How to fit a jump diffusion model to return prices

FRANCINE DIENER AND RATTIKAN SAELIM

Laboratoire J.A. Dieudonné
University of Nice-Sophia Antipolis (France)

October 2017

Black-Scholes model versus Merton model

Introduced by Merton in 1976, jump diffusion models are used in finance to capture discontinuous behavior in asset pricing or spot commodity pricing.

- In a Black-Scholes model, the prices evolve like a geometric Brownian motion with drift μ and volatility σ

$$S_t = S_0 \exp\left((\mu - \frac{\sigma^2}{2})t + \sigma B_t \right)$$

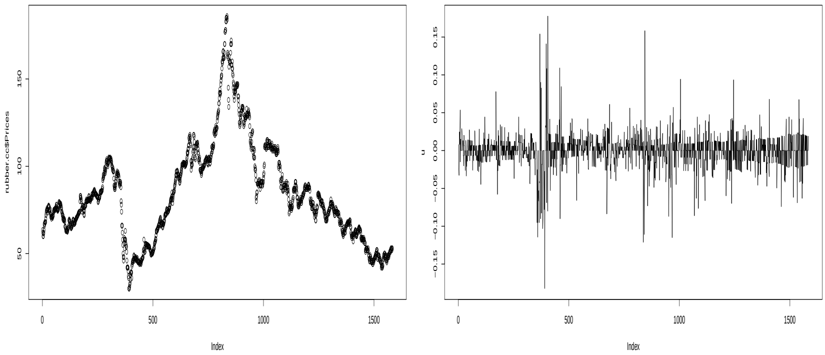
- In a Merton jump diffusion model, the prices evolve, between the jumps, like a geometric Brownian motion, and after each jump, the value of S_t is multiplied by e^{Y_i}

$$S_t = S_0 \exp\left((\mu - \frac{\sigma^2}{2})t + \sigma B_t + \sum_{i=1}^{N_t} Y_i \right)$$

where $(N_t)_{t\geq 0}$ is a Poisson process with intensity λ and the jump sizes $(Y_t)_{t\geq 0}$ a sequence of iid normally distributed r.v. with mean m and standard deviation s.

An exemple: Thai natural rubber prices

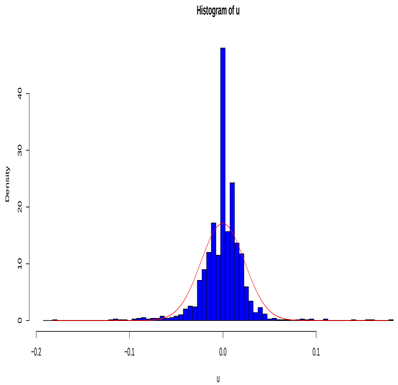
The first plot is the rubber prices from 03/01/2007 to 31/10/2016 on Hat Yat market(more than 1500 dates) and the second the rubber logreturns



When the prices S_t follow a geometric Brownian motion (BS model), the logreturns $R_t = \ln\left(\frac{S_{t+\delta t}}{S_t}\right)$ are normally distributed with mean $(\mu - \frac{\sigma^2}{2})\delta t$ and standard deviation $\sigma\sqrt{\delta t}$.

Distribution of returns

Rubber returns: the jumps appear through the heavy tails

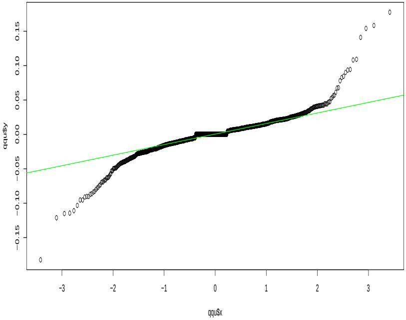


(the huge peak is probably due to the lack of liquidity (a totally different problem....))

How to extract the tails from the sample?

