ROCCHIA
Laurence

Feuille 2

Exercice 10

1 $$B_2S_3 + 6H_2O \longrightarrow 2H_3BO_3 + 3H_2S$$

2 $$x_1 PbN_6 + x_2 CrMn_2O_8 \longrightarrow x_3 PbO_4 + x_4 Cr_2O_3 + x_5 MnO_2 + x_6 N$$

On équilibre :

Pb : $x_1 = 3x_3$
N : $6x_1 = x_6$
Cr : $x_2 = 2x_4$
Mn : $2x_2 = x_5$
O : $8x_2 = 4x_3 + 3x_4 + 2x_5 + x_6$

On obtient un système du type :

$$\begin{cases} x_1 + 0 - 3x_3 + 0 = 0 & 1 \\ 6x_1 + 0 + 0 + 0 + 0 + 0 = 0 & 2 \\ 0 + x_2 + 0 - 2x_4 + 0 + 0 = 0 & 3 \\ 0 + 2x_2 + 0 + 0 - x_5 + 0 = 0 & 4 \\ 0 + 8x_2 - 4x_3 - 3x_4 - 2x_5 - x_6 = 0 & 5 \end{cases}$$

$$\begin{cases} x_1 - 3x_3 = 0 & & 1' \\ x_2 - 2x_4 = 0 & & 2' \\ 18x_3 - x_6 = 0 & 2 - 6\times 1 = & 3' \\ 4x_4 - x_5 = 0 & 4 - 2\times 3 & 4' \\ -4x_3 + 15x_4 - 2x_5 - x_6 = 0 & 5 - 8\times 3 & 5' \end{cases}$$

$$\begin{cases} x_1 = 3x_3 & (1)'' \\ x_2 = 2x_4 & (2)'' \\ x_3 = \dfrac{x_6}{18} & (3)'' \\ 4x_4 = x_5 & (4)'' \\ 135x_4 - 18x_5 - 11x_6 = 0 & (5)'' = 9 \times (5)' + 2 \times (3) \end{cases}$$

on fait le calcul :

$4 \times (5)'' - 135\,(4)''$ et on obtient :

$$63x_5 - 44x_6 = 0.$$

$\boxed{\text{1 variable libre : } x_6}$

$$\begin{cases} x_1 = \dfrac{x_6}{6} \\ x_2 = \dfrac{22}{63} \\ x_3 = \dfrac{x_6}{18} \\ x_4 = \dfrac{x_5}{4} = \dfrac{11}{63}x_6 \\ x_5 = \dfrac{44}{63}x_6 \end{cases}$$

$$S = \left\{ x_6\left(\frac{1}{6}, \frac{22}{63}, \frac{1}{18}, \frac{11}{63}, \frac{44}{63}, 1\right) \text{ tq } x_6 \in \mathbb{Q} \right\}$$

$$S = \left\{ x_6\,(21, 44, 9, 22, 88, 126)k, \text{ tq } k \in \mathbb{N} \right.$$

Feuille 2
exercice 17

ⓐ $E(\lambda)$: $\begin{cases} (1-\lambda)x + 2y - 2z = 0 & (E_1) \\ -x + (3-\lambda)y = 0 & (E_2) \\ 2y + (1-\lambda)z = 0 & (E_3) \end{cases}$

équivalent à $\begin{cases} -x + (3-\lambda)y = 0 & (E_1') = (E_2) \\ 2y + (1-\lambda)z = 0 & (E_2') = (E_3) \\ [2+(1-\lambda)(3-\lambda)]y - 2z = 0 & (E_3') = (E_1) + (1-\lambda)(E_2) \end{cases}$

$\begin{cases} -x + (3-\lambda)y = 0 & (E_1') \\ 2y + (1-\lambda)z = 0 & (E_2') \\ (5-4\lambda+\lambda^2)y - 2z = 0 & (E_3') \end{cases}$

$\begin{cases} -x + (3-\lambda)y = 0 & (E_1'') = (E_1') \\ 2y + (1-\lambda)z = 0 & (E_2'') = (E_2') \\ [(1-\lambda)(5-4\lambda+\lambda^2)+4]z = 0 & (E_3'') = (5-4\lambda+\lambda^2)(E_2') - 2(E_3') \end{cases}$

$\begin{cases} -x + (3-\lambda)y = 0 & (E_1'') \\ 2y + (1-\lambda)z = 0 & (E_2'') \\ (9-9\lambda+5\lambda^2-\lambda^3)z = 0 & (E_3'') \end{cases}$

