1. $(2+t)^{2/3} = 2^{2/3}\left(1+\frac{t}{2}\right)^{2/3} = 4^{1/3}\left(1+\frac{t}{3}+o(t)\right) = \sqrt[3]{4} + \sqrt[3]{4}\,\frac{(x-2)}{3} + \underset{2}{o}(x-2)$

$\cos x = 1 - \frac{x^2}{2} + o(x^4)$

$(1+\cos x)^{2/3} = \left(2 - \frac{x^2}{2} + o(x^2)\right)^{2/3} = 4^{1/3}\left(1 - \frac{x^2}{6} + o(x^2)\right)$

$$\frac{\sqrt[3]{4} - (1+\cos x)^{2/3}}{x^2 - x^3} = 4^{1/3}\,\frac{\frac{x^2}{6} + o(x^2)}{x^2 - x^3} \longrightarrow \frac{4^{1/3}}{6}$$

2 $\int \frac{x\,dx}{1+x^2} = \frac{1}{2}\ln(1+x^2)$

$\int_0^x \frac{t}{1+t^2}\,dt = \frac{1}{2}\ln(1+x^2) \underset{+\infty}{\sim} \ln(x)$

3 $\cancel{\int_1}$ $\ln\left(1-\frac{1}{x^3}\right) \sim \frac{1}{x^3}$ $\quad \int_2^{+\infty} \frac{dx}{x^3}$ cvge $\Rightarrow \int_2^{+\infty} \ln\left(1-\frac{1}{x^3}\right)dx$ cvge (règle de Riemann)

$\ln\left(1-\frac{1}{x^3}\right) = \ln\left(\frac{x^3-1}{x^3}\right) = \ln\left((x-1)\frac{x^2+x+1}{x^3}\right) = \ln(x-1) + \ln\left(\frac{x^2+x+1}{x^3}\right) = \ln(x-1) + \underset{x\to 1}{o}(1) \underset{1}{\sim} \ln(x-1) = o\left(\frac{1}{\sqrt{x-1}}\right)$ or $\int_1^2 \frac{dx}{\sqrt{x-1}}$ cvge

$\rightsquigarrow \int_1^2 \ln\left(1-\frac{1}{x^3}\right)dx$ cvge

4 $\ln\left(1-\frac{1}{n^3}\right) \sim -\frac{1}{n^3}$ $\quad$ ~~$\sum \frac{1}{n^3}$ cvge~~ $\rightsquigarrow \sum \ln\left(1-\frac{1}{n^3}\right)$ cvge par règle de Riemann ($\sum \frac{1}{n^3}$ cvge ~~$x-2>n$~~)

$\frac{(n+1)^2}{n+2n^3} \sim \frac{n^2}{2n^3} = \frac{1}{2n}$ $\quad \rightsquigarrow \sum \frac{(n+1)^2}{n+2n^3}$ diverge par règle de Riemann ($\sum \frac{1}{n}$ diverge)

5 $\sum_{n=2}^{N} \frac{1}{\ln(n)} - \frac{1}{\ln(n+1)} = \frac{1}{\ln 2} - \frac{1}{\ln(N+1)} \longrightarrow \frac{1}{\ln 2}$ qd $N \to +\infty$

$\ln(n+1) = \ln\left(n\left(1+\frac{1}{n}\right)\right) = \ln n + \ln\left(1+\frac{1}{n}\right)$

$\frac{1}{\ln(n+1)} = \frac{1}{\ln n}\,\frac{1}{1+\frac{\ln(1+\frac{1}{n})}{\ln(n)}} = \frac{1}{\ln(n)}\left(1 - \frac{\ln(1+\frac{1}{n})}{\ln(n)} + o\left(\frac{\ln(1+\frac{1}{n})}{\ln n}\right)\right) \rightsquigarrow \frac{1}{\ln n} - \frac{1}{\ln n+1} = \frac{\ln(1+\frac{1}{n})}{\ln^2 n} + o\left(\frac{\ln(1+\frac{1}{n})}{\ln n}\right)$

$\ln\left(1+\frac{1}{n}\right) = \frac{1}{n} + o\left(\frac{1}{n}\right) \rightsquigarrow \frac{\ln(1+\frac{1}{n})}{\ln n} = \frac{1}{n\ln n} + o\left(\frac{1}{n\ln n}\right) \quad \rightsquigarrow \frac{1}{\ln n} - \frac{1}{\ln n+1} = \frac{1}{n\ln^2 n} + o\left(\frac{1}{n\ln^2 n}\right) \sim \frac{1}{n\ln^2 n}$

$\sum \left(\frac{1}{\ln n} - \frac{1}{\ln n+1}\right)$ cvge & $\frac{1}{\ln n} - \frac{1}{\ln n+1} > 0 \rightsquigarrow \sum \frac{1}{n\ln^2 n}$ cvge à la m vitesse c.a.d à la vitesse $\frac{1}{\ln(n+1)} \sim \frac{1}{\ln n}$