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Scalar and vector potentials' coupling on nonmatching grids for the simulation of an electromagnetic brake

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Abstract

Purpose – We present a method for the simulation of the dynamical behavior of a coupled magneto-mechanical system given in terms of a conductor moving through an electromagnetic field.

Design/methodology/approach – For the magnetic part, we consider a model based on an electric vector and a magnetic scalar potential, whereas the mechanical part is modelled by the equation of a rigid body motion. A weak coupling is employed: at each time step the resulting forces are calculated yielding the new displacement of the conductor.

Findings – Numerical results are given for the simulation of an electromagnetic brake with axisymmetric geometry. They indicate that the proposed method is especially well suited for eddy current problems involving moving conductors.

Research limitations/implications – Further research should be undertaken toward the application of the proposed method to real 3D problems.

Originality/value – The spatial discretization of the problem relies on the use of two independent triangulations to approximate the two involved potentials. Whereas the scalar magnetic potential is discretized by means of nodal H^1-conforming finite elements on a grid covering the global computational domain, the vector electric potential is approximated by H^{curl}-conforming edge elements on another grid only covering the conductor. The coupling between the two grids is accomplished via the mortar finite element method. At each time step, only the coupling matrix has to be reassembled, all other involved matrices remain the same. Moreover, no remeshing is necessary when the conductor changes its position. The paper should be valuable for any researcher interested in the numerical simulation of eddy current problems.

Keywords Electromagnetism, Eddy currents, Mathematical modelling

Paper type Research paper

Introduction

In a coupled magneto-mechanical system, the forces due to the magnetic field contribute to the free structure motion and the resulting variation in the structure configuration modifies the distribution of the magnetic field and so of the induced forces. To take simultaneously into account the electromagnetic and mechanical equations it is necessary to compute the global magnetic force acting on the moving

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part of the system, through the numerical evaluation of the magnetic field. Here, we study a system where, attached to an elastic spring suspended from the lid of a rigid box, a conductor of mass m oscillates within the magnetic field supported by a ferromagnet.

The algorithm we consider is based on an "explicit" coupling procedure: at each time step, the magnetic force obtained from the field solution is inserted in the mechanical equation to compute the displacement. The latter is imposed to the moving part for the next step of the magnetic field calculation. A finite difference scheme is adopted to discretize the ordinary differential equation which accounts for the conductor position. The magnetic field is computed by applying the finite element method to the T-Ω formulation of the eddy current problem. This formulation allows to use a scalar function (Ω) in the computational domain and a vector quantity (T) only in the conducting parts. The application of the proposed method to an electrodynamic levitation device (see problem 28 of the TEAM workshop in Karl *et al.* (1997)) is in progress in order to provide a comparison with the experimental measurements.

In this paper we adopt a "piecewise Lagrangian description", where the spectator is attached to the considered part and describes the events from his material point of view. This approach makes disappear the explicit velocity term from Ohm's law provided that each part is treated in its own "co-moving" frame (one frame fixed to the computational domain and another frame co-moving with the conductor). If two different reference systems are used, one has to couple both by suitable transmission conditions at the conductor boundary. We stress out the fact that for the analysis of eddy current problems in domains with moving parts, there is some freedom in the choice of the reference frame, provided that the motion can be regarded as quasi-stationary with respect to electrodynamics, as it is the case for the considered example. This allows us to approximate the vector and scalar potentials on different meshes and to couple them by means of the mortar element method (Flemisch *et al.*, 2004; Maday *et al.*, 2003). The proposed method is a valid alternative to remeshing or boundary elements.

