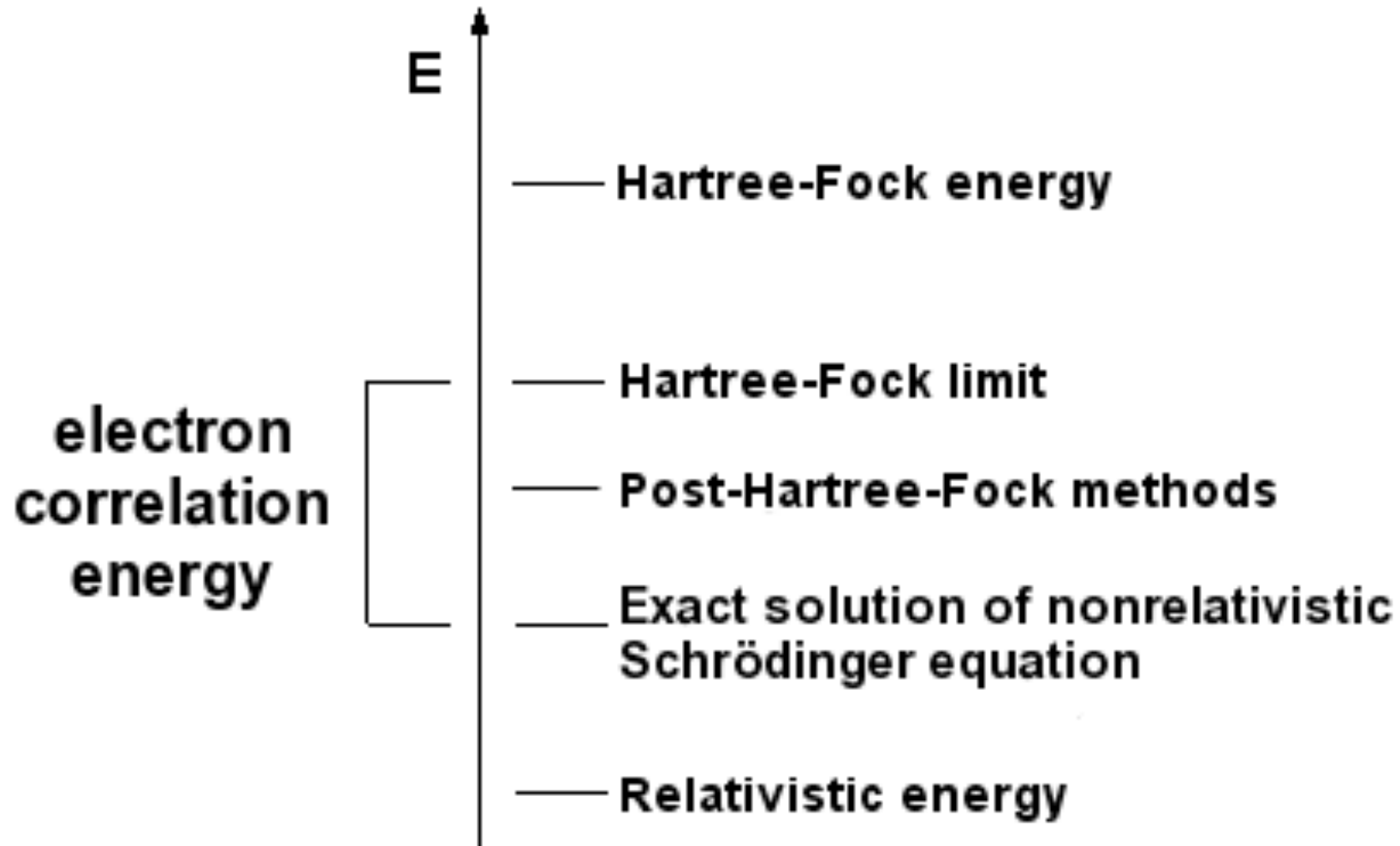
A brief summary

- Electron-Nuclei Mean field approximation
- EN-SCF/MCSCF/MRCI
- EN-FCI (direct product of electron and nucleus)
- Off-center basis sets

Contains

- Excited state and non-adiabatic problem
- EN-GMFCI
- ***icMRCI based on GUGA***

Electron correlation energy



Configuration interaction

$$|\Psi\rangle = \sum_R C_R |\Phi_R\rangle + \sum_{ai} C_{ai} |\Phi_{ai}\rangle + \sum_{ai,bj} C_{ai,bj} |\Phi_{ai,bj}\rangle \\ + \sum_{ia,jb,kc} C_{ai,bj,ck} |\Phi_{ai,bj,ck}\rangle + \dots$$

coefficients C can be calculated by solving eigenvalue problem

$$HC = EC$$

The dimension of CI space (Spin-adapted CSFs, neglect spatial symmetry) can be calculated from Weyl equation

$$D = \frac{2S+1}{m+1} \binom{m+1}{\frac{N}{2} - S} \binom{m+1}{\frac{N}{2} + S + 1}$$

CH₄, 6-31G**, 10 electron, 49 MOs. The CI dimension is
673,371,590,640

Electron correlation methods

- Truncated CI — (MR)CISD,(MR)CISDT,
- Cluster expansion — CCSD,CCSD(T),
- Multi-reference perturbation theory - CASPT2, NEVPT2
- DMRG-CI, by White, Chan, Wouter etc.
- QMC-CI by Alavi, Booth etc.

