Homework II : Finite difference approximation of $\frac{d^2T}{dx^2} = S(x)$

Numerical approximation is not an elegant subject. It is a collection of technical details and dirty work. However, is the more convenient way to solve real world problems.

WILL BE COLLECTED AT 25 MARCH 2009 , 10H (PAPER WRITING ONLY).

1. We consider a regular mesh $x_i = i\delta x$ on the real axis.
   (a) Find an finite difference approximation $\widetilde{\frac{d^2\tilde{T}}{dx^2}}(x_i)$ of $\frac{d^2T}{dx^2}$ at the point x_i by using Taylor expansion for

   stencil 1 : $T(x_{i-2})$ and $T(x_{i+2})$,

   stencil 2 : $T(x_{i-2})$ and $T(x_{i+1})$,

   stencil 3 : $T(x_{i-2})$ and $T(x_{i+2})$,

   stencil 4 : $T(x_{i-2}), T(x_{i-1}), T(x_{i+1})$ and $T(x_{i+2})$.

   (b) Then compute in each case the order of accuracy of the scheme defined as

   $$-\widetilde{\frac{d^2\tilde{T}}{dx^2}}(x_i) = S(x_i)$$

   (c) For the case "stencil 4" can you find a scheme that is 4 order accurate ?

2. Let us consider a general case where the mesh defined by the points $x_{i-1} < x_i < x_{i+1}$.
   (a) Find an finite difference approximation $\widetilde{\frac{d^2\tilde{T}}{dx^2}}(x_i)$ of $\frac{d^2T}{dx^2}$ at the point x_i by using Taylor expansion for

   stencil B1 : $T(x_{i-1})$ and $T(x_{i+1})$,

   stencil B4 : $T(x_{i-2}), T(x_{i-1}), T(x_{i+1})$ and $T(x_{i+2})$.

   (b) Then compute in each case the order of accuracy of the scheme defined as

   $$-\widetilde{\frac{d^2\tilde{T}}{dx^2}}(x_i) = S(x_i)$$

   (c) For the case "stencil B4" can you find a combination to obtain more accurate scheme ?