On the implicit Interest Rate in the Yunus Equation

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Abstract

In his book with Alan Jolis, Vers un monde sans pauvret (1997) Yunus gives the example of a microcredit loan of 1000BDT reimbursed via 50 weekly settlements of 22BDT and correctly claims that this corresponds to the annual interest rate of 20%. But this is without taking into account that if the borrower has good reasons not to pay at one installment, she can postpone of one week all remaining settlements, under the same conditions, so without extra cost. This of course leads to a lower implicit interest rate. Introducing a simple geometric law model for the time between settlements, this turns the implicit interest rate into a random variable, whose laws is still unknown but for which we provide simulated empirical distribution density function. Defining by actuarial expected rate the real number $\bar{r}$ that satisfies the expectation of the random Yunus equation, we compute this number as a function of the probability p of in-time installment. This allows in turn to compute the implicit probability p which is to the value of p corresponding to the observed 3% default rate, where in practice, "default" means "more than four weeks delay". The mathematical tool used is the probability generating function, the computer tool is the Scilab algebraic equation solver.

1 Introduction : microcredit

Microcredit has played important role in poverty alliviation, empowered those marginalized and it has been recognized as the most significant innovations in development policy. Microcredit is defined as the provision of very small loans (microloans) to very poor people designed to spur small businesses, entrepreneurship, or other generating income activities. The basic idea came from the finding that a large part of humanity has no access to traditional credit because banks require their borrowers to meet a range of criteria, such as being able to read and write, bears some identification documents, collateral, steady employment,

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a verifiable credit history, or to have already secured a minimum deposit. For the mico-borrowers, microcredit is often the only alternative to paying usury debt which is excessive interest rates charged by pownshops or unofficial moneylenders.

The first experiments date back to the 70s in Bangladesh as an initiative of Muhammad Yunus, then a professor of economics at Chittagong University. In 1974, he watched helplessly as a terrible famine in the little village Joha near to his University. He then with his students asks the craftsmen and peasants of the village in order to try to understand their needs and lists a demand for small loans for 42 women to whom he finally decides to pay himself a total of about 27 Euros. Then he spends nearly 10 years trying to persuade banks to take on these loans before finally deciding to start his own bank, the Grameen Bank in 1983. This bank and himself receive the Nobel Prize for Peace in 2006 for their effort to create economic and social development from below. Currently microcredit activity has spread to most countries in the world, it is ensured by close 10 000 Microfinance Institutions (MFIs) who lend 50 billions euros to almost 500 millions beneficiaries.

It has been noticed that the target clients are the wives and mothers of landless and small-scall farmers. The share of female debtors are especially in Asia as high as 99%[1]. The majority of women reflects the fact that women are more reliable debtors due to the stronger social and family ties; they often follow conservative investment strategy which result in lower default rates for MFIs. The repayment rate is very high close to 100%. The micro-lending institutions usually refrain from taking collateral. In order to ensure the borrowers' descipline, many MFIs apply the principle of grouplending as the group is jointly liable for the failure of any single member to repay the loan. This allows the MFIs lend a small loan to an individual who belongs to a group of 5 to 30 people. As the individual borrower proves reliable, the next credit is extended to additional members in the group. Some microlenders use dynamic incentive mechanism to borrowers by threatening to exclude defaulting borrowers from future access to loans and the possibility of a new loan granted automatically in case of timely repayments.

The microcredit has been accepted as a tool for assisting the poor in developing countries in the last 25 years; on the other hand, the interest rates charged on this credit is still one of the most discussed issues capturing the attention of governments, medias and industry analysts. The interest rates charged by MFIs are very high range from 15% to 70% annually[2] and vary significantly according to the geographic regions; for example, in India, microloands are usually granted at 15% to 30% per year;in Bangladesh and Indonesia, the main institutions keep interest rates below 50%, typically around 30%.

The MFIs claim that the main reason why microcredit interest rates are higher than those of non-microfinance institutions, such as traditional commercial banks is because of the higher operating cost necessary to deliver such small loans, including administrative and personel expenses; without taking into the consideration of some difficulties of the IMFs when the loans are not paid or there are some delays in paying back the loans with good reasons of the borrowers and without imposing any penalties or extra charges by the MFIs.