MRCISD - UC v.s. IC

Un-contracted MRCISD

$$|\Psi\rangle = \sum_R C_R |\Phi_R\rangle + \sum_{ai} C_{ai} |\Phi_{ai}\rangle + \sum_{ai,bj} C_{ai,bj} |\Phi_{ai,bj}\rangle$$

$$|\Phi_{ai}\rangle = E_{ai} |\Phi_R\rangle$$

$$|\Phi_{ai,bj}\rangle = E_{ai,bj} |\Phi_R\rangle$$

Internally contracted MRCISD

$$|\Psi\rangle = |\Psi_0\rangle + \sum_{ai} C_{ai} |\Phi'_{ai}\rangle + \sum_{ai,bj} C_{ai,bj} |\Phi'_{ai,bj}\rangle$$

$$|\Psi_0\rangle = \sum_R C_R |\Phi_R\rangle$$

$$|\Phi'_{ai}\rangle = E_{ai} |\Psi_0\rangle$$

$$|\Phi'_{ai,bj}\rangle = E_{ai,bj} |\Psi_0\rangle$$

CI matrix element

Iterative diagonalization

$$\sigma_{\mu}^{(n+1)} = \sum_{\nu} H_{\mu\nu} C_{\nu}$$

$$\begin{aligned} H_{\mu\nu} &= \langle \Phi_{\mu} | \hat{H} | \Phi_{\nu} \rangle \\ &= \sum_{p,q} (p|h_1|q) \Gamma_{pq} + \sum_{pr,qs} (pq|h_{12}|rs) \Gamma_{pq,rs} \end{aligned}$$

$$\Gamma_{pq}^{\mu\nu} = \langle \Phi_{\mu} | E_{pq} | \Phi_{\nu} \rangle$$

$$\Gamma_{pq,rs}^{\mu\nu} = \langle \Phi_{\mu} | E_{pr,qs} | \Phi_{\nu} \rangle$$

Coupling coefficients

$$(p|h_1|q)$$

$$(pr|h_{12}|qs)$$

Molecular integrals

Multi-electron basis

Slater Determinate

Configuration
state function

Spin adapted

NO

YES

Size

2-4 times

1

Calculation
of coupling
coefficients

easy

complicated

Graphic unitary group approach - GUGA

- 1950 Gelfand-Tsetlin
- 1962-1966 Biedenhavn, Moshisky, Boyu Hou
- 1974 Paldus: UGA
- 1978 Shavitt: GUGA
- 1979-1982 Schaefer, Sigbahn, Saxe, Payne
- 1984 Shepard, Lishika in Cloumbus
- 1986 - now, Wen and Wang, Xi'an-CI
- 2013 -now, Shepard, contracted GUGA

Paldus tabular, step vector and Distinct Row Table

CSF is represented as Paldus tabular

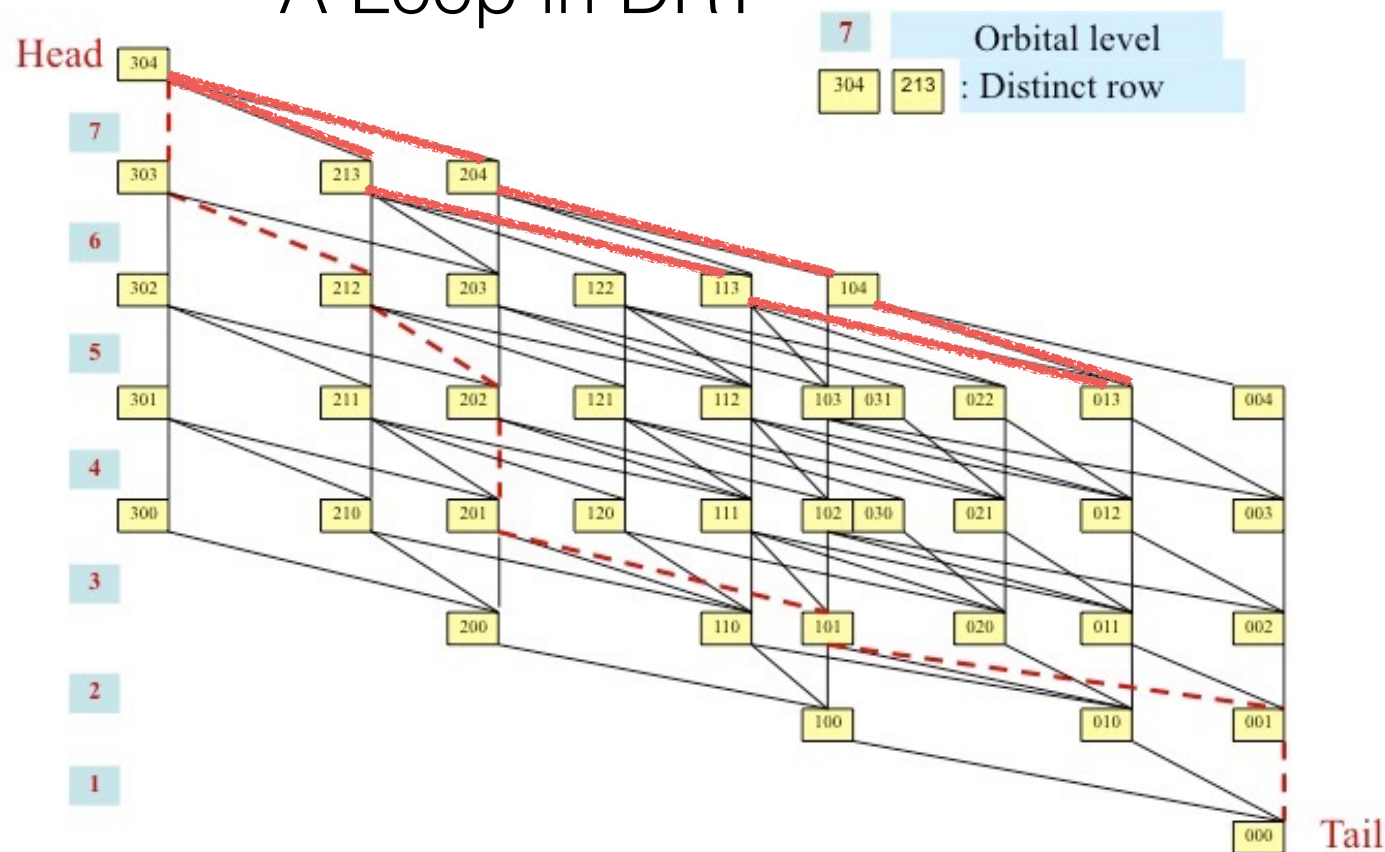
$$|(abc)\rangle = \left| \begin{array}{ccc} a_n & b_n & c_n \\ \dots & \dots & \dots \\ a_r & b_r & c_r \\ \dots & \dots & \dots \\ a_1 & b_1 & c_1 \end{array} \right\rangle$$



