WORKSHOP MFG

MARCH 28-29 2017

Abstracts

L. Campi (LSE). N-player games and mean field games with absorption.

Abstract : We introduce a simple class of mean field games with absorbing boundary over a finite time horizon. In the corresponding N-player games, the evolution of players' states is described by a system of weakly interacting Itô equations with absorption on first exit from a bounded open set. Once a player exits, her/his contribution is removed from the empirical measure of the system. Players thus interact through a renormalized empirical measure. In the definition of solution to the mean field game, the renormalization appears in form of a conditional law. We justify our definition of solution in the usual way, that is, by showing that a solution of the mean field game induces approximate Nash equilibria for the N-player games with approximation error tending to zero as N tends to infinity. This convergence is established provided the diffusion coefficient is non-degenerate. The degenerate case is more delicate and gives rise to counter-examples. This talk is based on a joint paper with Markus Fischer (Padua University).

M. Laurière (Paris UPMC). Mean field type control with congestion.

Abstract : The theory of mean field type control aims at describing the behaviour of a large number of interacting agents using a common feedback. This is tightly related to the theory of mean field games but is in general different. A phenomenon that has raised a lot of interest recently is called congestion: the agents want to move while avoiding crowded regions. We will present a system of partial differential equations arising in this setting: a forward Fokker-Planck equation and a backward Hamilton-Jacobi-Bellman equation describe respectively the evolution of the density of agents and the evolution of the value function. We are able to prove the existence and uniqueness of suitably defined weak solutions, which are characterized as the optima of two optimal control problems in duality. Then, based on this viewpoint, we propose an augmented Lagrangian algorithm solving numerically this mean field type control problem with congestion. This is joint work with Yves Achdou.

R. Foguen (Nice). On selection approaches for a linear quadratic mean field game model.

Abstract : We consider a simple mean field game model having a unique equilibrium in presence of common noise and many equilibriums in absence of common noise. When there is no common noise, the question of selection arises naturally. We consider three approaches. First, amongst all equilibriums, we search for the ones with the smallest cost. Secondly, we study the associated zero-noise limit problem. Thirdly, we study the limit of the associated N-players game's equilibrium as N tends to infinity. We find that the First approach selects an equilibrium that the two others do not select.

J. Li (Shandong University). Mean-field forward and backward SDEs with jumps. Associated nonlocal quasi-linear integral-PDEs.

Abstract : In this talk we consider a decoupled mean-field backward stochastic differential equation (BSDE) driven by a Brownian motion and an independent Poisson random measure. The existence and the uniqueness of the solution $(Y^{t,x,P_\xi}, Y^{t,\xi})$ of the decoupled equation are proved. We prove that under our assumptions the value function $V(t,x,P_\xi) := Y_t^{t,x,P_\xi}$ is regular, and it is the unique classical solution of the related quasi-linear integral-partial differential equation of mean-field type with the help of a new Itô formula.

A. Lorz (Paris UPMC & KAUST). On a Boltzmann mean field model for knowledge growth.

Abstract : We analyze a Boltzmann type mean field game model for knowledge growth, which was proposed by Lucas and Moll. It describes a population of agents structured by their knowledge level. Each agent optimizes their future earnings by choosing between producing with their current knowledge level or learning to increase their knowledge. We discuss the underlying mathematical model, which consists of a coupled system of a Boltzmann type equation for the agent density and a Hamilton-Jacobi-Bellman equation for the optimal strategy. We study the analytic features of the fully coupled system. Furthermore we focus on the existence of special solutions, which are related to exponential growth in time - so called balanced growth path solutions.

This is joint work with Martin Burger and Marie-Therese Wolfram

C. Mouzouni (EC Lyon). Prediction Mean field games models : self-organization and Learning.

Abstract : We introduce a mean field games model of players looking to predict and optimize their future states, and we prove that population of this model self-organizes and reaches exponentially fast the ergodic equilibrium. We provide sufficient conditions for existence and uniqueness to the MFG system introduced (more generally for a class of coupled systems of quasilinear elliptic and parabolic equations). We give the ideas of the derivation of the mean field games model from a game with N player. And we present the proof of exponential convergence in the case of quadratic Hamiltonian, and for more general Hamiltonian in the case of one dimension. Finally, we give an interpretation of our convergence result in terms of learning in a more general framework.

M. Nutz (Columbia). A Mean Field Game of Optimal Stopping.

Abstract : We formulate a stochastic game of mean field type where the agents solve optimal stopping problems and interact through the proportion of players that have already stopped. Working with a continuum of agents, typical equilibria become functions of the common noise that all agents are exposed to, whereas idiosyncratic randomness can be eliminated by an Exact Law of Large Numbers. Under a structural monotonicity assumption, we can identify equilibria with solutions of a simple equation involving the distribution function of the idiosyncratic noise. Solvable examples allow us to gain insight into the uniqueness of equilibria and the dynamics in the population.

D. Possamaï (Paris Dauphine). A tale of a Principal and many Agents.

Abstract: In this talk we will review recent progresses on contract theory with moral hazard problems involving finitely many interacting Agents, as well as the corresponding mean-field limits when the number of Agents goes to infinity. Based on joint works with Thibaut Mastrolia and Romuald Elie.

F. Silva (Limoges). On the variational formulation of some stationary second order MFGs.

Abstract : In this talk we consider an extension of the work by A. Mészàros and myself (15') dealing with variational stationary MFGs with density constraints. We consider general Hamiltonians, satisfying a suitable growth condition, and also rather general coupling terms. For systems with and without density constraint we establish the existence of solutions using a variational technique and we also improve some of our previous results.

D. Tonon (Paris Dauphine). Variational approach for the planning problem in the mean field games.

Abstract : We consider the planning problem for a class of mean field games. This is an optimal transport problem consisting in the exact controllability at time T of Fokker-Planck equations obtained using drifts arising as the optimal feedbacks from a coupled backward Hamilton-Jacobi-Bellman equation. The starting point of the current study is that in some cases the MFG system can be understood as the optimality system of two convex optimization problems in duality. This leads to a variational analysis strategy to study the well-posedness of the PDE system. Adapting this methodology to the planning problem, we will discuss the existence and uniqueness of weak solutions of some Mean Field Games where the initial and final densities are prescribed and regular. We will also discuss the open problem where the initial and final densities are not regular.

G. Turinici (Paris Dauphine). From gradient flows to vaccination mean field games numerical schemes.

Abstract : In this talk we focus on two models based on evolution equation in metric spaces: on one hand the gradient flows and on the other hand the vaccination mean field games. The first question is how to exploit the time regularity of the solution in order to accelerate the numerical resolution of the evolution equation. The endeavor is made complex by the absence, in a metric space, of vectorial calculus invoked by e.g., Runge-Kutta high order (in time) numerical schemes. With Guillaume Legendre we proposed recently several schemes which are variational forms of the implicit midpoint scheme in the same sense as the celebrated Jordan-Kinderlehrer-Otto (JKO) is a variational form of the implicit Euler scheme.

This high order numerical schemes are then translated into the frame of Mean Field Games; first we give a mathematical meaning of the flow of a 'best reply' (or 'fictitious play') algorithms that 'learns' a MFG equilibrium; then we present a recent work by L. Laguzet on high order numerical schemes in MFG.