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Answer-Sheet 1
First steps in Scilab and Allee model

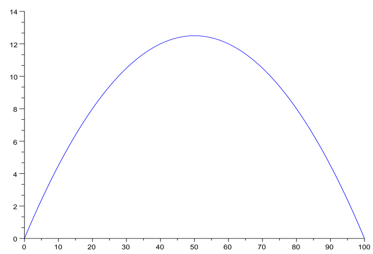
Exercice 1. : We want to study two differential equations, logistic equation

$$y' = ry\left(1 - \frac{y}{K}\right)$$

and Allee equation

$$y' = ry\left(\frac{y}{M} - 1\right)\left(1 - \frac{y}{N}\right).$$

Lets call *logistic function* the fonction $y \mapsto ry\left(1 - \frac{y}{N}\right)$ and *Allee function* the fonction $y \mapsto ry\left(\frac{y}{M} - 1\right)\left(1 - \frac{y}{N}\right)$.
Here the graph of the logistic fonction and the Scilab code used to plot it.




```
r=0.5 ;K=100 ;
function f=fLogistic(y) ; f=r*y.*(1-y/K) endfunction ;
y=0 :0.1 :K ;
plot(y,fLogistic(y)) ;
```

1. Type this code in Scilab and execute it. Look at the results together in the console and in the graphic window. Be carefull in typing `.*` instead of simply `*`, we need this to compute $f(y)$ not only for a single number y but for un vector `y=0 :0.1 :N` which means for example $y \in \{0, 0.1, 0.2, \ldots, 9.9, 10\}$ if $N = 10$.

   Modify the code in order to get the graph of the Allee function for $M = 10$ and $N = K$, first in the same window, then in a separate window.

   Outline the graph of the Allee function at the right of the one of the logistic function. Explain the difference.

2. Observe what happens if you type `a=gca() ;a.x_location="origin" ;` before the plot of the Allee function. Explain.

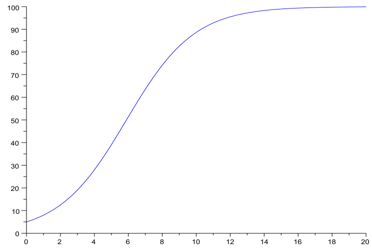
3. If (t_0, y_0) is an initial condition, the Scilab command `ode(y0,t0,t,f) ;` compute the (approximate, with 8 digits) value of the solution $y(t)$ of the equation $y' = f(t, y)$ such that $y(t_0) = y_0$, for t_0=`t0` et y_0=`y0`. Notice that `f` must be a function of two variables (here `t` and `y`), one needs to define first a *new* function

   `function f=fL(t,y) ; f=fLogistic(y) endfunction ;`

   Here is an exemple to see how it works for $y(0) = 5$ and $t \in [0..20]_{0.1}$.

   `t0=0 ;y0=5 ;t=0 :0.1 :20 ; yt=ode(y0,t0,t,fL) ; xset("window",1) ; plot(t,yt) ;`

   which gives you the following graph :



Plot yourself this picture and add to it several other solutions obtained with other initial conditions. Describe the dynamics of this model (what happens for the solution when $t \to +\infty$?).

4. Do the same for the Allee equation and explain why the dynamic is different.

5. What is the value, with 4 digits, when $t = 10$, of the solution of the logistic equation corresponding to $(t_0, y_0) = (0, 5)$?

   $y_{logis}(10) =$

6. Give the scilab code you used to compute it.

7. For the initial condition $(t_0, y_0) = (0, 200)$, is the solution of the logistique equation monotone ? croissante ? décroissante ? Explain.

8. What is the behaviour of the solution of the logistic equation when $(t_0, y_0) = (0, K)$? Explain.

9. What is the behaviour of the solution of the Allee equation when $(t_0, y_0) = (0, 15)$ for $t \in [0..20]_{0.1}$. Explain.

10. For $y_0 \geq 0$ describe the behaviour of the solution of the Allee equation of initial condition $(0, y_0)$ when $y_0 < M$, $y_0 = M$, $M < y_0 < N$, $y_0 = N$ or $N < y_0$.