The magnetic model

The mathematical model describing the eddy current problem in the conductor at low frequencies is given by the quasi-stationary Maxwell's equations (Albanese and Rubinacci 1990). In this paper, we restrict ourselves to the magnetic field approach. The space R^3 is decomposed into the conducting region V_c and the external region R^3/V_c. Denoted by H, B, J and E the magnetic field, the magnetic flux density, the current density and the electric field, respectively, the quasi-stationary Maxwell's equations in V_c read:

$$\nabla \times H = J, \quad \nabla \times E = -\partial_t B, \quad \nabla \cdot B = 0. \tag{1}$$

The densities and the fields are linked by the constitutive properties, i.e. $B = \mu H$, $J = \sigma E$, where $\mu > 0$ is the magnetic permeability with $\mu = \mu_c$ constant in V_c, and $\sigma \geq \bar{\sigma} > 0$ stands for the electric conductivity. Moreover, we assume that the material parameters are time independent and associated with linear isotropic media, and that the given external source J_s is zero within the conducting regions. As a result, we obtain the following field equations in R^3/V_c:

$$\nabla \times H = J_s, \quad \nabla \cdot B = 0, \quad B = \mu H. \tag{2}$$

The problem is well posed by adding regularity conditions at infinity and suitable interface conditions on ∂V_c. In particular, $[H]_c \times n_c = 0$, $[B]_c \cdot n_c = 0$, together with $[E]_c \times n_c = 0$, $[J]_c \cdot n_c = 0$, where n_c is the outer normal on ∂V_c, and $[v]_c$ stands for the jump of v on ∂V_c. Additionally, to the boundary conditions, we have to impose suitable initial values for the vector fields at a given time t_0. The initial condition on B has to satisfy $\nabla \cdot B = 0$ and $[B] \cdot n = 0$ at any interface. By introducing artificial boundary conditions, we can work on a bounded domain V. For simplicity, we assume that V_c is a simply connected polyhedral subdomain of V and $\bar{V}_c \subset V$ (note that the approach can be extended to the case of non-trivial conductors, see Albanese and Rubinacci, 1990). For the current density J, the condition $\nabla \cdot J = 0$ suggests the introduction of a vector potential $\tilde{T}$ such that $J = \nabla \times \tilde{T}$. Then in V_c, the difference between the vector potential $\tilde{T}$ and the magnetic field H can be written as the gradient of a scalar function $\tilde{\Omega}$, i.e. $H = \tilde{T} - \nabla\tilde{\Omega}$. A similar argument holds for the insulating region, where we assume knowing a vector potential T_s such that $J_s = \nabla \times T_s$.

Using an implicit Euler time stepping scheme, we have to solve an electromagnetic problem at each time step. Based on the decomposition of H into $\tilde{T} - \nabla\tilde{\Omega}$ in V_c and $T_s - \nabla\tilde{\Omega}$ in V/V_c, the variational formulation reads: find $(\tilde{T}, \tilde{\Omega}) \in H_0(\mathrm{curl}, V_c) \times H_0^1(V)$ such that

$$\begin{aligned} a(\tilde{\Omega}, v) + \tilde{b}_c(\tilde{T}, v) &= \int_{V \setminus V_c} f \nabla v, \quad \forall v \in H_0^1(V), \\ \tilde{b}_c(W, \tilde{\Omega}) + a_c(\tilde{T}, W) &= \int_{V_c} f_c W, \quad \forall W \in H_0(\mathrm{curl}, V_c), \end{aligned} \tag{3}$$

where the continuous bilinear forms are given by

$$\tilde{b}_c(W, v) = -\int_{V_c} W \nabla v, \quad \forall v \in H_0^1(V), \ \forall W \in H_0(\mathrm{curl}, V_c),$$

$$a(\tilde{\Omega}, v) = \int_V \beta \nabla\tilde{\Omega} \nabla v, \quad \forall \tilde{\Omega}, v \in H_0^1(V),$$

$$a_c(\tilde{T}, W) = \int_{V_c} (\alpha \nabla \times \tilde{T} \nabla \times W + \tilde{T} W), \quad \forall \tilde{T}, W \in H_0(\mathrm{curl}, V_c).$$

The coefficients α, $\beta > 0$ are assumed to be piecewise constant and depend on the material parameters σ, μ as well as on the time step size δt, in particular, $\alpha = \delta t / \mu\sigma$ and $\beta = \mu / \mu_c$. The unknowns $\tilde{T}$ and $\tilde{\Omega}$ denote the approximations at the current time step, f_c depends on the approximations of $\tilde{T}$ and $\tilde{\Omega}$ at the previous time step, and f denotes the scaled source term depending on T_s.

