From modelization to simulation

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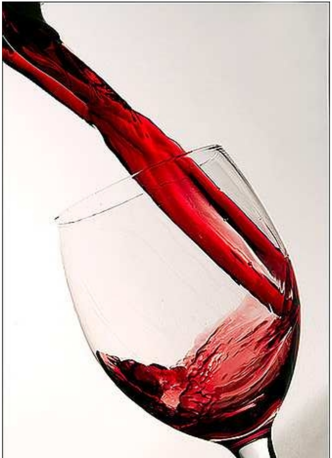
MathMods 2010

To enjoy and understand!



Introduction : Why we need computation?

To prevent some humanity disaster, at least we hope so!

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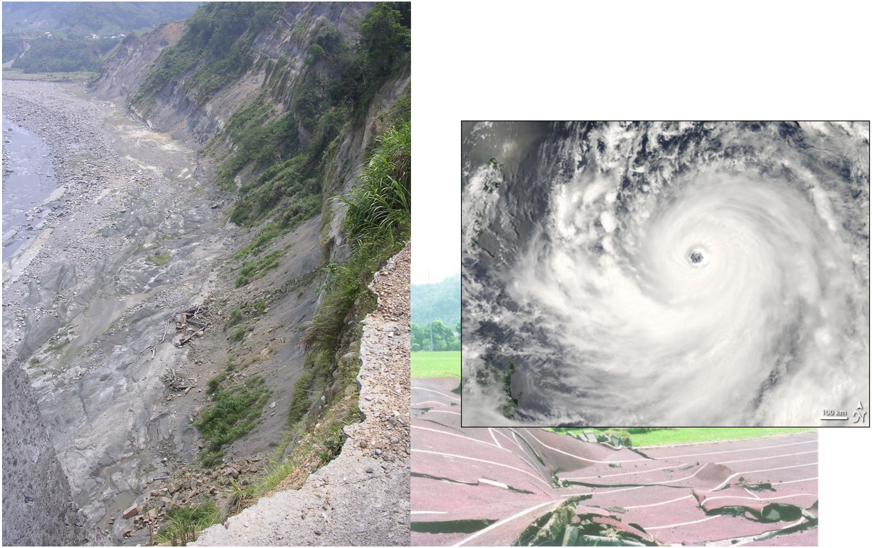
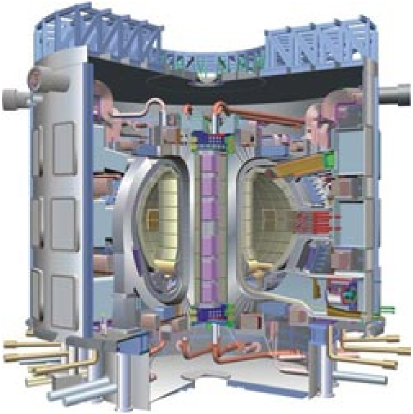


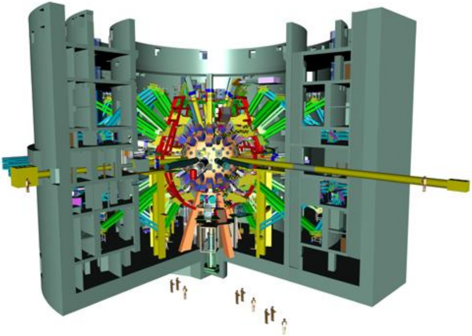
To prevent consequences of some natural disaster!

Introduction : Why we need computation?

ITER Int. Project.



LMJ & NIF : Laser/plasmas



To help meet mankind's future energy needs : Fusion

Are computations possibles?

Yes, Because we have

Fundamental Laws

Conservation of mass, Momentum, Energy.
Laws of the Thermodynamics: (1st, 2nd, 3th) Gauss, Ampère's, Faraday Laws

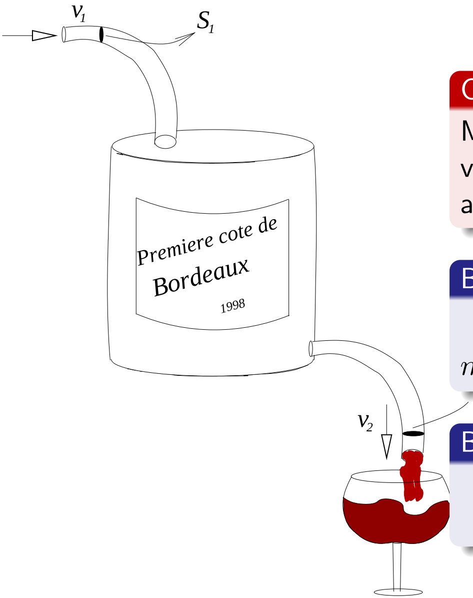
Computers are efficient

Tera-FLOPS computers availables 10^{12} FLoating point Operations Per Second.

Numerical approximation strategies

Finite volume, Finite element, Finite difference, Particles In Cells, ..., structured/unstructured mesh, Parallel programming (MPI),

Balance "relations" or equations!



Conservation Law: for Mass

Mass fluctuation, in a control volume, is the sum of outgoing and incoming mass:

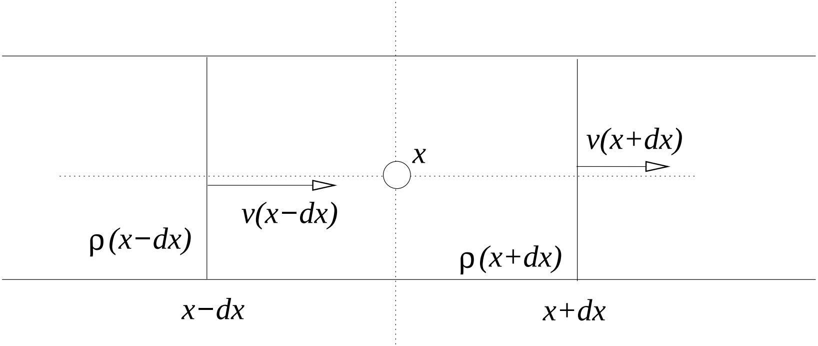
Balance Relation

$$m(t+dt) = m(t)+\rho_1 S_1 v_1 - \rho_2 S_2 v_2$$

Balance equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

Balance "relations" or equations!