[1]Benchmarking Asian Microfinance 2005.The MIX

[2]Deutsche Bank Research, December 19, 2007

2 The Yunus polynomial and equation

The following has been given by Muhammad Yunus [6][5]: Grameen lends 1000 BDT (Bangladesh Taka) to borrowers that pay back 22 BDT [3] each week during 50 weeks. Let us denote by r the annual continuously compound interest rate. The present value of the 22 BDT refunded after one week is $22e^{\frac{-r}{52}}$ this value of those of the next payment is $22e^{\frac{-2r}{52}}$ ... and so on. So, letting $q = e^{-\frac{r}{52}}$, as the 50 installments to balance the 1000 BDT immediately received by the borrower, we get following equation for q :

$$1000 = 22\sum_{k=1}^{50} q^k = 22\frac{q-q^{51}}{1-q} \tag{1}$$

that reduces to (the degree 51 polynomial equation) $\mathcal{Y}(q) = 0$ where $\mathcal{Y}$ denotes what we shall call *Yunus polynomial*

$$\mathcal{Y}(q) := 22q^{51} - 1022q + 1000. \tag{2}$$

We observe that $\mathcal{Y}$ has obviously $q = 1$ as a zero, has two other zeros $q_- < 0 < q_+ < 1$, and all other zeros are complex conjugate. An approximation of q_+ gives $q_+ = 0.9962107\ldots$ which leads to $r = 19,74\ldots$, so nearly 20%.

But some borrowers don't pay in time, so the n-th payment takes place at some random time $T_n = T_{n-1} + \frac{1}{52}X_n = \frac{1}{52}(X_1 + X_2 + \ldots + X_n)$

Lets assume $(X_i)_{i=1..50}$ i.i.d, $X_i \rightsquigarrow \mathcal{G}(p)$, the geometric distribution, $p = \mathbb{P}(X_i = 1)$, close to 1 ; in other words each week the borrower has probability p to be able to pay in time the 22 BDT installment that she should pay, weekly refunding accidents being assumed to be independent. We call the probability as *in-time installment probability* or shorter word *installment probability* which is the probability that she is no longer delay to pay the installment. So r becomes a random variable, $R = r(X_1, \ldots, X_{50})$, satisfying the "Yunus equation" :

$$1000 = \sum_{n=1}^{50} 22e^{-\frac{R}{52}(X_1+\ldots+X_n)} \left(= 22\sum_{n=1}^{50} v^{R(X_1+\ldots+X_n)} \quad , \quad v = e^{-\frac{1}{52}} \right). \tag{3}$$

For the sake of getting a better understanding of the risks faced by the lender under these new asumptions we wish to have informations on the probability law of the random variable R. The sequence of this paper is devoted on the results we got so far.

[3] The value of 100 Bangladesh Taka (BDT) is about 1 Euro.

3 Actuarial expected rate

Lets call *actuarial expected rate* the positive real number $\bar{r}$ such that, replacing R by $\bar{r}$, it satisfies the expectation of the Yunus equation :

$$
\begin{aligned}
1000 &= \mathbb{E}\left(\sum_{n=1}^{50} 22 v^{-\bar{r}(X_1+\ldots+X_n)}\right) \quad , \quad v = e^{-\frac{1}{52}} \\
&= 22 \sum_{n=1}^{50} \mathbb{E}\left(v^{-\bar{r}X_1}\right) \ldots \mathbb{E}\left(v^{-\bar{r}X_n}\right) \text{ , as } X_1 \ldots X_n \text{ are independent} \\
&= 22 \sum_{n=1}^{50} \bar{q}^n = 22 \; \frac{\bar{q} - \bar{q}^{51}}{1 - \bar{q}} \text{ , with } \bar{q} = \mathbb{E}(e^{-\frac{\bar{r}}{52} X_1}) = M_{X_1}\left(-\frac{\bar{r}}{52}\right),
\end{aligned}
$$

where $M_{X_1}(t) = \frac{pe^t}{1-(1-p)e^t}$ is the moment generatring function of $\mathcal{G}(p)$. So $M_{X_1}\left(-\frac{\bar{r}}{52}\right) = \bar{q} = q_+$, the positive non trivial zero of the Yunus polynomial, which leads to

$$
e^{-\frac{\bar{r}}{52}} = \frac{q_+}{q_+ + p(1-q_+)} = \frac{1}{1 + p\left(\frac{1}{q_+} - 1\right)}.
$$