The potentials satisfy at each time step the interface conditions, i.e. $\tilde{\Omega}$ is continuous at ∂V_c and $[\tilde{T}]_c \times n_c = 0$. Note that, if $(\tilde{T}, \tilde{\Omega})$ is a solution of the variational problem (3), then $(\tilde{T} + \nabla\varphi, \tilde{\Omega} + \varphi)$, $\varphi \in H_0^1(V_c)$, is a solution as well. In order to obtain a well posed problem, we choose φ such that $\Omega = \tilde{\Omega} + \varphi$ is harmonic on V_c, i.e. $\Omega_{|\partial V_c} = \tilde{\Omega}_{|\partial V_c}$ and

$$\int_{V_c} \nabla\Omega\nabla v = 0 \text{ for } v \in H_0^1(V_c).$$

This can be achieved by involving the harmonic extension operator H: $H^{1/2}(\partial V_c) \rightarrow H^1(V_c)$ and defining $\Omega_{|V_c} = H\tilde{\Omega}_{|\partial V_c}$. Changing the coupling bilinear form $\tilde{b}_c(\cdot,\cdot)$ to $b_c(\cdot,\cdot)$, defined by

$$b_c(W,v) = -\int_{V_c} W\nabla H v_{|\partial V_c}$$

we obtain a uniquely solvable variational problem: find $(T,\Omega) \in H_0(\text{curl}, V_c) \times H_0^1(V)$ such that

$$\begin{aligned} a(\Omega,v) + b_c(T,v) &= \int_{V\setminus V_c} f\nabla v, \qquad \forall v \in H_0^1(V), \\ b_c(W,\Omega) + a_c(T,W) &= \int_{V_c} f_c W, \qquad \forall W \in H_0(\text{curl}, V_c). \end{aligned} \tag{4}$$

Lemma 1. Problem (4) has a unique solution (T,Ω).

Proof. In the paper by Maday *et al.* (2003), we proved that the bilinear form $a_g((W,w),(V,v))$ given by $a_c(W,V) + b_c(W,v) + b_c(V,w) + a(w,v)$ where $V, W \in H_0(\text{curl}, V_c)$ and $v, w \in H_0^1(V)$ is continuous and elliptic on $H_0(\text{curl}, V_c) \times H_0^1(V)$. □

Lemma 2. The vector field T is divergence-free in V_c.

Proof. Taking $v \in H_0^1(V_c)$ in the first equation of (4), we get $b_c(T,v) = 0$. In fact, Ω harmonic implies $a(\Omega,v) = 0$ and

$$\int_{V\setminus V_c} f\nabla v = 0.$$

From the second equation in (3) with $\Omega = \nabla v$, $v \in H_0^1(V_c)$, we obtain $b_c(W,\Omega) = 0$ since Ω is harmonic and

$$a_c(T,W) = \int_{V_c} TW$$

due to the fact that curl(grad) is zero. So,

$$\int_{V_c} TW = \int_{V_c} f_c W$$

implying $\nabla \cdot T = 0$ as soon as $\nabla \cdot f_c = 0$, which is the case. □

Discretization method

We adopt two different quasi-uniform triangulations τ_H on V and τ_h on V_c. Here, H and h denote the maximal diameter of the elements in τ_H and τ_h, respectively. On τ_H, we use standard conforming finite elements of lowest order to approximate Ω. The associated discrete space having zero boundary conditions on ∂V is called $S_{0;H}(V)$. For the discretization of the vector field T, we use edge elements, see the book by Bossavit (1998), on τ_h. The associated discrete space is $X_h(V_c)$ and we set

$X_{0;h}(V_c) = X_h(V_c) \cap H_0(\text{curl}; V_c)$. Finally, we denote by $S_h(V_c)$ the space of standard conforming finite elements of lowest order associated with τ_h on V_c with no boundary conditions on ∂V_c, and its trace space on ∂V_c is called $W_h(\partial V_c)$.

