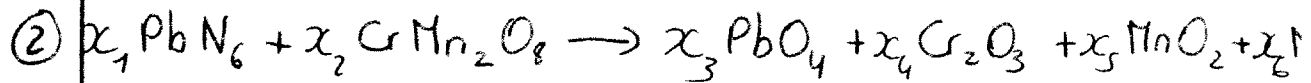
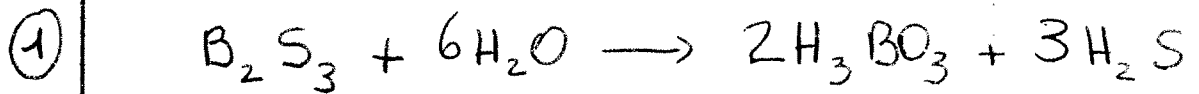


Exercice 10



On équilibre :

Pb:  $x_1 = 3x_3$

N:  $6x_1 = x_6$

Cr:  $x_2 = 2x_4$

Mn:  $2x_2 = x_5$

O:  $8x_2 = 4x_3 + 3x_4 + 2x_5 + x_6$

On obtient un système du type :

$$\begin{cases} x_1 + 0 - 3x_3 + 0 = 0 & \text{①} \\ 6x_1 + 0 + 0 + 0 + 0 + 0 = 0 & \text{②} \\ 0 + x_2 + 0 - 2x_4 + 0 + 0 = 0 & \text{③} \\ 0 + 2x_2 + 0 + 0 - x_5 + 0 = 0 & \text{④} \\ 0 + 8x_2 - 4x_3 - 3x_4 - 2x_5 - x_6 = 0 & \text{⑤} \end{cases}$$

$$\begin{cases} x_1 - 3x_3 = 0 & \text{①}' \\ x_2 - 2x_4 = 0 & \text{②}' \\ 18x_3 - x_6 = 0 & \text{②} - 6 \times \text{①} = \text{③}' \\ 4x_4 - x_5 = 0 & \text{④} - 2 \times \text{③} = \text{④}' \\ -4x_3 + 15x_4 - 2x_5 - x_6 = 0 & \text{⑤} - 8 \times \text{③} = \text{⑤}' \end{cases}$$

$$\begin{cases} x_1 = 3x_3 & \textcircled{1}'' \\ x_2 = 2x_4 & \textcircled{2}'' \\ x_3 = \frac{x_6}{18} & \textcircled{3}'' \\ 4x_4 = x_5 & \textcircled{4}'' \\ 135x_4 - 18x_5 - 11x_6 = 0 & \textcircled{5}'' = 9 \times \textcircled{5}' + 2 \times \textcircled{3} \end{cases}$$

on fait le calcul :

$$4 \times \textcircled{5}'' - 135 \textcircled{4}'' \text{ et on obtient :}$$

$$63x_5 - 44x_6 = 0.$$

1 variable libre:  $x_6$

$$\begin{cases} x_1 = \frac{x_6}{6} \\ x_2 = \frac{22}{63} x_6 \\ x_3 = \frac{x_6}{18} \\ x_4 = \frac{x_5}{4} = \frac{11}{63} x_6 \\ x_5 = \frac{44}{63} x_6 \end{cases}$$

$$S = \left\{ x_6 \left( \frac{1}{6}, \frac{22}{63}, \frac{1}{18}, \frac{11}{63}, \frac{44}{63}, 1 \right) \text{ tq } x_6 \in \mathbb{Q} \right\}$$

$$S = \left\{ x_6 (21, 44, 9, 22, 88, 126)k, \text{ tq } k \in \mathbb{N} \right\}$$

$$\textcircled{a} E(\lambda) : \begin{cases} (1-\lambda)\alpha + 2y - 2z = 0 & (E_1) \\ -\alpha + (3-\lambda)y = 0 & (E_2) \\ 2y + (1-\lambda)z = 0 & (E_3) \end{cases}$$

$$\text{équivalent à} \begin{cases} -\alpha + (3-\lambda)y = 0 & (E'_1) = (E_2) \\ 2y + (1-\lambda)z = 0 & (E'_2) = (E_3) \\ [2+(1-\lambda)(3-\lambda)]y - 2z = 0 & (E'_3) = (E_1) + (1-\lambda)(E_2) \end{cases}$$

$$\begin{cases} -\alpha + (3-\lambda)y = 0 & (E''_1) \\ 2y + (1-\lambda)z = 0 & (E''_2) \\ (5-4\lambda+\lambda^2)y - 2z = 0 & (E''_3) \end{cases}$$

$$\begin{cases} -\alpha + (3-\lambda)y = 0 & (E'''_1) = (E'_1) \\ 2y + (1-\lambda)z = 0 & (E'''_2) = (E'_2) \\ [(1-\lambda)(5-4\lambda+\lambda^2)+4]z = 0 & (E'''_3) = (5-4\lambda+\lambda^2)(E'_2) - 2(E'_3) \end{cases}$$

$$\begin{cases} -\alpha + (3-\lambda)y = 0 & (E''''_1) \\ 2y + (1-\lambda)z = 0 & (E''''_2) \\ (9-9\lambda+5\lambda^2-\lambda^3)z = 0 & (E''''_3) \end{cases}$$

$$\begin{cases} -\alpha + (3-\lambda)y = 0 & (E''''_1) \\ 2y + (1-\lambda)z = 0 & (E''''_2) \\ (3-\lambda)(\lambda-2\lambda+3)z = 0 & (E''''_3) \end{cases}$$