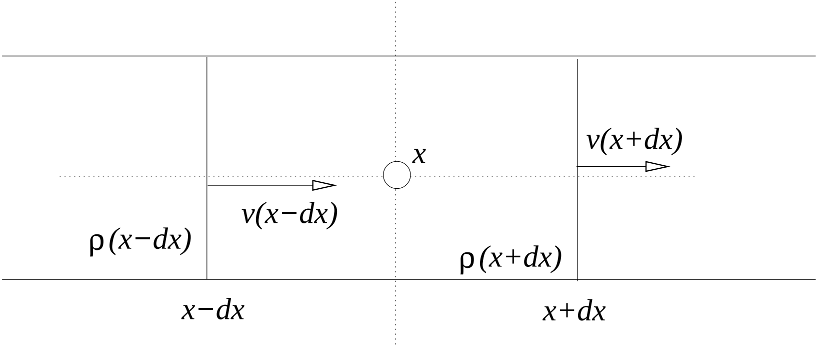


From relations to equations

$$m(x,t+dt) = m(x,t) \quad + S\int_t^{t+dt} f(x-dx,s)ds$$
$$\qquad\qquad\qquad\qquad - S\int_t^{t+dt} f(x+dx,s)ds$$

where the flux is defined by $f(x,t) = \rho(x,t)v(x,t)$

Balance “relations” or equations!



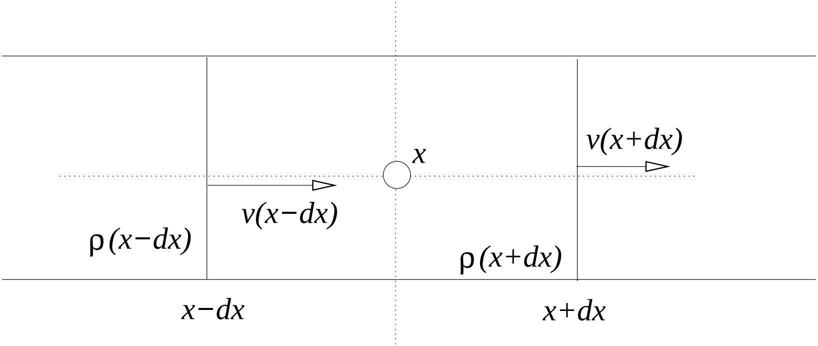
From relations to equations

$$m(x, t + dt) = m(x, t) + S dt f(x - dx, t) - S dt f(x + dx, t)$$

$$m(x, .) = 2 S dx \rho(x, .)$$

where the flux is defined by $f(x, t) = \rho(x, t) v(x, t)$

Balance "relations" or equations!



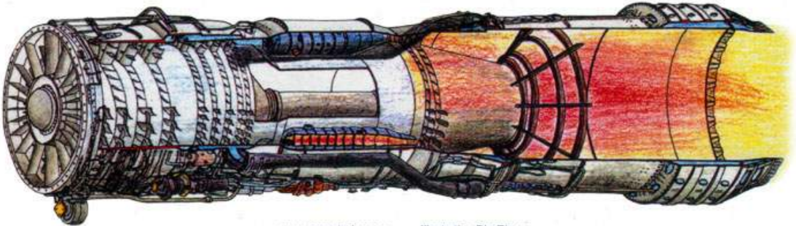
From relations to equations $dx \to 0$ and $dt \to 0$

$$\frac{\rho(x,t+dt)-\rho(x,t)}{dt} = -\frac{f(x+dx,t)-f(x-dx,t)}{2dx}$$

$$\Longrightarrow \frac{\partial \rho}{\partial t} + \frac{\partial f}{\partial x} = 0 \Longrightarrow \frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0$$

where the flux is defined by $f(x,t) = \rho(x,t)v(x,t)$

Balance equations : general 1D case

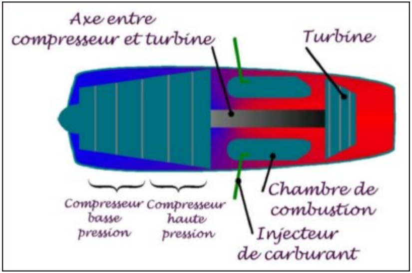


$$\frac{\partial \omega}{\partial t} + \frac{\partial f(\omega)}{\partial x} = \mathcal{S}$$

where $\mathcal{S}$ can be defined, for example, by the chemistry process, geometrical topology ...

$$\omega(x,t) = \begin{pmatrix} \rho \\ \rho Y \\ \rho u \\ E \end{pmatrix} \quad \text{and} \quad f(\omega) = \begin{pmatrix} \rho u \\ \rho Y u \\ \rho u^2 + p \\ (E+p)u \end{pmatrix}$$

Balance equations : general 1D case



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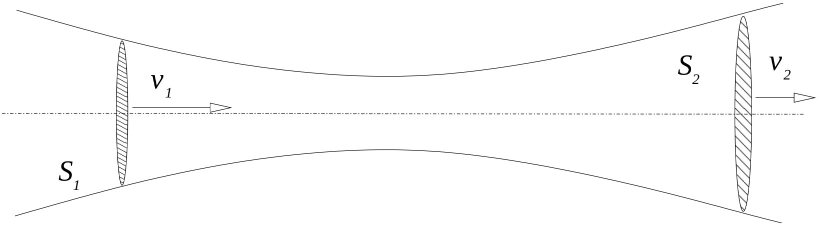