$\begin{cases} -x + (3-\lambda)y = 0 & (E_1'') \\ 2y + (1-\lambda)z = 0 & (E_2'') \\ (3-\lambda)(\lambda^2-2\lambda+3)z = 0 & (E_3'') \end{cases}$

Donc si $(3-\lambda)(\lambda^2-2\lambda+3) \neq 0$ on a $z = 0$ pour vérifier (E_3''), et donc $y = 0$ pour vérifier (E_2'') et $x = 0$ pour vérifier (E_1''). L'unique solution du système est alors $(0,0,0)$

ⓑ $(x,y,z) \in \mathbb{C}^3$ solution de $E(\lambda) \iff \begin{cases} (1-\lambda)x + 2y - 2z = 0 \\ -x + (3-\lambda)y = 0 \\ 2y + (1-\lambda)z = 0 \end{cases}$

$\iff \begin{cases} \overline{(1-\lambda)x + 2y - 2z} = \bar{0} = 0 \\ \overline{-x + (3-\lambda)y} = \bar{0} = 0 \\ \overline{2y + (1-\lambda)z} = \bar{0} = 0 \end{cases}$

$$\Leftrightarrow \begin{cases} \overline{(1-\lambda)}\,\overline{x} + 2\overline{y} - 2\overline{z} = 0 \\ -\overline{x} + \overline{(3-\lambda)}\,\overline{y} = 0 \\ 2\overline{y} + \overline{(1-\lambda)}\,\overline{z} = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} (1-\overline{\lambda})\overline{x} + 2\overline{y} - 2\overline{z} = 0 \\ -\overline{x} + (3-\overline{\lambda})\overline{y} = 0 \\ 2\overline{y} + (1-\overline{\lambda})\overline{z} = 0 \end{cases}$$

La bijection demandée si donc :

$$S_{E(\lambda)} \longrightarrow S_{E(\overline{\lambda})}$$
$$(x,y,z) \longmapsto (\overline{x},\overline{y},\overline{z})$$

$\Leftrightarrow (\overline{x},\overline{y},\overline{z})$ solution de $E(\overline{\lambda})$

© $(3-\lambda)(\lambda^2-2\lambda+3)=0$ soit $(3-\lambda)=0$ d'où $\lambda=3$

soit $(\lambda^2-2\lambda+3)=0$

$\Delta = 4-12 = -8 = (2\sqrt{2}i)^2$ $\quad \lambda_1 = \dfrac{2-2\sqrt{2}i}{2} = 1-\sqrt{2}i$

$\lambda_2 = 1+\sqrt{2}i$

1/ $\lambda=3$ $\quad E(3)$: $\begin{cases} -2x+2y-2z=0 \\ -x = 0 \\ 2y-2z=0 \end{cases}$ d'où $\begin{cases} x=0 \\ y=z \end{cases}$

donc $S_{E(3)} = \{ y(0,1,1)\ y \in \mathbb{C} \}$

droite passant par $(0,0,0)$ de vecteur directeur $(0,1,1)$

2/ $\lambda = 1+\sqrt{2}i$ $\quad E(1+\sqrt{2}i)$: $\begin{cases} -i\sqrt{2}x + 2y - 2z = 0 \\ -x + (2-i\sqrt{2})y = 0 \\ 2y - i\sqrt{2}z = 0 \end{cases}$ d'où $\begin{cases} x = (2-i\sqrt{2}) \times \frac{\sqrt{2}}{2}iz = (1+i\sqrt{2})z \\ y = \frac{\sqrt{2}}{2}z \end{cases}$

donc $S_{E(1+\sqrt{2}i)} = \{ z(1+i\sqrt{2}, \frac{\sqrt{2}}{2}i, 1)\ z \in \mathbb{C} \}$

3/ D'après Ⓑ on a $S_{E(1-\sqrt{2}i)} = S_{E(\overline{1+\sqrt{2}i})} = \{ z(\overline{1+i\sqrt{2}}, \overline{\frac{\sqrt{2}}{2}i}, \overline{1})\ z \in \mathbb{C} \}$

$\lambda = 1-\sqrt{2}i$

non mais en bijection

$= \{ z(1-i\sqrt{2}, -\frac{\sqrt{2}}{2}i, 1)\ z \in \mathbb{C} \}$