$$|d_1 d_2 \dots d_{n-1} d_n\rangle$$

Step vector

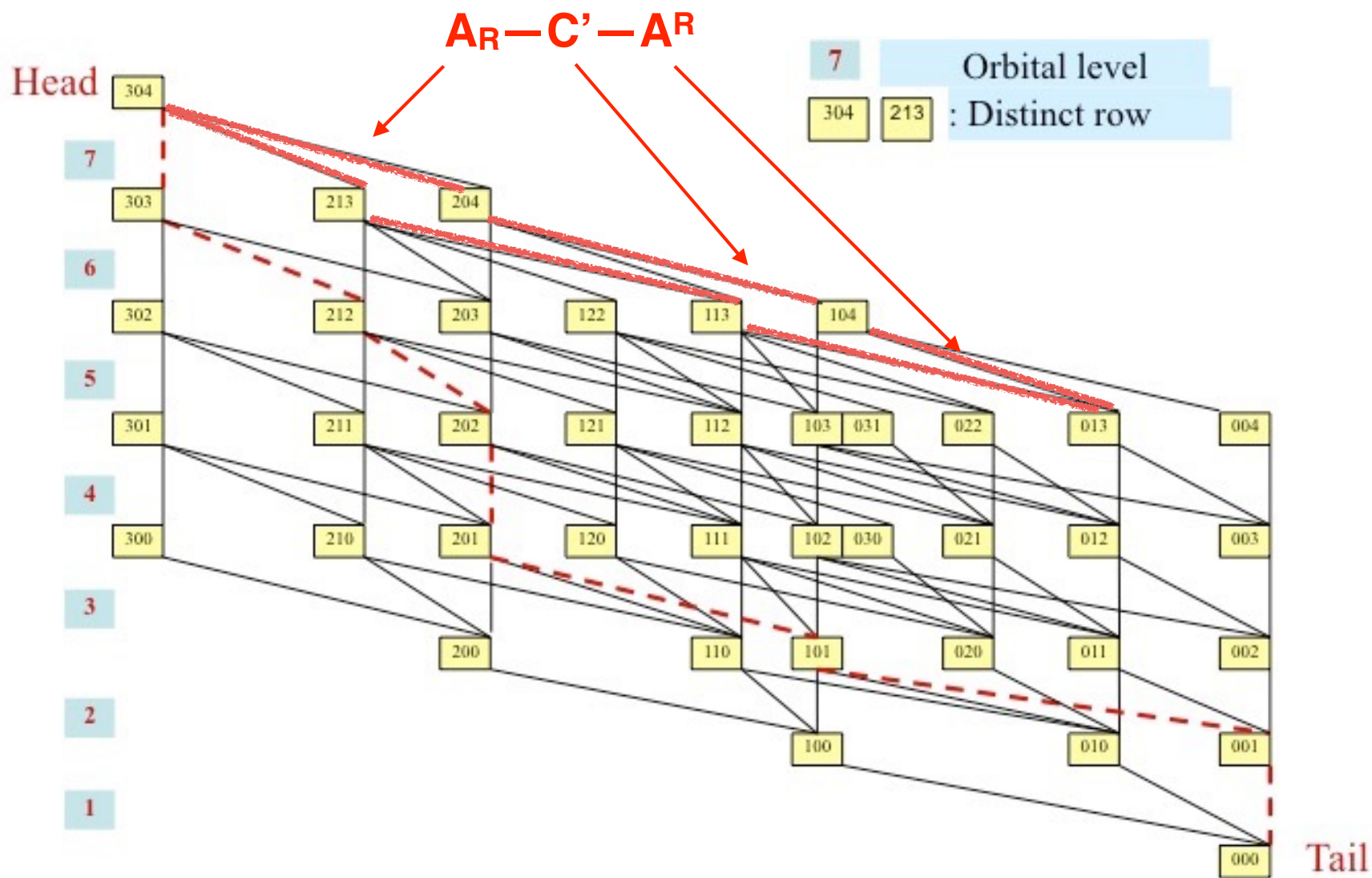
A Loop in DRT



Coupling coefficients are calculated as

$$\begin{aligned}\Gamma_{pq}^{\mu\nu} &= \langle d'_\mu | E_{pq} | d_\nu \rangle \\ &= \prod_{r=\min(p,q)}^{\max(p,q)} W(Q_r, d'_r, d_r, \Delta b_r, b_r)\end{aligned}$$

