COMSAT Insitut for Information Technology
Islamabad 20 April - 4 May 2012

Discrete Models for Finance and Microlending

Teaching using Scilab and the Method of Exercices Leaflets

Francine Diener, Marc Diener
Université de Nice Sophia-Antipolis (UNS)
Laboratoire UNS-C.N.R.S J-A Dieudonné,

Hedging price of a derivative contract on a stock

- Unknow future price/value of a asset
- Babylon
- Black and Scholes and TI pocket computer
- B&S and CBOT
- Nobel price in Ecomomy 1997

Hedging price of an option

· An *option* on a stock S of payoff function φ is a contract that pays $\varphi(S_T)$ at exercise time T that depends ont the (unknowen on *subscription* time $t = 0$) value S_T of the stock at this time $t = T > 0$.

· Here a trivial model and the "only rationnal" resulting *hedging* price : $S_0 = 120$, $S_T = 180$ or $S_T = 60$, derivative contract="Call" with "strike price $K = 80$ which pays $S_T - K$:

Hedging price of an option

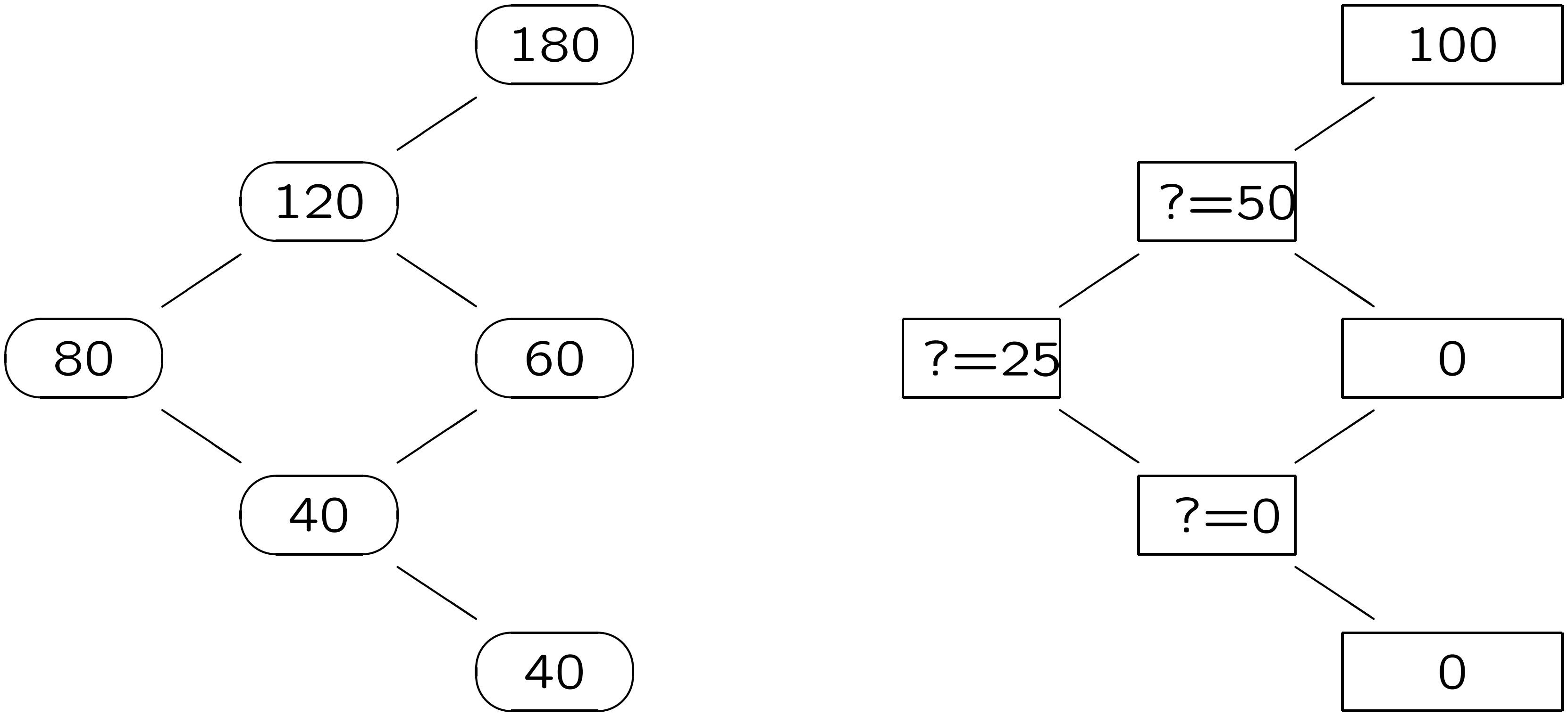
· $S_0 = 120$, $S_T = 180$ or $S_T = 60$, Call with $K = 80$ which pays $S_T - K$:

· Hedge with a portfolio balanced between a stocks and b cash (0 interest)

· Solve the linear system : you get $a = 5/6$ and $b = -5O$ (borrow cash)

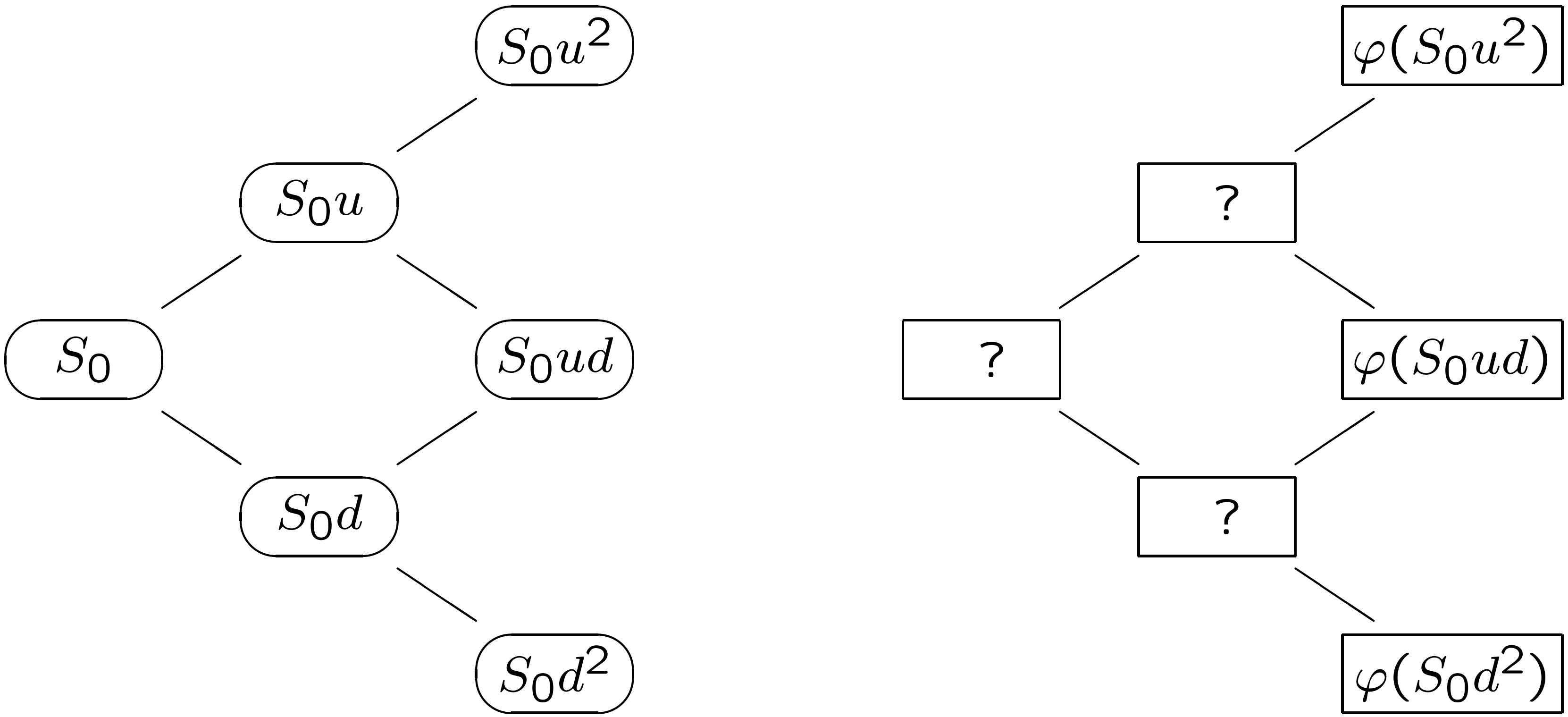
$$a * 120 + b = 120 * 5/6 - 50 = 50$$

A slightly more realistic model



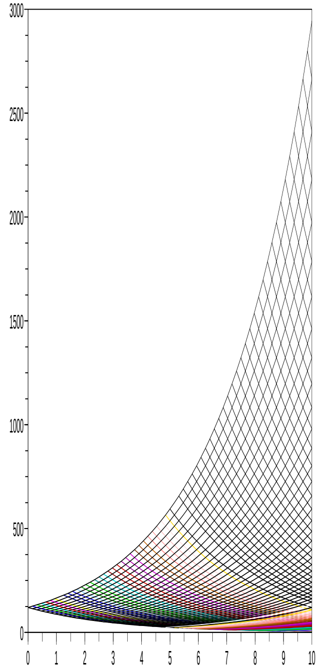