So

$$
\bar{r} = 52 \ln\left(1 + p\left(\frac{1}{q_+} - 1\right)\right) \tag{4}
$$

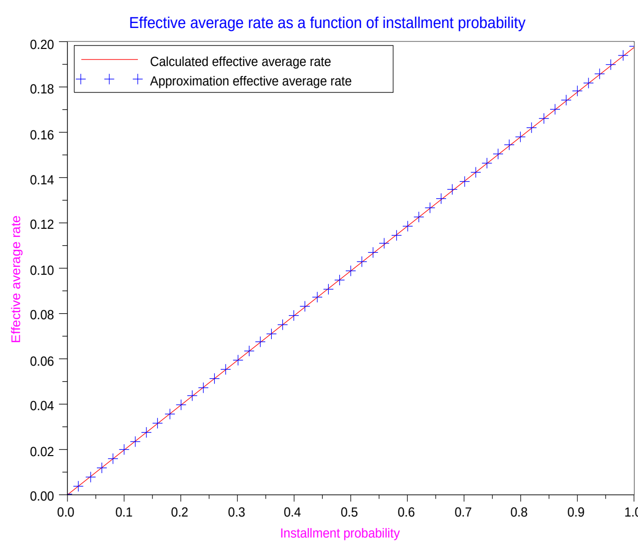

Figure 1: Expected interest rate as a function of installment probability

In figure 1, the graph looks like a straight line, since $\ln\left(1 + p\left(\frac{1}{q_+} - 1\right)\right) \approx p\left(\frac{1}{q_+} - 1\right)$ as p tends to zero.

4 Random Yunus Equation and experimental results

In the process of finding the law of *the random interest rate* R in the equation (3), with the assumption that the time to start repaying after the n^{th} installment follows the geometic law with the installment probability p, we used Scilab to simulate the distribution of the interest rates by giving different values to p and we observe the below of each histograms as in figure 2.

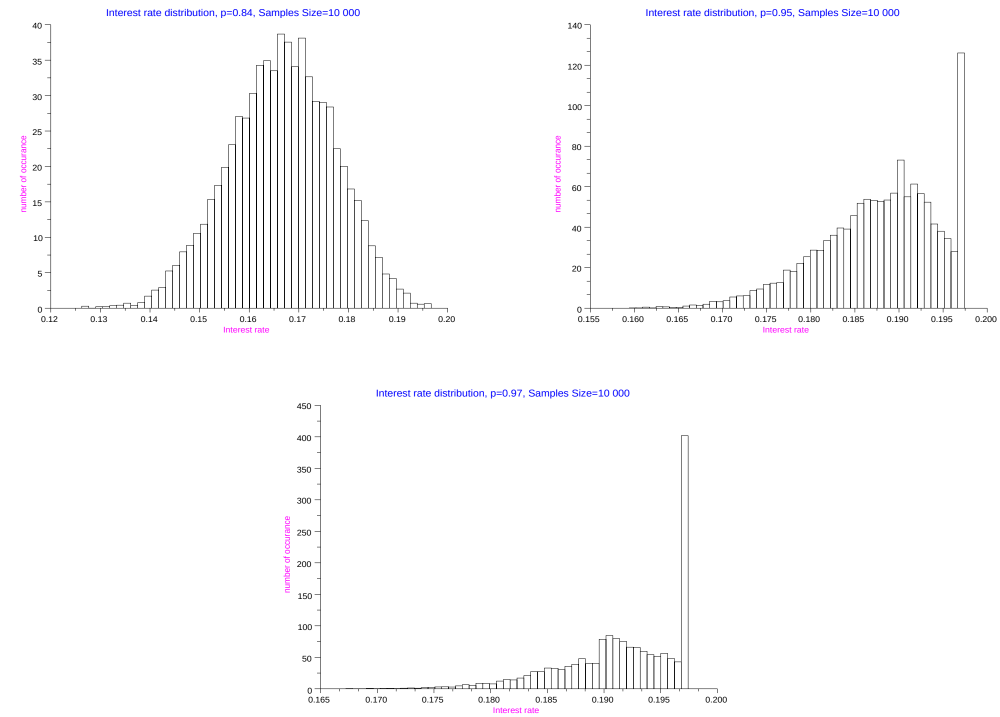

Figure 2: Interest rate distributions, $p = 0.84, 0.95$ and 0.97, sample size =10 000

So the question is (and stays, up to here) : what is the law of R ?

