Numerical simulation of the fiber motion in a viscous fluid flow

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Context:

Processing Flow → Fibres : • **orientation** • length • curvature • dispersion → Mechanical properties Shrinkage, warpage Aspect

Objectives: To evaluate fiber orientation and rheological equations

Micromechanics approach →

Method: Direct simulation of a fiber population motion in an elementary volume

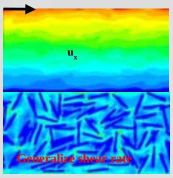
Numerical procedure

Computation of Velocity field

1) Characteristic function

$$1_{\Omega_j}(x,t)=\begin{cases}1 & x\in\Omega_j\\ 0 & x\notin\Omega_j\end{cases}$$

j = fluid or solid (fibers)



2) Velocity field

$\nabla\sigma = 0$

$$\int_\Omega 1_{\Omega_f} 2\eta\varepsilon(u):\varepsilon(v)\,d\Omega + \int_\Omega 1_{\Omega_s}\, r\,\varepsilon(u):\varepsilon(v)\,d\Omega$$

$$-\int_\Omega p\nabla.v\,d\Omega = 0$$

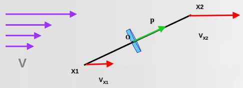
Penalization ~ $10^3\,\eta$

$\varepsilon(u) = 0$

$\nabla u = 0$ $\quad -\int_\Omega q\nabla.u\,d\Omega = 0$

Update fiber position and orientation

Particle method



$$X_i(t+\Delta t) = X_i + V_{X_i}\Delta t$$

Rigid motion

$$\vec{p} = \frac{X_1X_2}{\|X_1X_2\|}$$ Orientation vector

$$O = \frac{X_1 + X_2}{2}$$ Fiber center

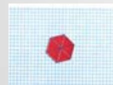
Advantages

- Perfect rigid motion of each fiber
- Conservation of the fiber's length
- No mumerical diffusion

Computation of Characteristic Function : "voxelization"

Cavity mesh

Voxelisation



Add fiber mesh

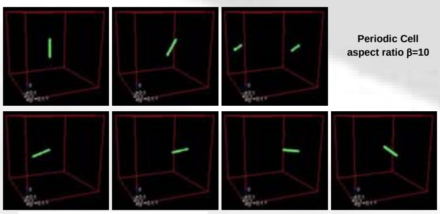
Switch on voxels

Get values

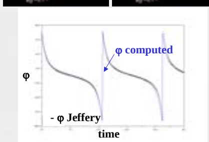
$t = t + \Delta t$

1 or 2 particles

Motion of a Single Fiber in a shear flow



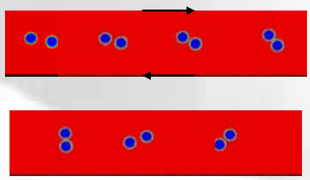
Periodic Cell
aspect ratio β=10



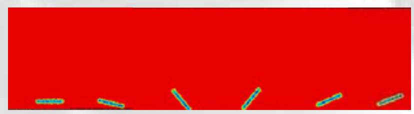
Find Jeffery's orbit for a shear flow
Periodic motion

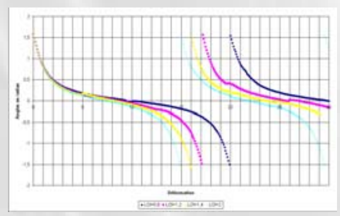
Hydrodynamics Interactions

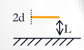
Sphericals particles in Couette Flow



Fiber – Wall interaction

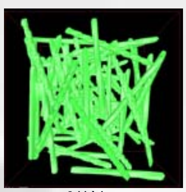




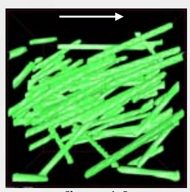


Rotation velocity ⇗
as L ⇗

Many fibers



Initial time



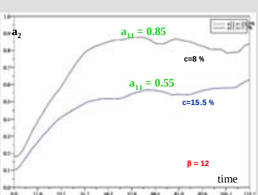
Shear strain 5

Shear strain 10

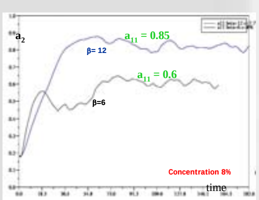
Aspect ratio = 12

Evolution of $a_2 = \frac{1}{60}\sum_{k=1}^{60} p_k \otimes p_k$

Effect of the concentration



Effect of the aspect ratio

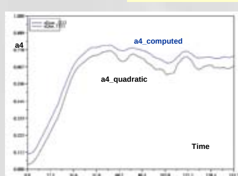


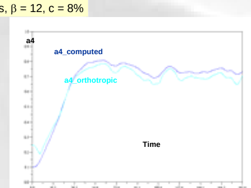
$\dot{\gamma} = 1\,s^{-1}$

Closure approximation $a_4 = f(a_2)$

- $$\frac{D\underline{\underline{a}}_2}{Dt} = \Omega\underline{\underline{a}}_2 - \underline{\underline{a}}_2\Omega + \lambda(\dot{\varepsilon}\underline{\underline{a}}_2 + \underline{\underline{a}}_2\dot{\varepsilon} - 2\dot{\varepsilon}:\underline{\underline{a}}_4) + 2C_I\dot{\bar{\varepsilon}}(\underline{\underline{I}}_d - 3\underline{\underline{a}}_2)$$
- Comparison $a_{1111computed}$ and a_{1111}= closure ($a_{ij\ computed}$)

Shear Flow 60 fibres, β = 12, c = 8%





Orthotropic closure : more accurate than quadratic closure

Viscosity evolution in shear flow

$$\eta_{apparent} = \frac{\sigma_{imposed}}{\int_\Omega \|\nabla u + u\nabla\|/2}$$