· Observe that the hedging price is always the average of the hedging price at the next step.

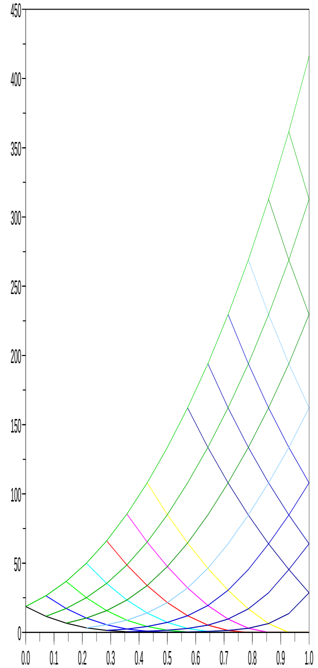
A slightly more general model

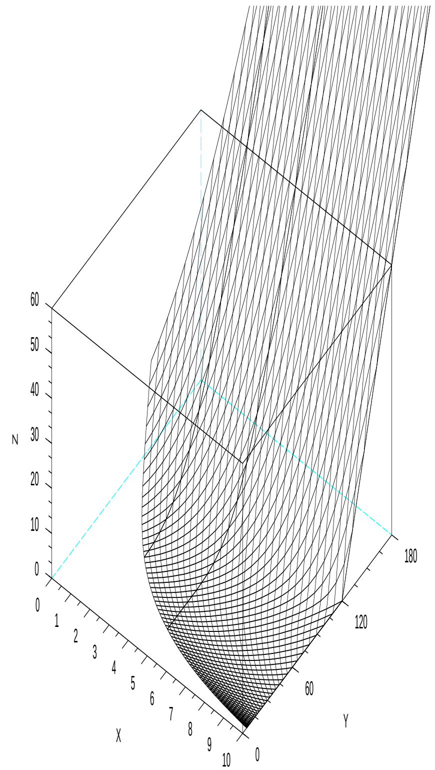


· It is easy to verify that here again the hedging price is the weighted average of the next step, using waits $p = \frac{1-d}{u-d}$ and $q = \frac{u-1}{u-d}$.

