On the Implicit Interest Rate in the Yunus Equation

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What is Microcredit?

The provision of very small loans (microloans) to very poor people designed to spur small businesses, entrepreneurship, or other generating income activities.

- The first experiment dated back to the 70s in Bangladesh as an initiative of Prof. Muhammad Yunus.
- Yunus received the Nobel Prize for Peace in 2006.
- Around 10 000 microfinance institutions to most countries in the world.
- About 50-billion euro loans have been lended to almost 500 millions beneficiaries.

The main characteristics of microcredit

- Very small loans over a short periods.
- Borrower has no colleteral to secure the loan.
- Benificiaries are mostly women.
- Usually loans with jointly liability of a group of borrowers (5 to 30)
- Interest rates are high, range from 20% to 70%.
- Repayment rate closes to 100%.

Yunus Equation

1000BDT(Bangladesh Taka) is lent to a borrower that she pays back 22BDT each week for 50 weeks. $22e^{-\frac{rk}{52}}$ is the present value of each week installment

- r is annual continuously compound interest rate
- k is the number of weeks, $k = 1, 2, \cdots, 50$

The equation can be written as

$$1000 = 22\sum_{k=1}^{50}\left(e^{-\frac{r}{52}}\right)^k = 22\sum_{k=1}^{50} q^k = 22\frac{q - q^{51}}{1-q}, \qquad q = e^{-\frac{r}{52}}$$

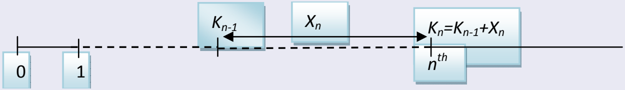
which reduce to *the Yunus Equation*

$$22q^{51} - 1022q + 1000 = 0$$

The interest rate is found to be about 20% (19.74175%)

- The poor clients have faced with high interest rates charged by microfinance institutes (MFIs).
- Meanwhile, some borrowers are late to repay the installments to MFIs.
- When the borrower does not pay back the installment on schedule, what is the risk faced by a lender (MFIs) and what is the law of interest rate?

Random Yunus Equation

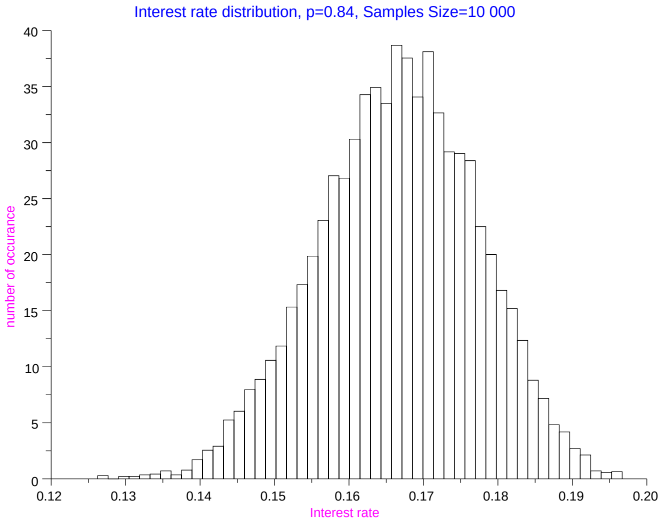


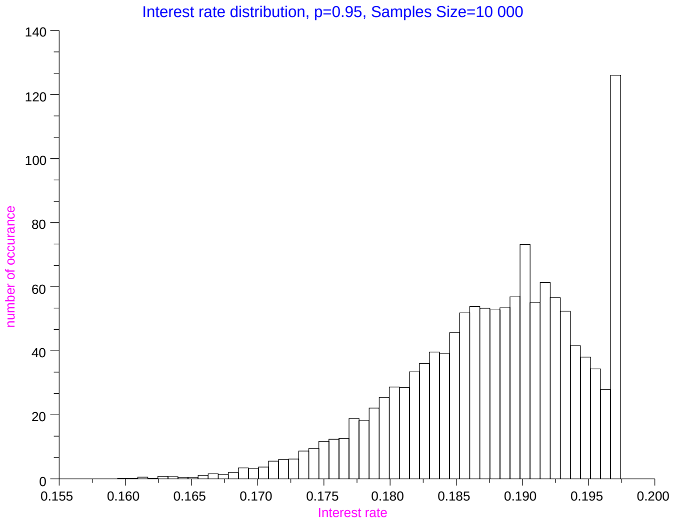
The n^{th} installment takes place at *random time*
$K_n = K_{n-1} + X_n = X_1 + X_2 + \cdots + X_n$ where X_i's are i.i.d. and $X_i \rightsquigarrow G(p), p = \mathbb{P}(X_i = 1)$ is *an installment probability* i.e. the probability that the borrower is able to pay in time the 22BDT.