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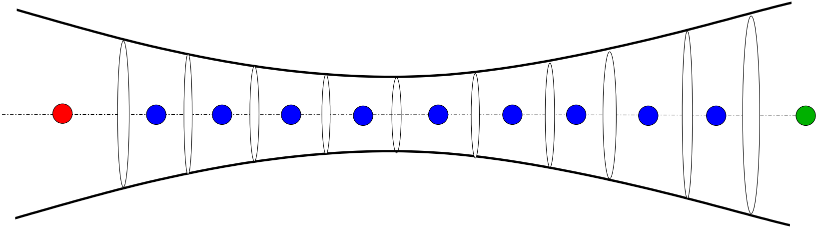
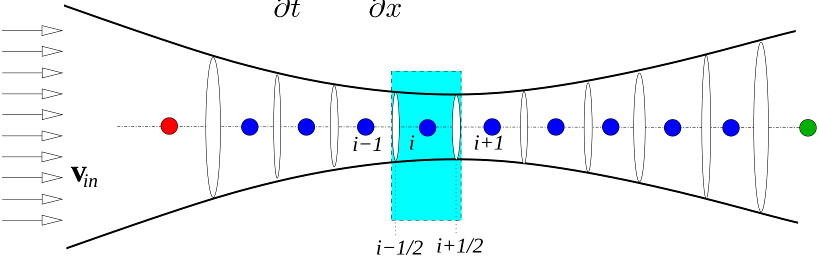


Mesh and Contol volumes

Finite volume Scheme: 1D case

$$\frac{\partial \omega}{\partial t} + \frac{\partial f(\omega)}{\partial x} = \mathcal{S}$$

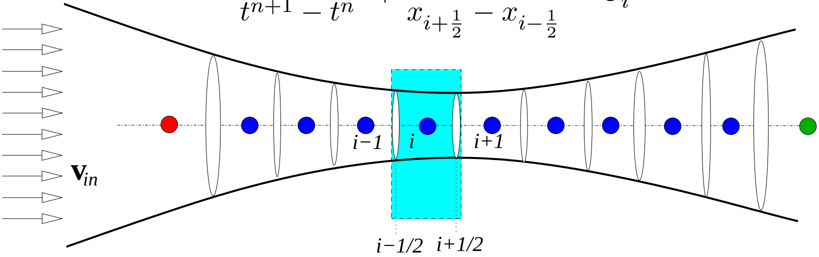


$$\frac{\omega_i^{n+1} - \omega_i^n}{t^{n+1} - t^n} + \frac{\phi^m_{i+\frac{1}{2}} - \phi^m_{i-\frac{1}{2}}}{x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}} = \mathcal{S}_i^m$$

Balance relations on a Contol volume

Finite volume Scheme: 1D case

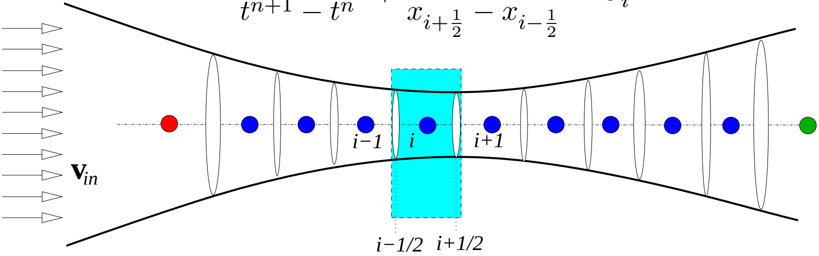
$$\frac{\omega_i^{n+1} - \omega_i^n}{t^{n+1} - t^n} + \frac{\phi^m_{i+\frac{1}{2}} - \phi^m_{i-\frac{1}{2}}}{x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}} = \mathcal{S}_i^m$$



How to define the numerical flux $\phi \simeq f$

Finite volume Scheme: 1D case

$$\frac{\omega_i^{n+1} - \omega_i^n}{t^{n+1} - t^n} + \frac{\phi^m_{i+\frac{1}{2}} - \phi^m_{i-\frac{1}{2}}}{x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}} = \mathcal{S}_i^m$$



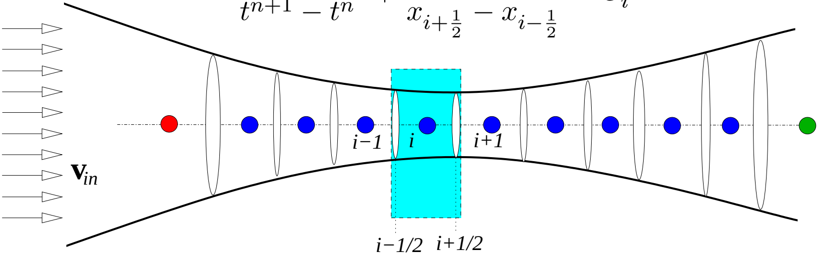
Centered scheme:

$$\phi_{i+\frac{1}{2}} = \frac{f_i + f_{i+1}}{2}$$

Accurate but Unstable!

