LAST NAME : Date : .
FIRST NAME : Groupe :

Mathematics for Biology: Answer Sheet of TD 2
Evolution towards a stationary distribution

We will answer questions as clearly as possible in the spaces provided and will give the answer sheet at the end of the session to the teacher in charge of the Course/TD.

Exercice 1. [1] **:** An individual lives in an environment where it is likely to catch a disease by insect bites. It may be in one of three states: neither sick nor immune (R), ill (M) or immune (I). One month to another, its status may change according to the following: being immunized, it can be with a probability of 0.9, or go to the state R with probability 0.1, being ill, he may remain with probability 0.2 or become imunis with probability 0.8, and finally being in the R state, he can stay with probability 0.5, or become sick with probability 0.5.

1. Describe a Markov chain with state space $S = \{R, M, I\}$ to model the population belongs to this individual.

2. Calculate the proportion of diseased individuals in the population after one month if 1% of individuals were ill at first and not sick or other immune.

3. If one calculates with the computer power P^{20} of the transition matrix, we find (keeping only the 3 decimal places) $\mathbb{P}^{20} = \begin{pmatrix} 0.151 & 0.094 & 0.755 \\ 0.151 & 0.094 & 0.755 \\ 0.151 & 0.094 & 0.755 \end{pmatrix}$ Deduce the approximate value of π^* a stationary distribution for this Markov chain . The transition matrix is a matrix-it primitive?

4. Indicate what will be the proportion of sick individuals in this population over time. The proportion of immunized individuals did reduced or increased?

5. What would happen under this model in a region where the proportion of patients would be 50% at baseline? Can we predict an epidemic in this case?

[1](Exercise inspired on the text online at http ://www.apprendre-en-ligne.net/graphes/markov/index.html)

Exercice 2. **:** We want to study the effect of the presence of a pair of lions in a savannah portion in which three people live animals including lions feed. We model the prey, antelopes (a), wildebeest (g) and zebrafish (z) as the states of a Markov chain whose trajectories are successions of prey eaten by lions, for example (gzzaggaa). We assume that the probability that a lion eats a prey (or g or z) after eating a prey g (or a or z) depends only on a (or g or z) and not that 'he has eaten before (and that this probability is invariant over time). Hence the modeling by a Markov chain state space $S = \{a, g, z\}$ which we propose the following transition matrix:

$$\mathbb{P} = \begin{pmatrix} 0.5 & 0.1 & 0.4 \\ 0.2 & 0.3 & 0.5 \\ 0.2 & 0.2 & 0.6 \end{pmatrix}$$

1. What is, according to this model, the probability that the lions eat a zebra after eating a antelope?

2. Draw the diagram by points and arrows.

3. Trajectories of the two following $(zaag)$ and $(Zaga)$, which is most likely? Justify your answer by calculation.

4. The following π_0 distribution is a stationary distribution for this chain of Markov? Justify your answer.

S	a	g	z
π_0	$\frac{6}{21}$	$\frac{4}{21}$	$\frac{11}{21}$

5. If this portion of the population of savannah antelope is initially much higher than the other two types of prey, will she, according to this model, decrease, increase or remain dominant? Explain. If in this portion of the population of savannah antelopes is initially much higher than that of the two other types of prey, , according to this model, decrease, increase or remain paramount? Explain.