LAST NAME : Date : .
FIRST NAME : Groupe :

Applied Mathematics to biology: Answer Sheet of TD 1
Introduction to Markov chains

We will answer questions as clearly as possible in the spaces provided and will give the answer sheet at the end of the session to the teacher in charge of the Course/TD.

Exercice 1. : We study the evolution over time of vegetation on a vast territory by breaking them down to simplify into three categories , *heath*, *scrub* and *forest*. We model this dynamic by a Markov chain X_t state space $S = \{l, m, f\}$ and transition matrix:

$$\mathbb{P} = \begin{pmatrix} 0,4 & 0,6 & 0 \\ 0 & 0,2 & 0,8 \\ 0,35 & 0 & 0,65 \end{pmatrix}$$

1. Draw the diagram by points and arrows associated.

2. What is, according to this model, the probability that the population changes from the scrub to the state *forest*?

3. Calculate the probability of a trajectory of the type $X_0 = f, X_1 = f, X_2 = l, X_3 = m$ according to $\pi_0(f)$.

4. Give an example of trajectory of probability zero.

5. What is, according to this model, the probability that the population moves from l to m in one step? In two-steps?

6. Knowing the initial allocation $\pi_0 = (0,3 \;\; 0,4 \;\; 0,3)$, calculate the allocation to the next step π_1. The three vegetation types, which are progressing, which are decreasing?

Exercice 2. **:** It is assumed[1] that you are interested in a forest composed of two species of trees, E1 and E2. When a tree dies, a new grows in its place but it can be of either of the two species. Those of the first species with a long life, it is assumed that only 2% of them die each year while the rate is 4.5% for the second species. But the latter growing more quickly will succeed more often to occupy a vacant place: it is assumed that 65% of vacancies are made by a second species of the tree against only 35% for the first tree species.

1. We model the dynamics of this forest by a Markov chain $(X_t)_{t\geq 0}$ with two states $E1$ and $E2$. Justify the following formula $P(X_{t+1} = E_1/X_t = E_1) = 0,98 + 0,02 \cdot 0,35$ and then calculate the same $P(X_{t+1} = E_2/X_t = E_2)$.

2. Derive the transition matrix $\mathbb{P}$ of the Markov chain.

3. Draw the diagram by points and arrows associated.

4. If one begins with a population of 100 trees of the species E1 and 900 of the species E2, how many will there be trees of the species E1 after one step, after two steps?

5. Calculate the image of the distribution $\pi_0 = (0,1 \ \ 0,9)$ by this chain of Markov.

6. Repeat the last two questions assuming that there is initially a proportion of five trees in the first specises against three of the second.

[1]Extract from the book "Mathematical Models in Biology", E.S. Allman and J.A. Rhodes, Cambridge University Press, 2004