Finite volume Scheme: 1D case

$$\frac{\omega_i^{n+1} - \omega_i^n}{t^{n+1} - t^n} + \frac{\phi_{i+\frac{1}{2}}^m - \phi_{i-\frac{1}{2}}^m}{x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}} = \mathcal{S}_i^m$$



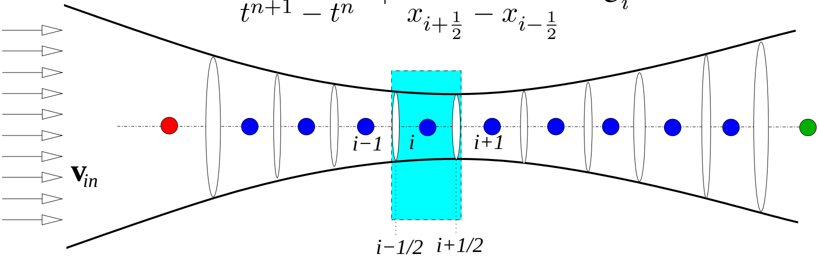
Upwind scheme (in this case):

$$\phi_{i+\frac{1}{2}} = f_i \quad \text{and} \quad \phi_{i-\frac{1}{2}} = f_{i-1}$$

Less accurate but Stable √

Finite volume Scheme: 1D case

$$\frac{\omega_i^{n+1} - \omega_i^n}{t^{n+1} - t^n} + \frac{\phi^m_{i+\frac{1}{2}} - \phi^m_{i-\frac{1}{2}}}{x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}} = \mathcal{S}_i^m$$

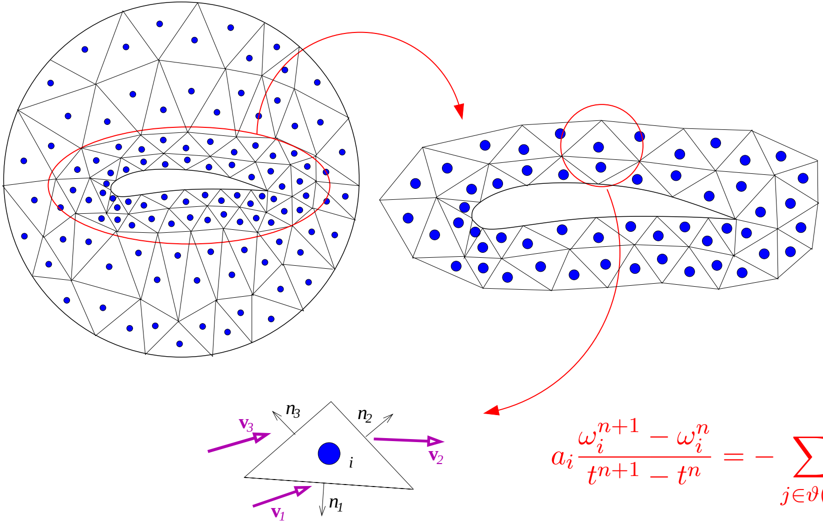


General rule:

The numerical flux should be consistent with the physics

Upwind : Follows informations traveling in the correct direction.

Finite volume Scheme: 2D case



$$a_i \frac{\omega_i^{n+1} - \omega_i^n}{t^{n+1} - t^n} = - \sum_{j \in \vartheta(i)} \Phi_{i,j}$$

$\phi_{i,j}$ is now the flux crossing an interface from the cell i to j.

Need properties for $\phi_{i,j}$

$$\phi_{i,j} = \phi(\omega_i, \omega_j)$$

can be obtained by the resolution of a simplify problem: Riemann Problem.

Properties we want the numerical flux to satisfy

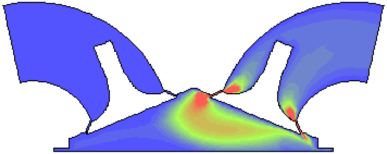
- Consistency, Stability, Convergence
- Positivity and maximum principles $\rho \geq 0$, $||v|| \leq c$.
- Second thermodynamic law (entropy production).
- Accuracy : have a better result with a given mesh.

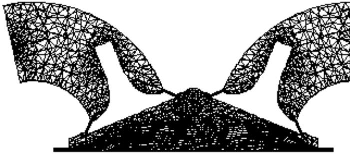
Lax Theorem for conservative systems

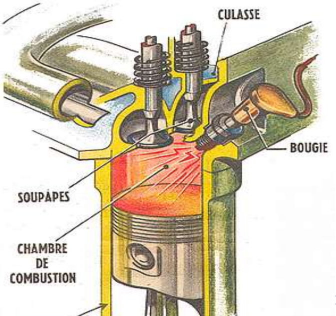
Consistency + Stability = Convergence

Four valves Diesel engine flow : Moving Boundaries

B. Nkonga (97).







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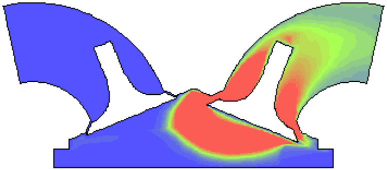

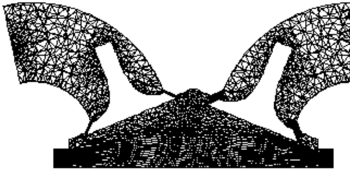

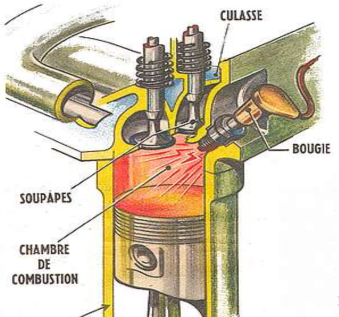

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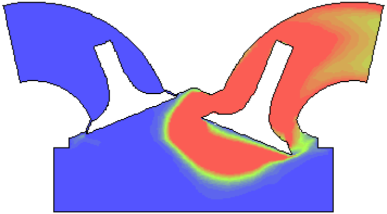

