Zeta functions and oscillatory integrals for meromorphic functions

Abstract: This is joint work with Wilson Zuniga-Galindo. In the 70's Igusa developed a uniform theory for local zeta functionsand oscillatory integrals attached to polynomials with coefficients in a local field of characteristic zero. Over the real and complex field, there is related work of Varchenko. We extend this theory to the case of rational functions, or, more generally, meromorphic functions f/g, with coefficients in any local field of characteristic zero. This generalization is far from straightforward, due to the fact that several new geometric phenomena appear. Also, the oscillatory integrals have two different asymptotic expansions: the 'usual' one when the norm of the parameter tends to infinity, and another one when the norm of the parameter tends to zero. The first asymptotic expansion is controlled by the (negative) poles of all the twisted local zeta functions associated to the meromorphic functions f/g - c, for certain special geometric values c. The second expansion is controlled by the (positive) poles of all the twisted local zeta functions associated to f/g.