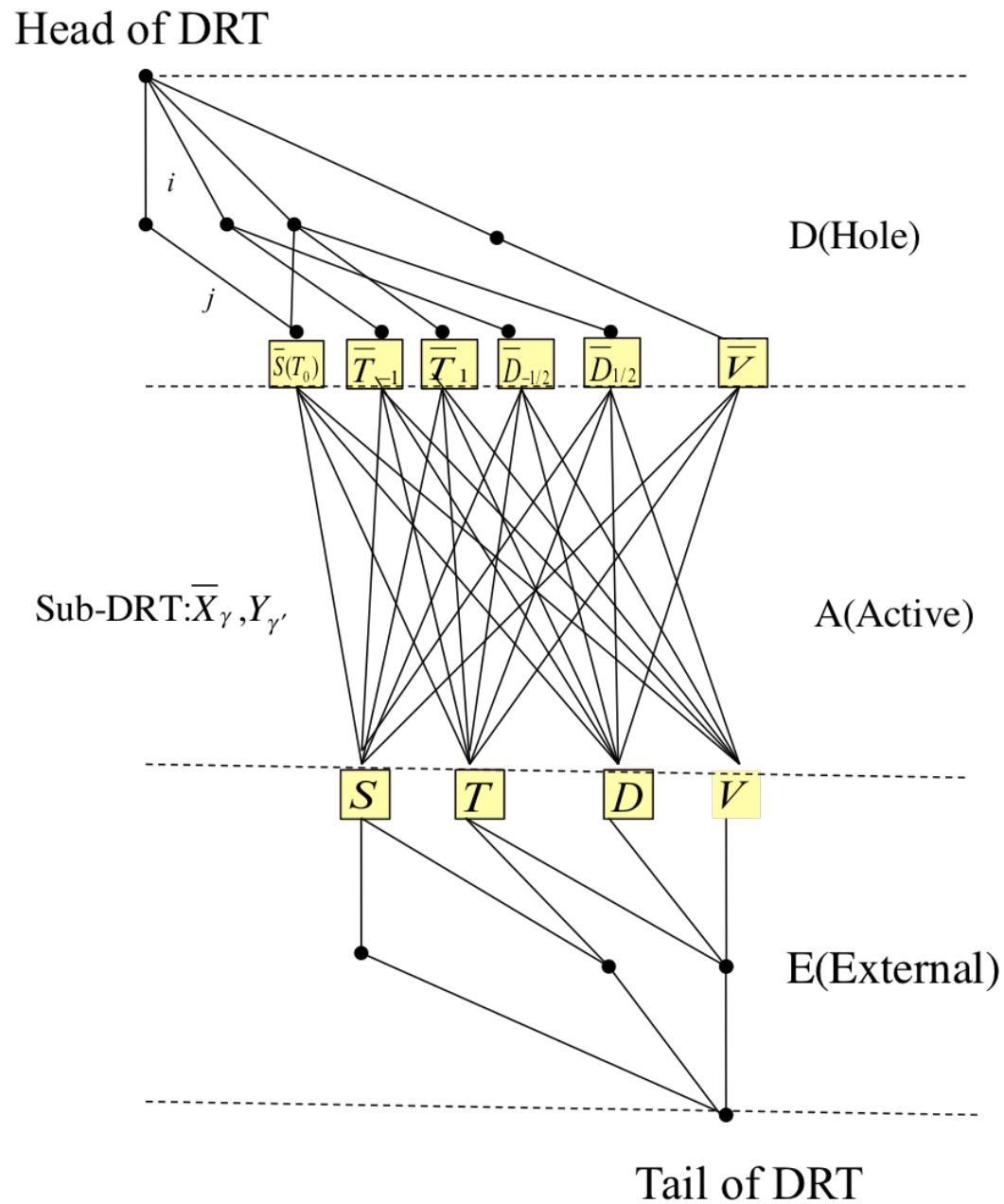
$$\begin{aligned}\Gamma_{pq,rs}^{\mu\nu} &= \langle d'_\mu | E_{pq,rs} | d_\nu \rangle \\ &= \sum_{J=0,1} \omega_J \prod_{r=\min(pq,rs)}^{\max(pq,rs)} W(Q_r, d'_r, d_r, \Delta b_r, b_r, J)\end{aligned}$$