A more and more realistic model : the n time-steps CRR model



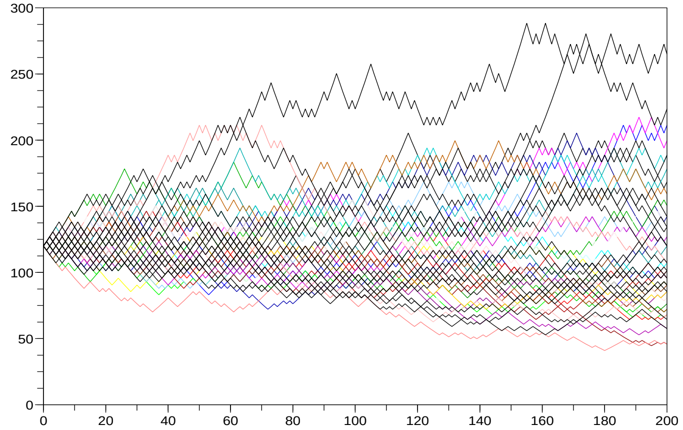


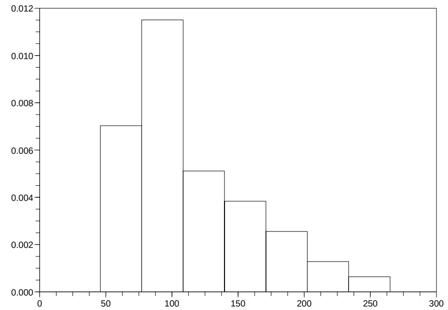


This is the grid on which the stock prices evolves with time

The n time-steps Cox Ross Rubinstein model

Here some trajectories, if at each time step we chose an "up" movement with probability 0.5 and "down" with probability 0.5.





The n time-steps CRR model

· But what is the correct probability to chose ? This will be clear as soon as we have fully discribed our model.

· Let $\delta t = T/n$; define $u = e^{\sigma\sqrt{\delta t}}$ (called "up") and $d = e^{-\sigma\sqrt{\delta t}} = 1/u$ (called "down").

·S_0 (observed "spot" price) ; $S_{t+\delta t} = S_t U_{t+\delta t}$ with $U_{t+\delta t} \in \{u, d\}$

· σ is called the *volatility* of the stock. The choice of this parametre is critical : it can be shown that the larger σ the larger the price : this allows to deduce the market-value of σ from liquid options on S. Actually, the fact that σ does not change is not realistic. Better model would introduce stochastic volatility (but the model is no longer "complete", ie does not provide exact hedging strategy, which, in turn, is very realistic).

· To the contrary to σ, the choice of n is less critical : of course too small n is not realistic. Sometimes one suggests to take $n = T =$ number of days ; in pratice, people let n tend to infinity : this leads to the Black-Scholes model that will be considered in the next session.

The n time-steps CRR model : the risk-neutral probability

· The trick is to chose $p = \frac{1-d}{u-d}$ to be the probability of $\{U_t = u\}$ and $q = \frac{u-1}{u-d} = 1-p$ to be the probability of $\{U_t = d\}$, called the *risk-neutral probability*.

· In this way the "weighted average" property observed for the price C_t of the hedging portfolio can be written $C_t = E_t^*(C_{t+\delta t})$ where E_t^* stands for *expectation, for the risk-neutral probability, conditionaly to all what has been observed between* O and t.

· Deciding that all U_t are independent allows to express the initial price of the hedging portfolio C_0. It is given by $C_0 = E^*(\varphi(S_T))$, and, when there is an interest r to pay on borrowed money, the formula becomes

$$C_0 = e^{-rT}E^*(\varphi(S_T)).$$

`Scilab`

- It's time now to check the ideas on computer numbers and pictures.
- We shall use `Scilab`, a free analog of `Matlab`.
- It can be downloaded from `http://www.scilab.org`
- ... or just ask Google for `download scilab` .

Thank You for your attention

.