In order to formulate the discrete version of problem (4), we have to replace the harmonic extension operator H in the definition of $b_c(\cdot,\cdot)$ by a discrete one H_h: $W_h(\partial V_c) \rightarrow S_h(V_c)$. Note that the restriction of $v \in S_{0;H}(V)$ on ∂V_c is, in general, not an element in $W_h(\partial V_c)$, since the two triangulations τ_H and τ_h are neither nested nor coincident on ∂V_c, but completely independent from each other (Figure 1).

To overcome this difficulty, we introduce a projection operator Π_h: $H^1(V) \rightarrow W_h(\partial V_c)$ onto the boundary ∂V_c. This operator is well known in the mortar finite element context (Bernardi *et al.*, 1994; Wohlmuth, 2001), and can be defined in terms of a Lagrange multiplier space $M_h(\partial V_c)$ as follows:

$$\int_{\partial V_c} (\Pi_h v - v)\nu = 0, \qquad \forall \nu \in M_h(\partial V_c). \tag{5}$$

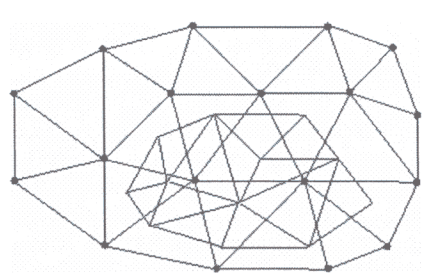
A proper choice of the space $M_h(\partial V_c)$ is of crucial importance to obtain a well defined and stable operator Π_h. There exist several alternatives of which we mention two: one possibility is to take the trace space $W_h(\partial V_c)$ (Bernardi *et al.*, 1994). The other is to use dual Lagrange multipliers (Wohlmuth, 2001), which satisfy a biorthogonality relation yielding a trivial to invert diagonal mass matrix.

In terms of H_h and Π_h, the discrete variational problem reads: find $(T_h, \Omega_H) \in X_{0;h}(V_c) \times S_{0;H}(V)$ such that

$$\begin{aligned} a(\Omega_H, v) + b_h(T_h, v) &= \int_{V \setminus V_c} f \nabla v, \quad \forall v \in S_{0;H}(V), \\ b_h(W, \Omega_H) + a_c(T_h, W) &= \int_{V_c} f_c W, \quad \forall W \in X_{0;h}(V_c), \end{aligned} \tag{6}$$

where

$$b_h(W, v) = -\int_{V_c} W \nabla H_h \Pi_h v, \quad \text{for } v \in S_{0;H}(V) \text{ and } W \in X_{0;h}(V_c).$$



Notes: The mortar projection operator Π_h acts from the values of Ω at the mesh nodes of τ_H in V (solid line with dots) to the values of Ω at the end points of the bounbdary mesh edges of τ_h on V_c (solid lines without dots)

Figure 1. Overlapping decomposition and nonmatching grids on V_c in two dimensions

This approach is stable in the time domain and characterized by an optimal error estimate in the spatial domain (Maday *et al.*, 2003). The only theoretical constraint appearing is that the ratio of the two meshsizes h/H has to be small enough. In what follows, we show how problem (6) can be solved efficiently by means of a Gauß-Seidel iterative scheme.