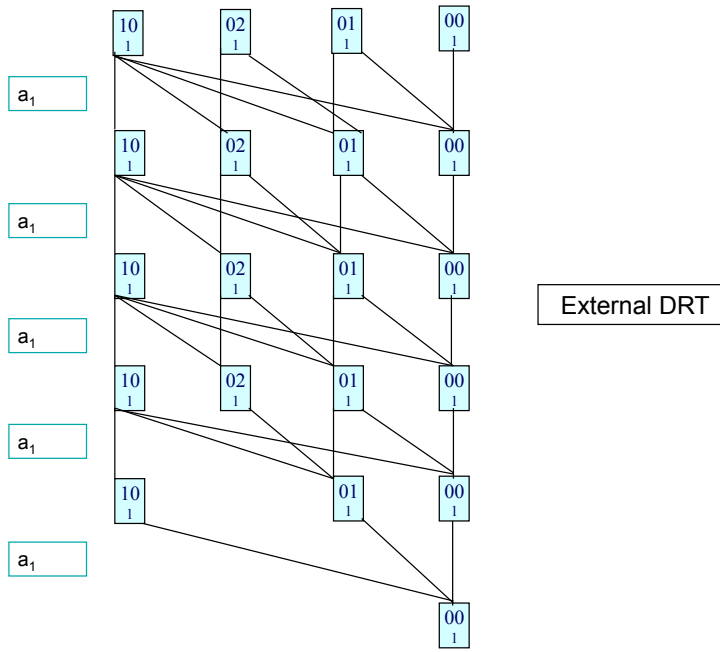


$$\Gamma_{pq,rs}^{\mu\nu} = \langle d'_\mu | E_{pq,rs} | d_\nu \rangle$$

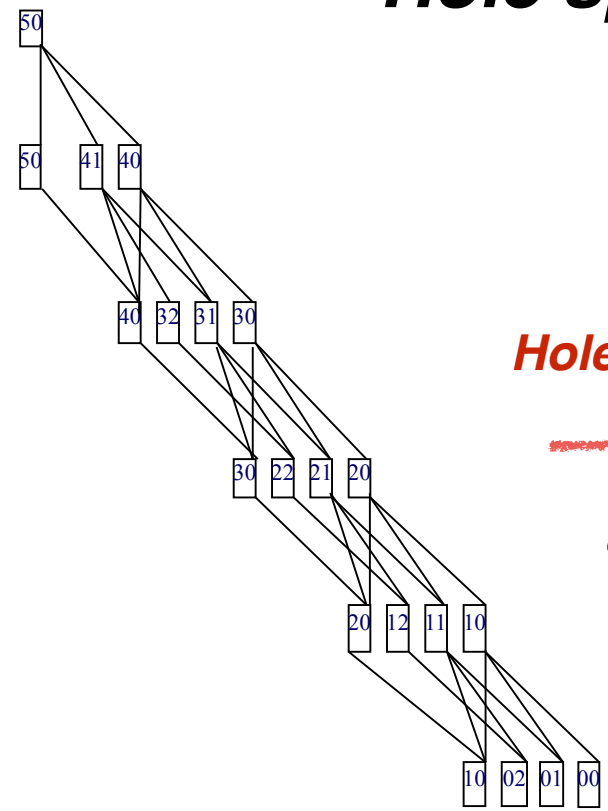
$$= \sum_{J=0,1} \omega_J \prod_{r=\min(pq,rs)}^{\max(pq,rs)} W(Q_r, d'_r, d_r, \Delta b_r, b_r, J)$$

Distinct Row Table of MRCISD





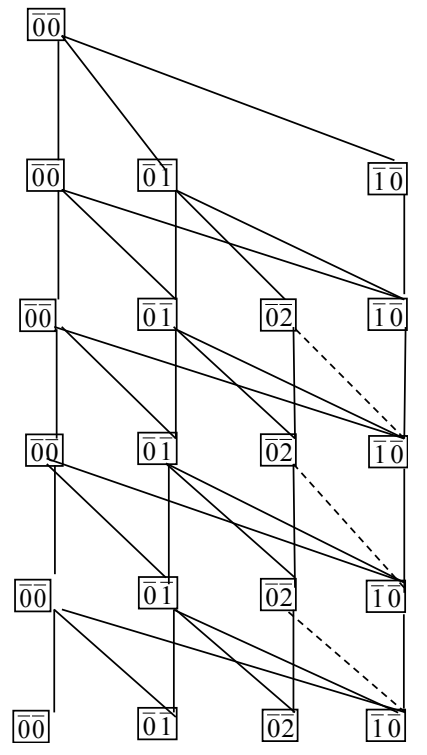
Hole space DRT



Hole-particle trans



$$\tilde{d} = 3 - d$$



$$\Gamma_{pq}^{\mu\nu} = \langle (d')_e (d')_a (d')_h | E_{pq} | (d)_e (d)_a (d)_h \rangle$$