Donc si  $(3-\lambda)(\lambda-2\lambda+3) \neq 0$  on a  $z=0$  pour vérifier  $(E''''_3)$ , et donc  $y=0$  pour vérifier  $(E''''_2)$  et  $\alpha=0$  pour vérifier  $(E''''_1)$ . L'unique solution du système est alors  $(0,0,0)$

$$\textcircled{b} (\alpha, y, z) \in \mathbb{C}^3 \text{ solution de } E(\lambda) \Leftrightarrow \begin{cases} (1-\lambda)\alpha + 2y - 2z = 0 \\ -\alpha + (3-\lambda)y = 0 \\ 2y + (1-\lambda)z = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \frac{(1-\lambda)\alpha + 2y - 2z}{(1-\lambda)} = \bar{0} = 0 \\ \frac{-\alpha + (3-\lambda)y}{(1-\lambda)} = \bar{0} = 0 \\ \frac{2y + (1-\lambda)z}{(1-\lambda)} = \bar{0} = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} (1-\lambda)\bar{x} + 2\bar{y} - 2\bar{z} = 0 \\ -\bar{x} + (3-\lambda)\bar{y} = 0 \\ 2\bar{y} + (1-\lambda)\bar{z} = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} (1-\bar{\lambda})\bar{x} + 2\bar{y} - 2\bar{z} = 0 \\ -\bar{x} + (3-\bar{\lambda})\bar{y} = 0 \\ 2\bar{y} + (1-\bar{\lambda})\bar{z} = 0 \end{cases}$$

la bijection demandée  
est donc :

$$S_{E(\lambda)} \longrightarrow S_{E(\bar{\lambda})}$$

$$(x, y, z) \longmapsto (\bar{x}, \bar{y}, \bar{z})$$

$$\Leftrightarrow (\bar{x}, \bar{y}, \bar{z}) \text{ solution de } E(\bar{\lambda})$$

$$\textcircled{c} (3-\lambda)(\lambda^2 - 2\lambda + 3) = 0$$

$$\text{soit } (3-\lambda) = 0 \text{ d'où } \lambda = 3$$

$$\text{soit } (\lambda^2 - 2\lambda + 3) = 0$$

$$\Delta = 4 - 12 = -8 = (2\sqrt{2}i)^2 \quad \lambda_1 = \frac{2 - 2\sqrt{2}i}{2} = 1 - \sqrt{2}i$$

$$\lambda_2 = 1 + \sqrt{2}i$$

$$1^\circ \lambda = 3 \quad E(3) : \begin{cases} -2x + 2y - 2z = 0 \\ -x = 0 \\ 2y - 2z = 0 \end{cases} \text{ d'où } \begin{cases} x = 0 \\ y = z \end{cases}$$

$$\text{donc } S_{E(3)} = \left\{ y(0, 1, 1) \mid y \in \mathbb{C} \right\}$$

droite passant par (0,0,0) de vecteur directeur (0,1,1)

$$2^\circ \lambda = 1 + \sqrt{2}i \quad E(1 + \sqrt{2}i) : \begin{cases} -i\sqrt{2}x + 2y - 2z = 0 \\ -x + (2 - i\sqrt{2})y = 0 \\ 2y - i\sqrt{2}z = 0 \end{cases} \text{ d'où } \begin{cases} x = (2 - i\sqrt{2}) \times \frac{\sqrt{2}iz}{2} = (1 + i\sqrt{2})z \\ y = \frac{\sqrt{2}}{2}z \end{cases}$$

$$\text{donc } S_{E(1 + \sqrt{2}i)} = \left\{ z(1 + i\sqrt{2}, \frac{\sqrt{2}}{2}i, 1) \mid z \in \mathbb{C} \right\}$$

$$3^\circ \text{ D'après } \textcircled{b} \text{ on a } S_{E(1 - \sqrt{2}i)} = S_{E(1 + \sqrt{2}i)} = \left\{ z(1 + i\sqrt{2}, \frac{\sqrt{2}}{2}i, 1) \mid z \in \mathbb{C} \right\}$$

$$\lambda = 1 - \sqrt{2}i$$

non  
mais on  
bijection

$$= \left\{ z(1 - i\sqrt{2}, -\frac{\sqrt{2}}{2}i, 1) \mid z \in \mathbb{C} \right\}$$