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Dynamical Systems
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Dynamical Systems : Answer-sheet 4
Qualitative study of differential systems

Exercise 1 : We consider the following differential system :

$$\left\{ \begin{array}{rcl} x' & = & x(1-\frac{x}{2}) - \frac{1}{3}xy \\ y' & = & y(1-\frac{y}{2}) - \frac{1}{3}xy. \end{array} \right. \tag{1}$$

1. Draw, in $\{x \geq 0, y \geq 0\}$, the two *isoclines* : $x' = 0$ (in red) and $y' = 0$ (in blue or black), so as the directions of the vector field in the four region separated by these isoclines.

2. Compute the coordinates of equilibria of the system

3. Compute the linearized system near the equilibrium that has no zero coordinate.

4. Infere the nature of this equilibirum.

Exercise 2 : Two companies X and Y which at each time t produce $x(t)$ and $y(t)$ respectively, share an emerging market. They do not produce the same product but a consumer who buys one with not buy the other.

We assume that the expansion of each company, if the other would not exist, exhibits an intrinsic expansion rate and a carrying capacity related with the limitations of the market. We also assume that the existence of the other company reduces the expansion rate which depend linearly of the production of the other company.

So this leads to the following system to model the dynamic (r, s, K, L, k, and l are all positive constants) :

$$\begin{cases} x' &= rx(1-\frac{x}{K}) - kxy \\ y' &= sy(1-\frac{y}{L}) - lxy \end{cases} \tag{2}$$

1. How are called the constants K and L ? How are denoted the intrinsic rate of expansion of each company ?

2. How is called is the dynamic of company X in absence of company Y ? How is modelled the interaction of the two companies ?

3. Give the limiting production of X when the company Y has no production ; check that this equilibrium is stable.

4. In the special case when $K = 2 = L$, $k = \frac{1}{3} = l$, and $r = 1 = s$, compute the linearized system at the equilibrium with no zero coordiante. Can you see if, for these constants, both companies will survive or not ?