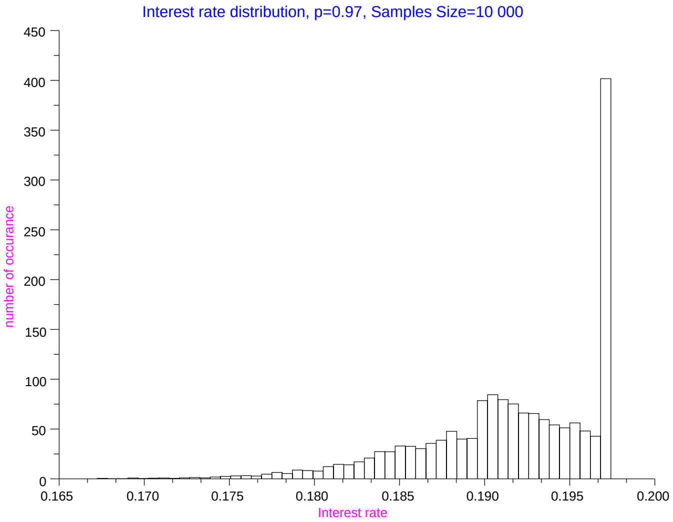
Thus, r becomes a random variable denoted by R, the random Yunus equation can be written as

$$1000 = 22 \sum_{n=1}^{50} e^{-\frac{R}{52}(X_1+X_2+\cdots+X_n)}$$

- What is the law of random interest rate R?


Interest rate distribution, p=0.84, Samples Size=10 000
number of occurance
Interest rate
0
5
10
15
20
25
30
35
40
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.20


Interest rate distribution, p=0.95, Samples Size=10 000
number of occurance
Interest rate
0
20
40
60
80
100
120
140
0.155
0.160
0.165
0.170
0.175
0.180
0.185
0.190
0.195
0.200

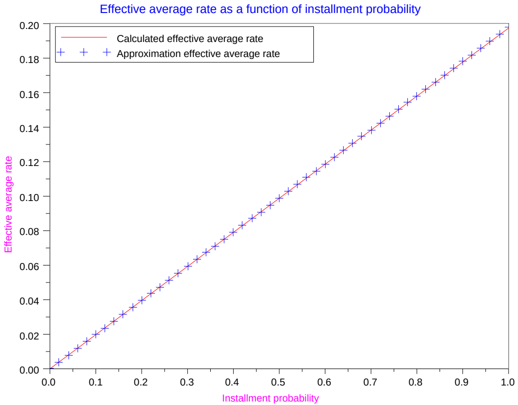

Interest rate distribution, p=0.97, Samples Size=10 000
450
400
350
300
250
200
150
100
50
0
number of occurance
0.165
0.170
0.175
0.180
0.185
0.190
0.195
0.200
Interest rate

Actuarial Expected Rate

- Assuming that the expected radom interest rate in the Random Yunus equation is equal to *the actuarial expected rate* $\bar{r}$, the expectation of the Yunus equation becomes

$$1000 = \mathbb{E}\left(\sum_{n=1}^{50} 22e^{-\frac{\bar{r}}{52}(X_1+X_2+\cdots+X_n)}\right)$$

- Using the moment generating function of geometric distribution to the above equation.
- The solution is obtained to be $\bar{r} = 52\ln\left(1 + p(\frac{1}{q_+} - 1)\right)$ where q_+ is the positive non trivial zero of the Yunus equation.



$\ln(1 + p(\frac{1}{q_+} - 1)) \approx p(\frac{1}{q_+} - 1)$ as $p \to 0$; therefore, the graph looks like a straight line.

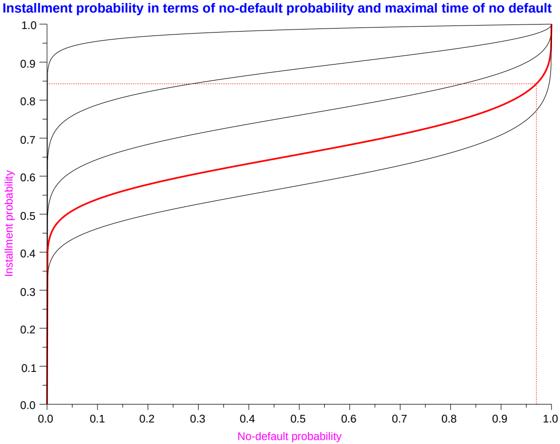
Relationship between Installment Probability and Non-default Probability

- Default is the failure of borrowers' repayment.
- Let d be *the maximal time of no default*
- γ be the repayment rate, named *non-default probability*.

The probability of non-default can be written as

$$\gamma = \mathbb{P}\left(Max\left\{X_1, \ldots, X_{50}\right\} \le d\right)$$
$$= \mathbb{P}\left(\bigcap_{i=1}^{50}\left\{X_i \le d\right\}\right), \qquad X_i\text{'s are i.i.d and } X_i \rightsquigarrow \mathcal{G}(p)$$

which lead to the installment probability $p = 1 - (1 - \gamma^{\frac{1}{50}})^{\frac{1}{d}}$.



The maximal time of no default $d = 1, 2, 3, 4, 5$

Remarks

- In reality, the average on-time repayment of the loan is around 97%.
- For $d = 4$ weeks, $\gamma = 97\%$, then $p = 84\%$ which leads to $\bar{r} \approx 16.59\%$.
- When there is a lateness in repayment, the expected rate tends to decrease with respect to non-default probability.
- High interest rate because of the high operating costs necessary to deliver such small loans.
- The borrower sometimes does not use the loan for her business target, instead she uses for the fee of hospital when there is any member in her family falling sick!

Thanks for your attention!