$$= E \cdot D \cdot A$$

$$E = \sum_{r=1}^{n_e} W(Q_r; d'_r d_r, \delta b_r, b_r)$$

Partial Loops in external space

$$A = \sum_{r=n_e+1}^{n_e+n_a} W(Q_r; d'_r d_r, \delta b_r, b_r)$$

Partial Loops in active space

$$D = \sum_{r=n_e+n_a+1}^n W(Q_r; d'_r d_r, \delta b_r, b_r)$$

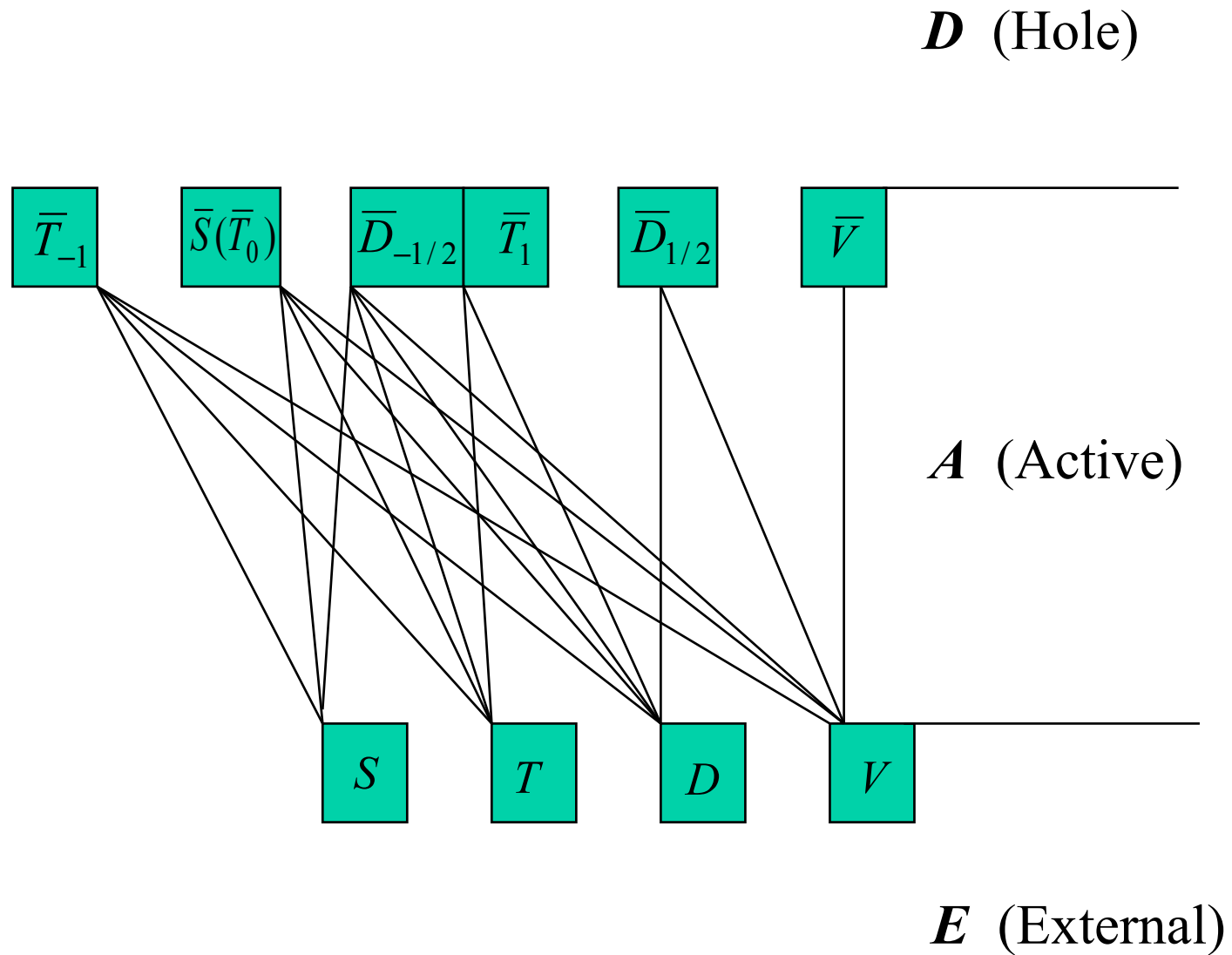
Partial Loops in hole space

E: 109 pLoop shapes

D: 244 pLoop shapes

A: Searching in active space

Programming



$$\text{Coupling coefficient} = \mathbf{D} \cdot \mathbf{A} \cdot \mathbf{E}$$

Application of Hole-particle symmetry

Molecules	CH ₂ F ₂		CH ₂ Cl ₂		CH ₂ Br ₂	
No. orbitals	55(11+4+40) ⁽¹⁾		63(19+4+40)		85(37+4+44)	
Symmetry	C _{2v}					
Active space	4 electrons in orbitals: a ₁ a ₁ b ₂ b ₂					
No. of CSFs	695,302		1,944,118		8,508,449	
	A ⁽²⁾	B	A	B	A	B
No. Partial loops	1,155,011	100,756	5,122,447	100,756	33,883,641	100,756
Storage required (Kb)	51,574	3,376	231,646	3,376	1,528,500	3,376
Time per iteration (sec) ⁽³⁾	39	24	176	101	1417	795

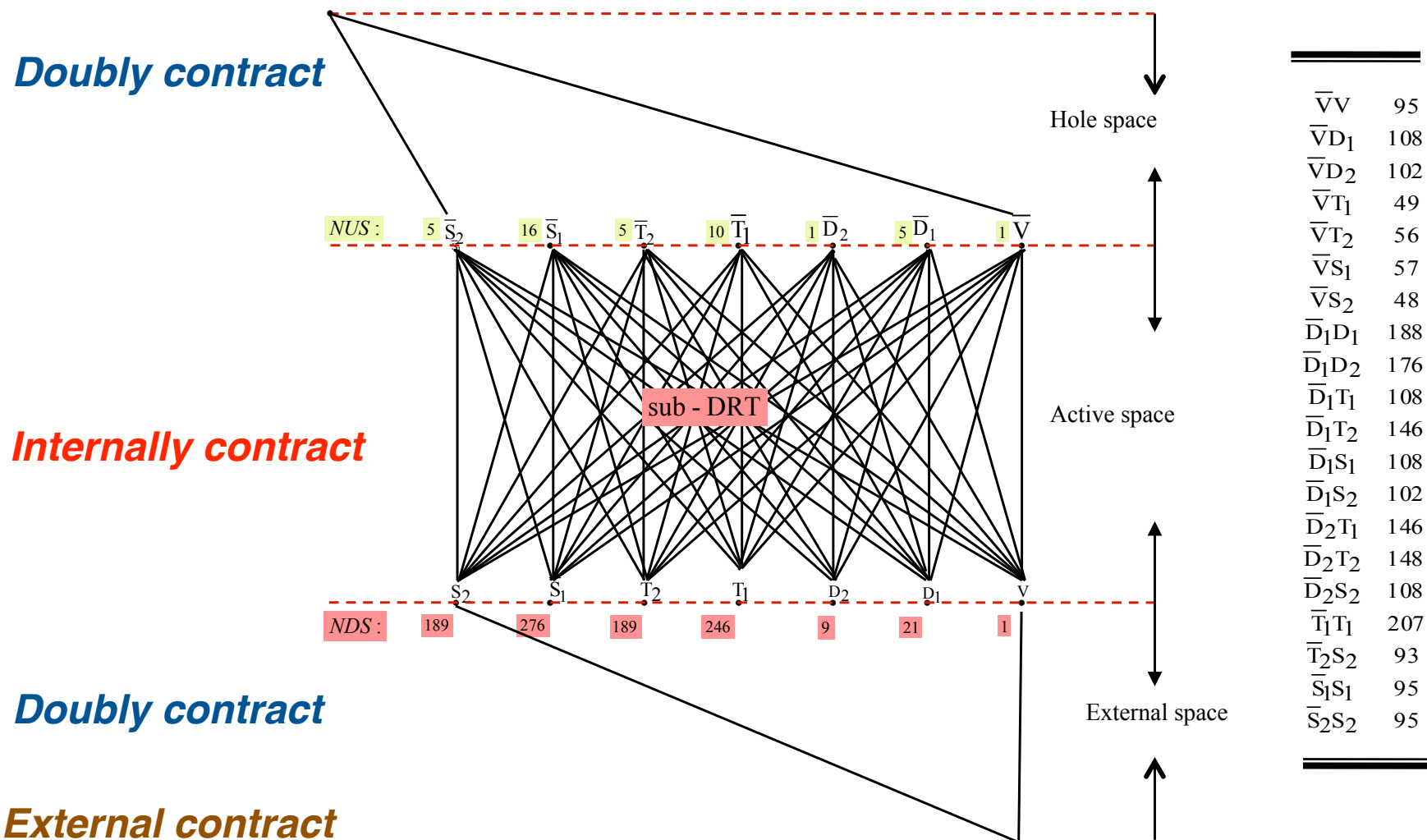
(1) In the parenthesis there are numbers of hole, active and external orbitals, respectively.