- In order to be able to estimate the 5 parameters of the Merton jump diffusion model, the two of the diffusion part μ and σ, and the three of the jump part, λ, m and s, we would like to separate the returns corresponding to Brownian increments from the returns corresponding to jumps.
- It is possible to decide on a threshold ... but this seems arbitrary!
- The main problem is how to distinguish small jumps from large Brownian increments?
- This difficulty is one of the main reasons why practitioners give up using jump diffusion models, even in commodity pricing where the existence of jumps looks quite realistic.

What can we learn from a qqplot?

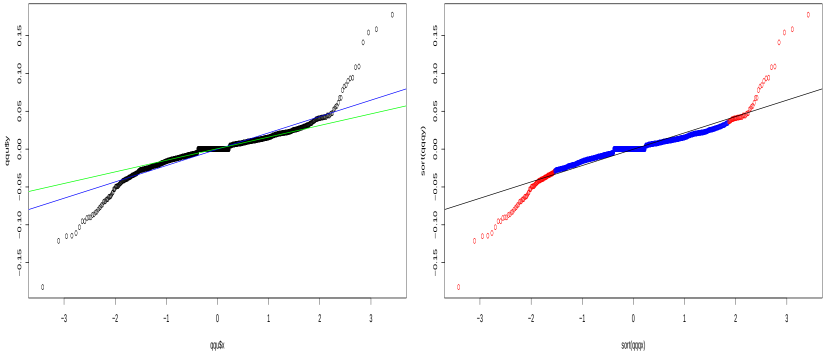


(Here the peak has become a plateau)

For a gaussian distribution all points should be on the line

Each positive jump corresponds to a point clearly above the green line (and similar for negative jumps)

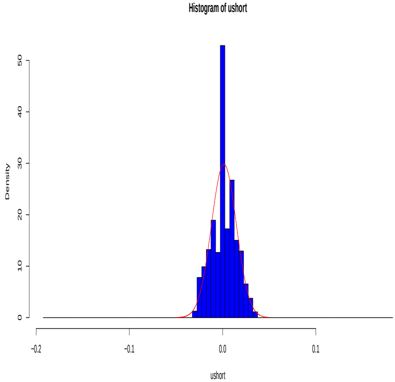
Result



Left: the same qqplot but with the (blue) linear model in addition to the (green) "qqline"

Right: according to our computations, 147 logreturns out of 1583 are considered as jumps (namely the first 99 and the last 50).

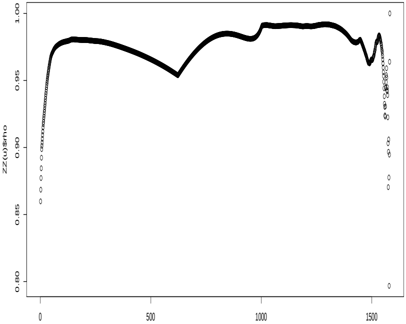
New distribution for the diffusion part (without the jumps)



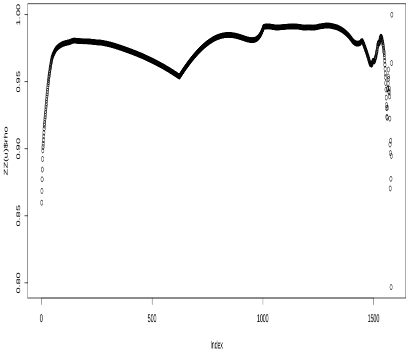
Algorithm we use

- Compute R^2 for all logreturns except the first and do the same for all logreturns except the last.
- Remove either the first logreturn or the last one depending on what leeds to the highest R^2.
- Repeat for the sample with one data less until the sample is reduced to 2 points.

Keeping the R^2 at each step, we get the following evolution:



When to consider all jumps are removed?



We chose the first maximum of this evolution of R^2 as a fonction of the number of steps.

How the method is working on simulations?

- With this algorithm, we were able to compute from a sample of prices an estimate of the 5 parameters of a possible jump diffusion model of their logreturns
- We have tested the algorithm on simulations of trajectories of a jump diffusion model and it seems to work well
- We do not have yet any proof of the convergence of these estimators (work in progress)
- I do not know if the method can really become useful for practitioners ... but, in any case, it already gave us interesting exercices for students enrolled in a program in Mathematical Finance.