Implementation details and iterative solver

The implementation of the proposed method involves three delicate steps that we are going to analyze in the following. First step, the mortar coupling to define the values of $v^+ = v_H \in S_{0;H}(V)$ at the nodes of τ_h lying on ∂V_c, i.e. $v^- = \Pi_h v_H$ at ∂V_c. The mortar condition (5) gives

$$\int_{\partial V_c} v^- \nu^- = \int_{\partial V_c} v^+ \nu^-, \quad \text{for } \nu^- \in M_h(\partial V_c).$$

Using the same notation for the discrete functions and their algebraic representations, condition (5) in matrix form reads $Q_- v^- = Q_+ v^+$, where Q_- and Q_+ are node mass matrices on ∂V_c associated with τ_h. Note that Q_- is easy to compute since its elements are integrals of products of functions defined with respect to the same mesh. This is not the case for Q_+ whose elements involve integrals of products of functions defined on different meshes. For the latter, the exact integration is feasible in two dimensions, but may become difficult in three dimensions unless quadrature formulas are used (Wohlmuth, 2001).

Denoted by Q the rectangular matrix $Q_-^{-1} Q_+$, condition (5) reads $v^- = Qv^+$. The inversion of Q_- is not expensive with respect to the rest of the algorithm. Moreover, in the case of dual Lagrange multipliers, Q_- is diagonal, and the inversion becomes a trivial task. The matrix Q is the only matrix involved that has to be rebuilt at each new position of the conductor V_c.

Second step, the extension of v^- from ∂V_c to the interior of V_c, i.e. $v^* = H_h v^-$ in V_c. This is done by solving a Dirichlet boundary problem in V_c for the Laplace operator with zero source term and a given boundary condition. In matrix form, we have $v^* = Sv^-$, where S is the matrix associated to H_h.

Final step, the bilinear form

$$b_h(W, v^*) = -\int_{V_c} W \nabla v^*,$$

which represents the coupling term in the problem between the local vector potential and the global scalar one. Note that $W \in X_{0;h}(V_c)$; as it is classical, the vector ∇v^* can be decomposed on the same basis as W. The coefficients of the decomposition are circulations along the edges of τ_h defined from nodal values at the end points of the edges. The passage from nodal values to associated circulations can be done efficiently in terms of G, the node-to-edge incidence matrix (Bossavit, 1998). For an edge $e = \{p, q\}$ of τ_h, oriented from node p to node q, we have $G(e, p) = -1$, $G(e, q) = 1$, and $G(e, r) = 0$ for all other nodes r (Figure 2). So, for a function $v^* \in S_h(V_c)$, the vector $\nabla v^* \in X_{0;h}(V_c)$ is algebraically given by Gv^*. The edge-to-node application is simply given by G^t, where the upper index t denotes the transposed operator. With the bilinear form $b_h(W, v)$, $W \in X_h(V_c)$ and $v \in S_h(V_c)$ we associate the rectangular matrix $B = -MG$, where M is the edge element mass matrix on V_c.

To get the final matrix problem, we introduce the stiffness matrix A associated with the bilinear form $a_c(\cdot,\cdot)$ on $X_h(V_c)\times X_h(V_c)$. Decomposing the edges of τ_h into interior and boundary ones, we obtain

$$A=\begin{pmatrix} A_{II} & A_{IB} \\ A_{BI} & A_{BB}\end{pmatrix},\quad A_0=\begin{pmatrix} A_{II} & A_{IB} \\ 0 & Id_{BB}\end{pmatrix},$$

where A_0 is the matrix associated with the Dirichlet problem on $X_{0;h}(V_c)$ defined in terms of the bilinear form $a_c(\cdot,\cdot)$ Let us further denote by $K_{0;v}$ the stiffness matrix associated with the bilinear form $a(\cdot,\cdot)$ on $H_0^1(V)\times H_0^1(V)$. The algebraic form of the discrete problem (6) reads: find two vectors T_h and Ω_H solving the linear system