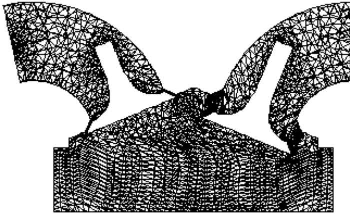

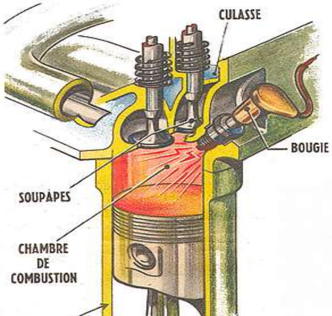

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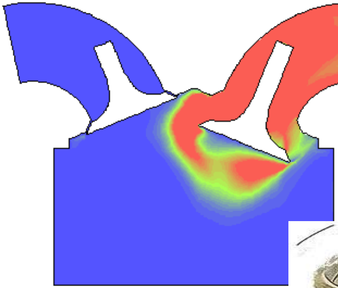

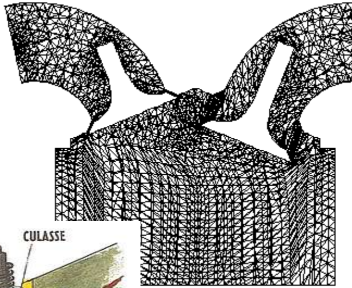

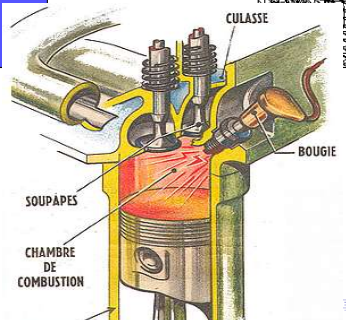

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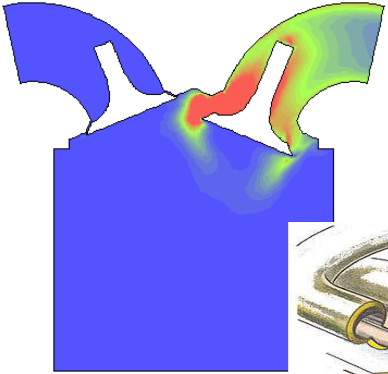

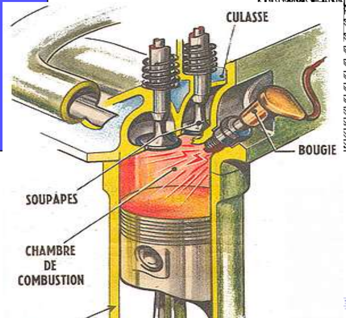

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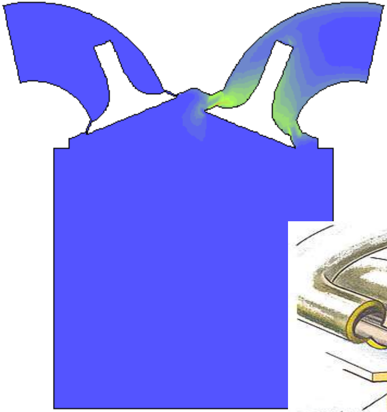
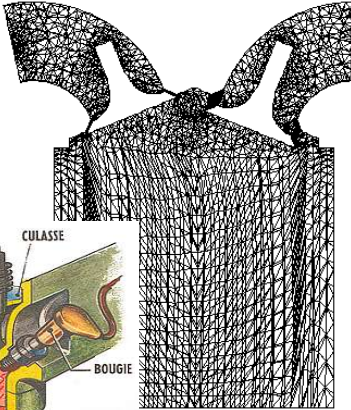
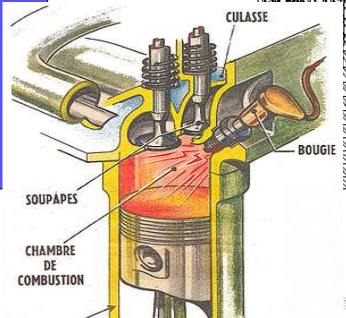


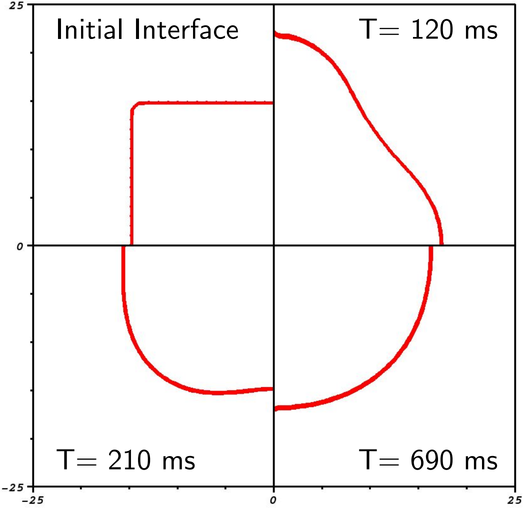


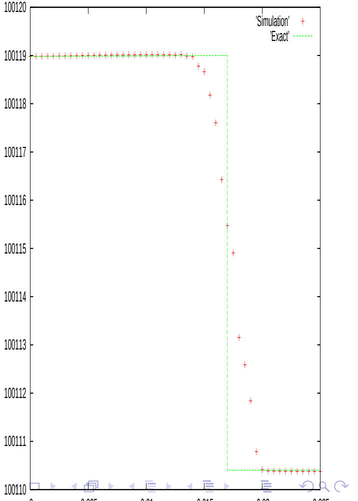


Low Mach interface flow with surface Tension

B. Nkonga & B. Braconnier (07)

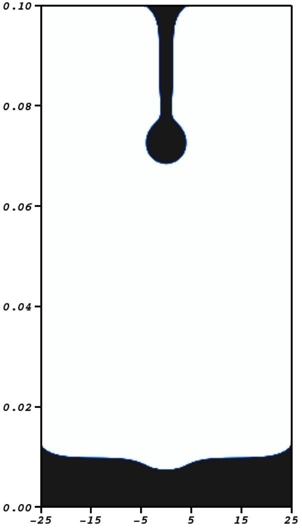
Laplace Law Recovered : $\delta p = \sigma \kappa$