5 Relationship beween the installment probability and no-default probability

Default risk is one of the vital components for lending decision and it is the same for micro-lending and even more obvious when the clients are lack of collateral or other security pledged for the loan. *Default* in microcredit is defined as the failure of borrowers' repayment and usually the lender is not able to recover the debt. For many MFIs who ultilize the method of collecting the weekly installment, if the debtor fails to repay the amount after a certain number of weeks (usually four weeks), the loan is treated as a default. While there is a concern in the default rates of microloans, there are claims that the credits have historically high repayment rates up to 97%. We call the repayment rate as *no-default probability* which is the complement of the probability of default.

Here, the relationship between the installment probability and probability of no-default is taken into the consideration. Let d be *the maximal time of no default* i.e. the maximum number of weeks that the borrower does not yet pay the installment till the credit is treated as a default, and γ be the *no-default probability.* For our model, the probability of no-default can be written as

$$\begin{aligned}
\gamma &= \mathbb{P}\left(Max\left\{X_1,\ldots,X_{50}\right\} \leq d\right) \\
&= \mathbb{P}\left(\bigcap_{i=1}^{50}\{X_i \leq d\}\right) \\
&= \prod_{i=1}^{50}\mathbb{P}\left(\{X_i \leq d\}\right) \qquad \text{since } X_i\text{'s are i.i.d and } X_i \rightsquigarrow \mathcal{G}(p) \\
&= \prod_{i=1}^{50}\left[p(1-p)^0 + p(1-p)^1 + \ldots + p(1-p)^{d-1}\right]
\end{aligned}$$

Therefore, we obtain the installment probability :

$$p = 1 - (1 - \gamma^{\frac{1}{50}})^{\frac{1}{d}} \tag{5}$$

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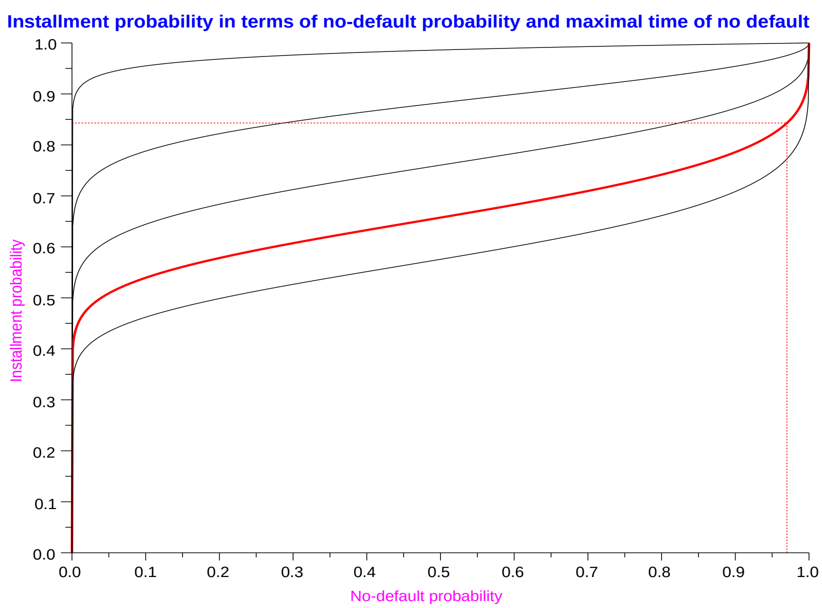

Figure 3: Installment probability as a function of no-default probability, $d = 1, 2, 3, 4\ ,5$

In practice, $d = 4$ weeks, $\gamma = 97\%$, then $p = 84\%$ which leads to the actuarial expected interest rate $r \approx 16.59\%$.

Conjecture 1. *The random interest rate is equal to the effective expected interest rate i.e.*

$$\mathbb{E}(R) = \bar{r} \tag{6}$$

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