$$A_0T_h+PBSQ\Omega_H=F_c,\qquad K_{0;V}\Omega_H+Q^tS^tB^tT_h=F. \tag{7}$$

The right hand side vectors take into account the homogeneous Dirichlet boundary conditions and P is a cut-off matrix, i.e. $P(v_I,v_B)^t=(v_I,0)^t$. The application of P is necessary to guarantee the boundary condition $T_h\times n_c=0$ on Γ. As an iterative solver for (7), we propose a block Gauß-Seidel scheme. Starting from Ω_H^n, we first compute T_h^{n+1} and then Ω_H^{n+1} by

$$A_0T_h^{n+1}+PBSQ\Omega_H^n=F_c,\qquad K_{0;V}\Omega_H^{n+1}+Q^tS^tB^tT_h^{n+1}=F. \tag{8}$$

The convergence of the algorithm is guaranteed by the following Lemma proved in Maday *et al.* (2003).

Lemma 3. Let $e^n=\Omega_H-\Omega_H^n$ be the iteration error in the n-th step, then there exists a constant $0<\theta<1$ not depending on H and h such that $a(e^{n+1},e^{n+1})<\theta a(e^n,e^n)$. The convergence of Ω_H^n to Ω_H yields the one of T_h^n to T_h.

In each iteration step, the solution of four subproblems is required. In particular, the application of the two matrices S and S^t involves the solution of two Dirichlet problems on V_c. Since the gauge is arbitrary, one may think of different extension operators, which are easier to apply. However, numerical tests show that using the trivial extension results in a loss of the positive definiteness of the system, and the Gauß-Seidel method (8) diverges. As an alternative to (8), the iteration scheme can be equivalently reformulated in terms of a preconditioned Richardson iteration in Ω_H (Flemisch *et al.*, 2004). As a consequence, only three subproblems have to be solved per iteration step.

If the number of time steps is moderate, we recommend to use highly efficient preconditioning algorithms like multigrid or domain decomposition methods for the inversion of the matrices A_0 and $K_{0;v}$, as well as for the application of the extension S and restriction S^t. However, if the problem demands a large number of time steps,

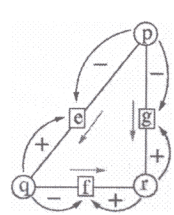


Figure 2. Computation and action of the node-to-edge incidence matrix G in two dimensions

it may become feasible to use LU-decompositions instead, which have to be carried out only once, since the involved matrices do not change during the whole computation.

The mechanical model

For the mechanical part, we use the simple model of a damped harmonic oscillator. Let L be the distance between the two equilibrium positions of V_c which correspond to take or not to take into account the gravity force, respectively. We denote by m the mass of V_c and by z the vertical position of the center of V_c at time t, with the initial conditions $z(0)$, $\dot{z}(0)$. Then at any given time, there are four forces acting on m: the gravity force $-mg$ pulling downward, the spring force $k(L-z)$, the damping force $-b\dot{z}$ and the external magnetic force $(F_m)_z$. Neglecting any movement parallel to the (x,y)-plane, the Newton's law of motion for a point mass (the center of V_c) reads:

$$m\ddot{z} = -mg + k(L-z) - b\dot{z} + \left(\int_{V_c} (\nabla \times H) \times \mu H\right)_z, \tag{9}$$

where we assume that V_c has free space permeability μ_0, and therefore, no forces due to magnetization need to be taken into account. If we write equation (9) at the equilibrium position, i.e. $z = 0$ and $F_m = 0$, we get $mg = kL$. Then, equation (8) simplifies into

$$m\ddot{z} + b\dot{z} + kz = \left(\int_{V_c} (\nabla \times H) \times \mu H\right)_z. \tag{10}$$

To discretize equation (10), we apply a second-order explicit finite difference scheme of time step size δt.