Water drop in air, gravity and Surface tension

B. Nkonga & B. Braconnier (07)



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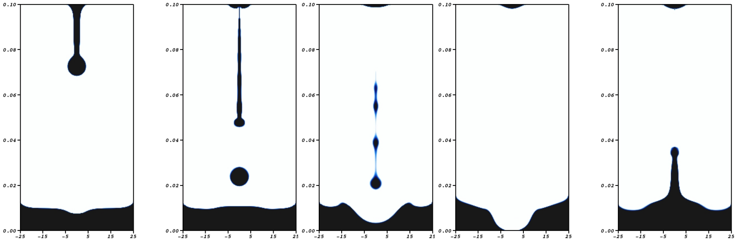
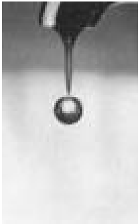
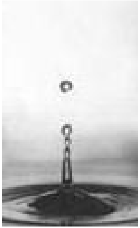
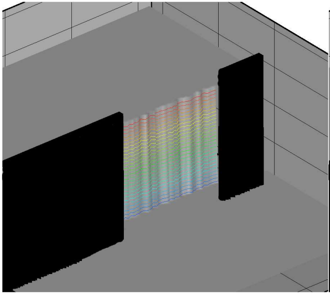


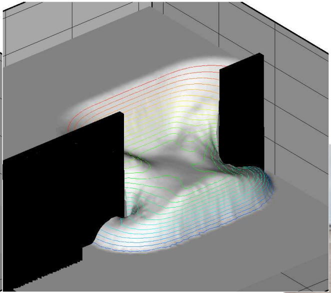


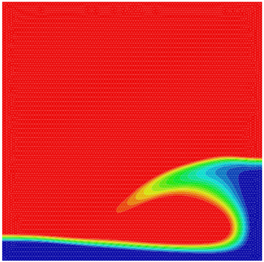


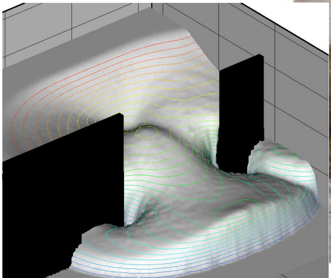
Dam Break : Shalow water and Multiphase flow