(2) A: without hole-particle symmetry; B: with hole-particle symmetry.

(3) CPU: P-4-1.6 GHz, memory: 512 Mb

Different contraction schemes based on GUGA

DRT for O_3 ()/CAS(6,6)/cc-pVDZ



icMRCI based on hole-particle symmetry

Internal contracted functions are defined **within sub-DRTs**

$$|\Phi_{ij;\bar{X}Y}^{ab}\rangle = \hat{E}_{bj,ai}|\Phi_0\rangle$$
$$|\Phi_0\rangle = \sum_{R \in ref} c_R |\Phi_R\rangle$$

Insert an identity

$$\begin{aligned}\Psi_{ij;\bar{X}Y}^{ab} &= \sum_R c_R \hat{E}_{ij}^{ab} |\Phi_R\rangle \\ &= \sum_M |\Phi_M\rangle \sum_R c_R \langle \Phi_M | \hat{E}_{ij}^{ab} | \Phi_R \rangle \\ &= \sum_M a^M |\Phi_M\rangle\end{aligned}$$

Here

$$|\Phi_M\rangle \in \bar{X}Y \quad a^M = \sum_R c_R \langle \Phi_M | \hat{E}_{ij}^{ab} | \Phi_R \rangle \quad \text{contaction coefficient}$$

icMRCI based on hole-particle symmetry

$$|\Phi_M\rangle = |(d)_e(d)_a(d)_h\rangle, |\Phi_R\rangle = |(0 \cdots 0)_e(d)_a(3 \cdots 3)_h\rangle$$

Contact coefficients can be calculated as

$$\begin{aligned} a^M &= \sum_R c_R \langle (d)_h (d)_a (d)_e | \hat{E}_{ai,bj} | (0 \cdots 0)_e (d)_a (3 \cdots 3)_h \rangle \\ &= ELS(YV) \cdot \left(\sum_R c_R \langle (d_\mu)_a | \hat{E}_{ai,bj} | (d_\nu)_a \rangle \right) \cdot HLS(\bar{X}\bar{V}) \\ &= ELS(YV) \cdot a^\mu \cdot HLS(\bar{X}\bar{V}) \end{aligned}$$

$ELS(YV)$

Segment factors in external and hole space

$HLS(\bar{X}\bar{V})$

a^μ is the fragment of a^M in the active space

Test 1 - O_3 $^1A'$ (C_s) cc-pVTZ CAS(12,9)

Method	Dimension	Energy(a.u.)	Time(min)/Iters ^c
ic-MRCI (WK) ^a	2284304	-225.083654	11.24/9
ic-MRCI (WK-like) ^a	2284304	-225.083658	79.83/14
ic-MRCI (WK) ^b	6114533	-225.125812	74.22/9
ic-MRCI (WK-like) ^b	6114533	-225.125802	146.79/14
uc-MRCI(FOIS) ^a	165247124	-225.093292	2042.87/20
uc-MRCI ^a	199281704	-225.093999	2799.00/20

a 3 frozen orbital

b No frozen orbital

c Xeon x5657 3.06 GHz, 1 CPU core

Y. Wang, H. Han. etc. JCP 141,164114,2014

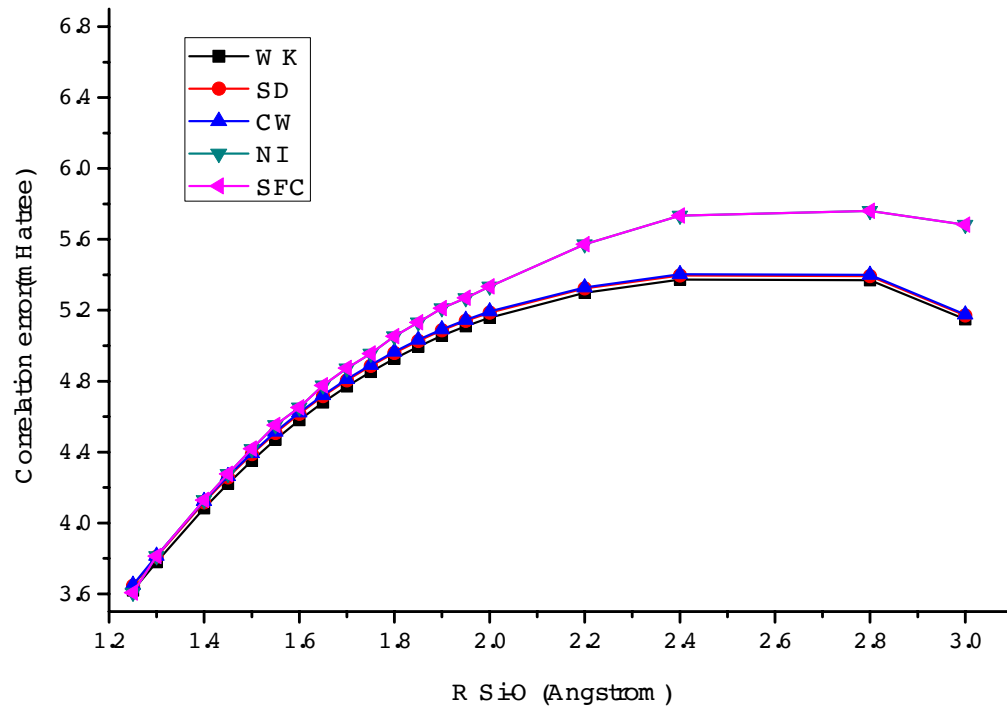
A flexible internally contracted MRCI scheme