Numerical results

We present some numerical results for problem (6) explicitly coupled with the discrete form of equation (10). The considered domain V is the cube $(-0.1\,\text{m}, 0.1\,\text{m})^3$ containing a ferromagnet of relative magnetic permeability $\mu_r = 1{,}000$, and a cylindrical conductor V_c having permeability $\mu_0 = 4\pi \times 10^{-7}$ H/m. The geometry parameters are shown in Figure 3. The ferromagnet contains a coil (shadowed part in the right picture

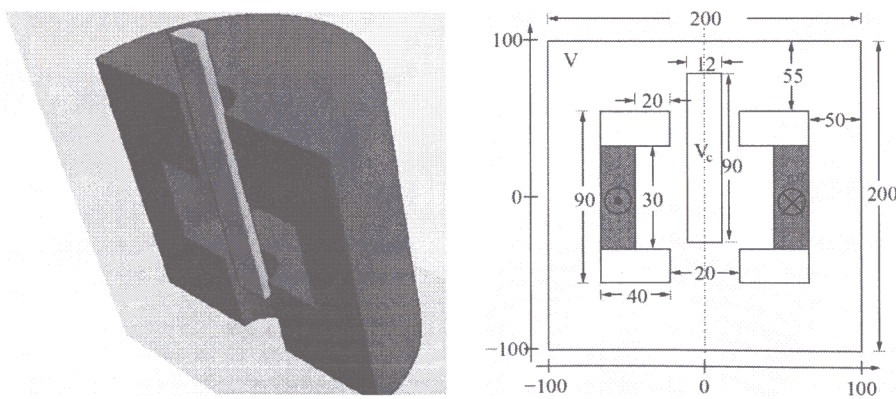


Figure 3.
(Left) Domain geometry. (Right) (x,z)-plane, dimensions in mm

of Figure 3) consisting of 500 windings each of which carries a constant current of 50 A, which yields a source current density $J_s = 41.67\,\mathrm{A/mm^2}$. The conductor V_c moves with a velocity $v//\vec{e}_z$ determined by the solution of the coupled problem. Its barycenter has the initial position $\vec{x}_0 = (0, 0, 0.02\,\mathrm{m})^t$.

Both geometry and source term are axisymmetric with respect to the vertical axis. Therefore, by transformation to cylindrical coordinates, we can reduce the problem dimension and choose the right half of the (x,z)-plane as computational domain (Henrotte *et al.*, 1999). The left picture of Figure 4 shows a zoom of the computational grids, being quadrilateral in V and triangular in V_c. The mechanical parameters are chosen to give a damped oscillating system (i.e. $b^2 - 4km < 0$). Here, $m = 0.1\,\mathrm{kg}$, $b = 0.05\,\mathrm{Ns/m}$, and $k = 20\,\mathrm{N/m}$. Using as initial conditions $z(0) = 0.02\,\mathrm{m}$ and $\dot{z}(0) = 0$, we perform 200 time iterations of step size $\delta t = 0.01\,\mathrm{s}$. We run five different tests, thereby varying the conductivity σ of V_c from 10^6 to $10^8\,\mathrm{S/m}$. Each test needs approximately 5 min on a 1.8 GHz PC.

The results are shown in the right picture of Figure 4, plotting the vertical position z of the center of the conductor against the time t. We observe that for $\sigma = 10^6\,\mathrm{S/m}$, the motion almost coincides with that described by the homogeneous form of equation (10). In this case, the magnetic force related to the induced currents in V_c is rather weak and within the computational time interval, the conductor cannot hold the equilibrium position. As soon as $\sigma \geq 5 \times 10^6\,\mathrm{S/m}$, the contrasting effect of the eddy currents to the motion is more visible and the amplitude of the oscillation decreases. For the large conductivities $5 \times 10^7\,\mathrm{S/m}$ and $10^8\,\mathrm{S/m}$, the conductor stays within the equilibrium position after roughly 1 and 0.5 s, respectively.