R. Abgrall & M. Ricchiuto (05)



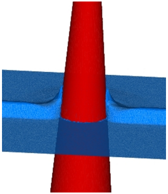


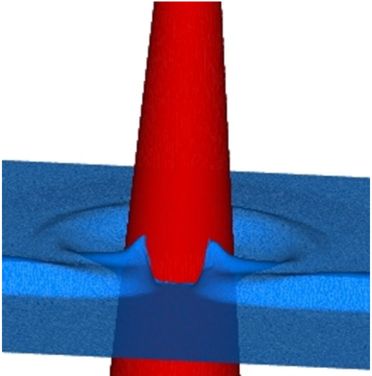


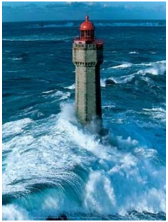


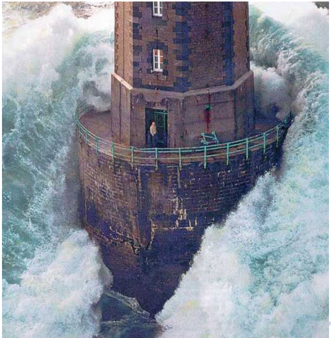
Shalow water and Multiphase flow

R. Abgrall & M. Ricchiuto (05)



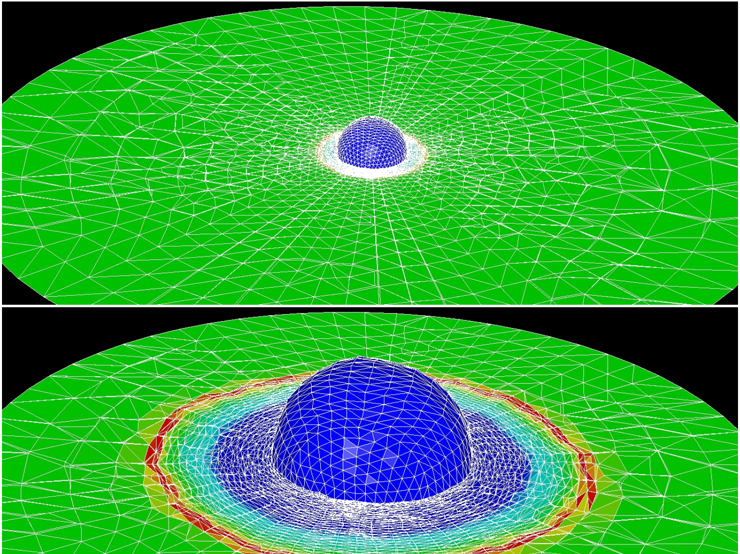






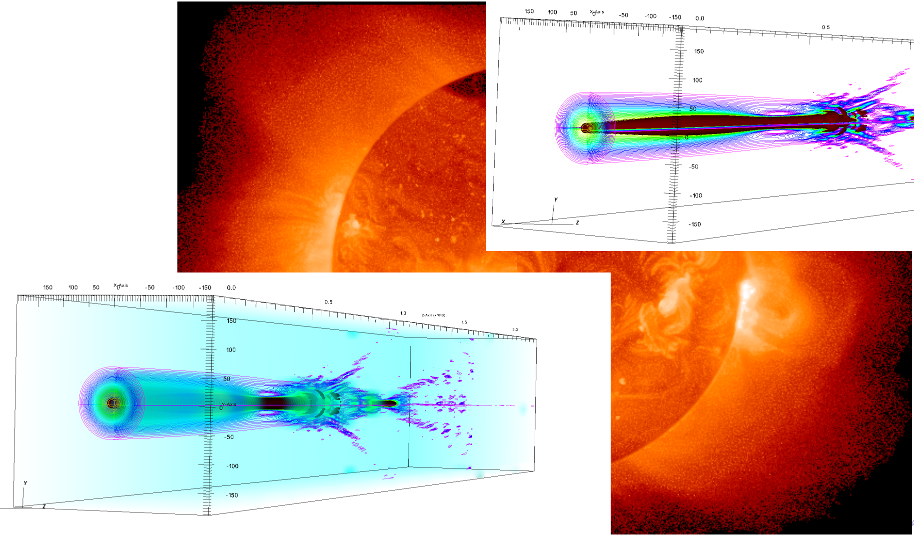
Explosion and propagation : Mesh follows the shock font.

R. Abgrall & P-H Maire & B. Nkonga (08)

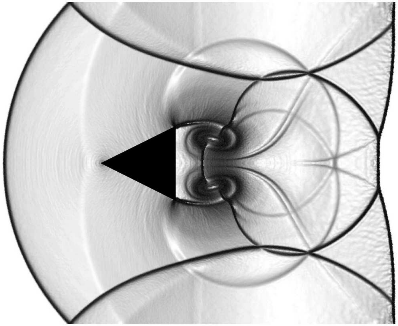


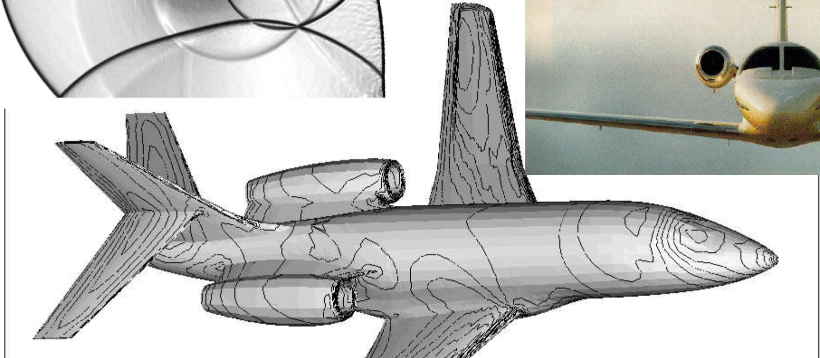
Laser / Plasmas Interactions for Nuclear Fusion

A. Bellue & A. Bourgeade & B. Nkonga & S. Weber (08)



Computation and reality : Aerodynamic flow



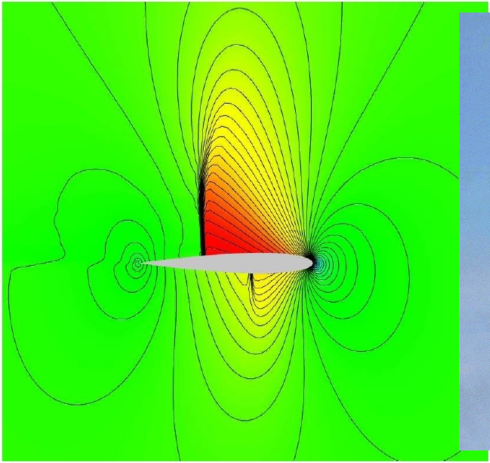


R. Abgrall & M. Ricchiuto (05)

European ADIGMA project

High speed aerodynamics : Choc and condensation

INRIA Bordeaux Sud-Ouest Team & ADIGMA

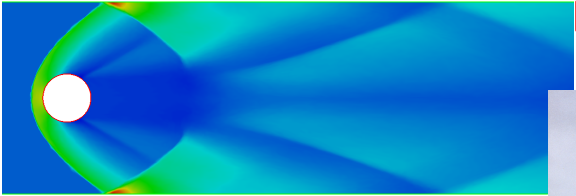


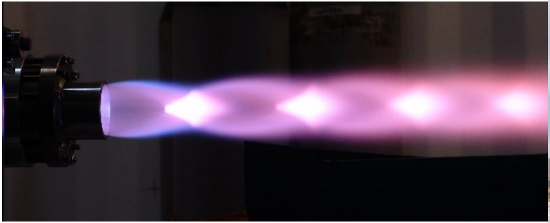
Condensation gives a view of the choc wave profile.

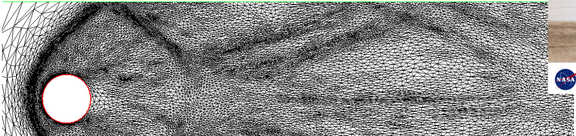
Shock waves interactions : Diamond structure