Contraction Scheme	Contracted Sub-CI spaces	Abbrv.
Werner and Knowles	$\bar{S}(\bar{T})S(T), \bar{D}S(T), \bar{V}S(T)$	WK
SD contraction	WK contraction plus $\bar{S}(\bar{T})D$	SD
Celani and Werner	SD contraction plus $\bar{D}D, \bar{S}(\bar{T})V$	CW
No internal excitation contraction	SD contraction plus $\bar{D}D, \bar{V}D$	NI
Semi-full contraction	CW contraction plus $\bar{D}V, \bar{V}D$	SFC

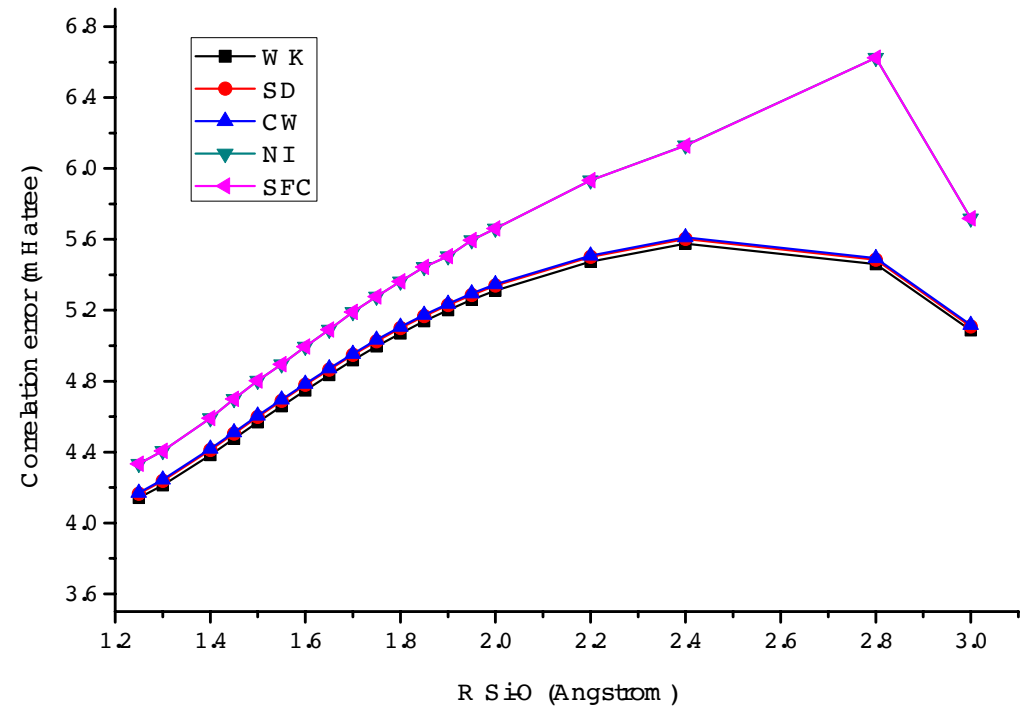
CFBr calculated by different icMRCI schemes

	<i>UC</i>	<i>WK</i>	<i>SD</i>	<i>CW</i>	<i>NI</i>	<i>SFC</i>
	CI dimension					
$1A'$	13,679,604,286	132,561,907	27,053,899	3,638,743	1,819,342	1,525,246
$3A''$	27,203,671,923	246,089,715	49,661,259	7,289,103	4,046,766	3,608,930
$1A''$	13,292,817,282	132,322,497	27,000,969	3,628,989	1,811,310	1,524,626
	Total energy (a.u.)					
$1A'$		-0.35201711	-0.35201169	-0.35200696	-0.35196129	-0.35196114
$3A''$		-0.30810780	-0.3080997	-0.30809192	-0.30762112	-0.30762102
$1A''$		-0.25015951	-0.25014786	-0.25013715	-0.24897901	-0.24897896
	Relative energy (eV)					
$1A'$		0.00	0.00	0.00	0.00	0.00
$3A''$		1.19	1.20	1.20	1.21	1.21
$1A''$		2.77	2.77	2.77	2.80	2.80

Correlation errors of PECs of SiO



$X^1\Sigma^+$



1Δ

d-d transitions of $[\text{Fe}(\text{H}_2\text{O})_6]^{3+}$

State	CASPT2(5,5)	CASPT2(5,10)	icMRCI(5,5)	icMRCI(5,10)	Expt.
6A_g	0.00	0.00	0.00	0.00	0.00
${}^4B_{1g}$	2.91	2.94	2.90(2.52)	2.53(2.31)	1.56
${}^4B_{1g}$	3.34	3.27	3.62(3.26)	3.25(3.05)	2.29
4A_g	3.74	3.77	3.97(3.66)	3.64(3.49)	3.01
4A_g	3.79	3.81	4.01(3.71)	3.68(3.53)	3.05
ΔE_{abs}	0.97	0.96	1.15(0.81)	0.80(0.61)	

Conclusion and Future development

- EN-GMFCI — treat nuclear and electron in same footing.
- icMRCI based on GUGA — much useful than uncontracted MRCI due to highly efficient.
- EN-GMF-icMRCI

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