Conclusion

Our proposed method is well adapted for eddy current problems involving moving conductors. The discretization approach relies on the use of two grids to approximate the two potentials. The transfer of the physical information from one mesh to the other is done by means of the mortar element technique which results in a coupling matrix Q that has to be reassembled at each new position of the moving conductor. We stress

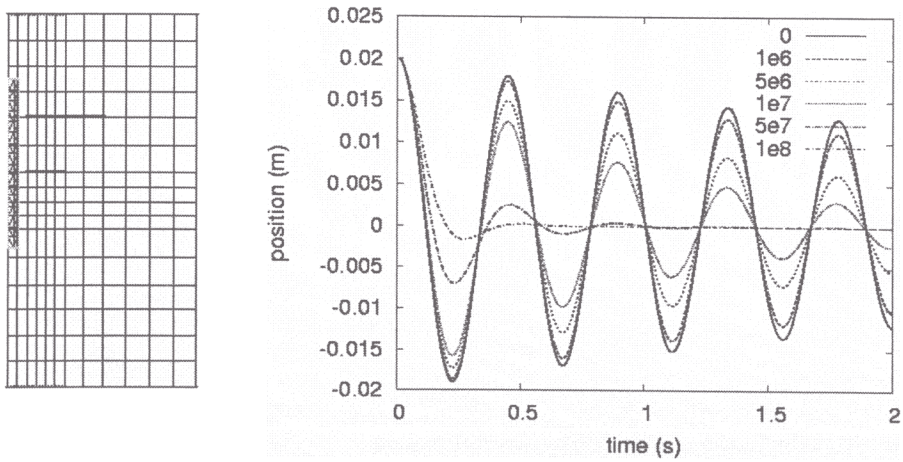


Figure 4. (Left) Overlapping grids (zoom). (Right) Vertical position z of V_c with respect to time for different values of σ (given in S/m)

the fact that no other involved matrix has to be reassembled during the whole computation. Moreover, no remeshing is necessary: the foreground and the conductor grid can be chosen completely independent to each other.

References

Albanese, R. and Rubinacci, G. (1990), "Formulation of the eddy-current problem", *IEE Proceedings*, Vol. A137, pp. 16-22.

Bernardi, C., Maday, Y. and Patera, A.T. (1994), "A new nonconforming approach to domain decomposition: the mortar element method", in Brezis, H. and Lions, J.L. (Eds), *Nonlinear Partial Differential Equations and Their Applications*, Pitman, Essex, pp. 13-51.

Bossavit, A. (1998), *Computational Electromagnetism: Variational Formulations, Complementarity, Edge Elements*, 1st ed., Academic Press, London.

Flemisch, B., Maday, Y., Rapetti, F. and Wohlmuth, B.I. (2004), "Coupling scalar and vector potentials on nonmatching grids for eddy currents in a moving conductor", *J. Comput. Appl. Math.*, Vol. 168 Nos 1/2, pp. 192-205.

Henrotte, F., Meys, B., Hedia, H., Dular, P. and Legros, W. (1999), "Finite element modelling with transformation techniques", *IEEE Trans. on Magn.*, Vol. 35, pp. 1434-7.

Karl, H., Fetzer, J., Kurz, S., Lehner, G. and Rucker, W. (1997), "Description of TEAM workshop problem 28: an electrodynamic levitation device", *Proceedings of the TEAM Workshop in the Sixth Round*, Rio de Janeiro, pp. 48-51.

Maday, Y., Rapetti, F. and Wohlmuth, B.I. (2003), "Mortar element coupling between global scalar and local vector potentials to solve eddy current problems", *Enumath'01 Proceedings*, Springer, pp. 847-65.

Wohlmuth, B.I. (2001), *Discretization Methods and Iterative Solvers Based on Domain decomposition*, Lecture Notes in Computer Sciences and Engineering, Springer, Heidelberg, Vol. 17.