

A polyhedral characterization of quasi-ordinary singularities

Abstract: Given an irreducible hypersurface singularity of dimension d (defined by a polynomial $f \in K[[x]][[z]]$) and the projection to the affine space defined by $K[[x]]$, we construct an invariant which detects whether the singularity is quasi-ordinary with respect to the projection. The construction uses a weighted version of Hironaka's characteristic polyhedron and successive embeddings of the singularity in affine spaces of higher dimensions. This permits to see a quasi-ordinary singularity as an "overweight deformation" of a toric variety. A parametrization of $\{f = 0\}$ is then obtained by lifting a parametrization of the toric variety; this yields the roots of f as a polynomial in z and hence a Newton type proof of the Abhyankar-Jung Theorem (Since the 80's there were many attempts to find a proof of the latter type). Moreover, when f is quasi-ordinary and $K = \mathbb{C}$ then our invariant encodes the embedded topology of the singularity $\{f = 0\}$ in a neighbourhood of the origin.

Joint work with Bernd Schober