C. Dobrisinsky & B. Nkonga & M. Ricchiuto (07)

FluidBox

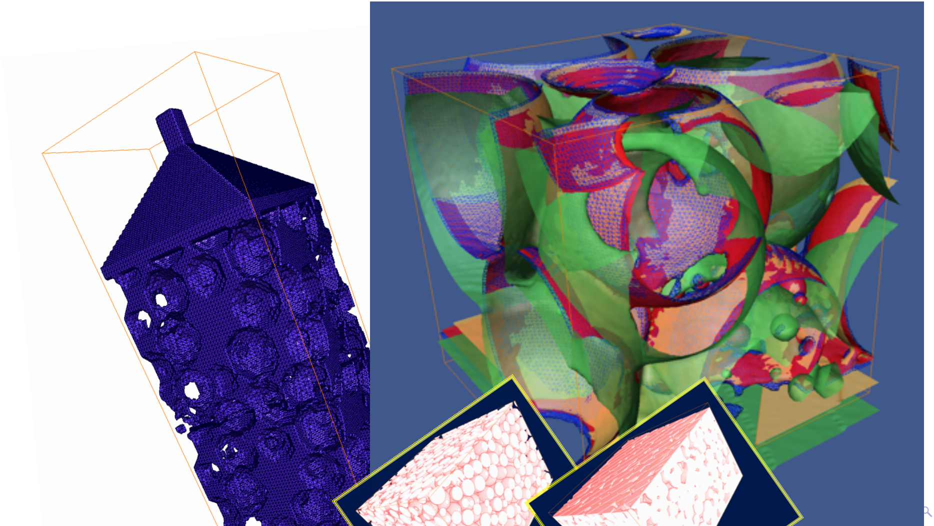






Material & Numerics

(P. Charrier, D. Bernard, B. Nkonga, M. Ricchiuto, G. Vignoles)



From modelization to simulation

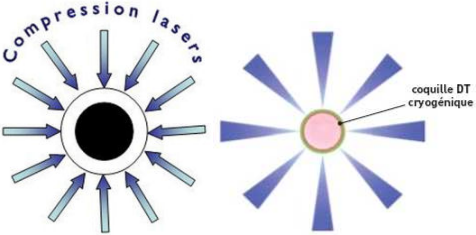
B. Nkonga

Univ. Nice & INRIA Sophia-Antipolis

MathMods 2010

Inertial Confinement Fusion : Compression Ignition

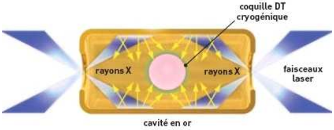
- Ignition by lasers compression



- Interfaces flow
- Large ratio computational domain change.

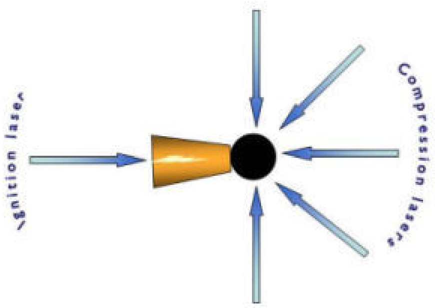
Needs : Lagrangian Hydrodynamics Scheme...

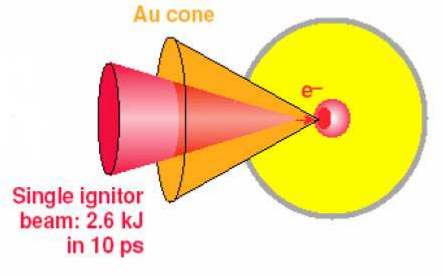
- Ignition by X-rays compression



Needs : Compute Focusing Beams interactions...

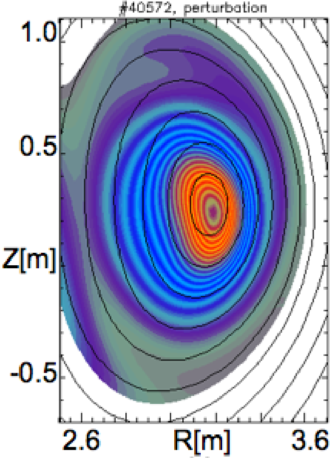
Inertial Confinement Fusion : Fast laser ignition

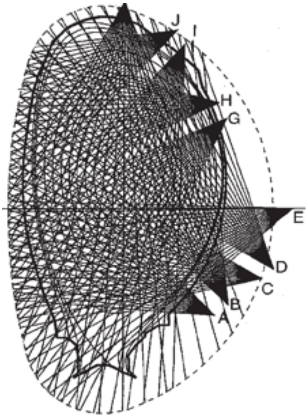




Need : Nonlinear laser/plasmas model, non-local Transport...

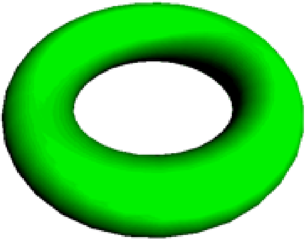
Tokamak Plasma : Avoid large scale instabilities.

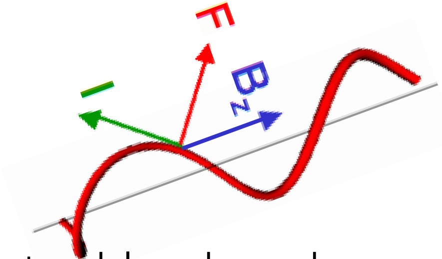




Tomographic reconstruction of X-ray emission. JET SXR cameras (1998).

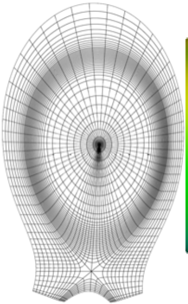
Tokamak Plasma : Kink instability (MHD, Jorek).

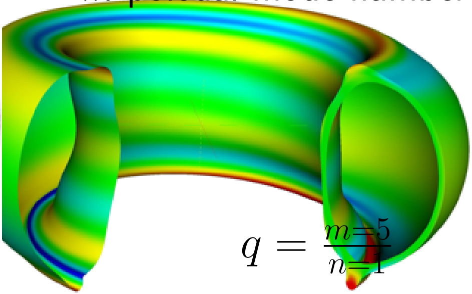




n torodal mode number
m polodal mode number.

$$q = \frac{m{=}3}{n{=}1}$$



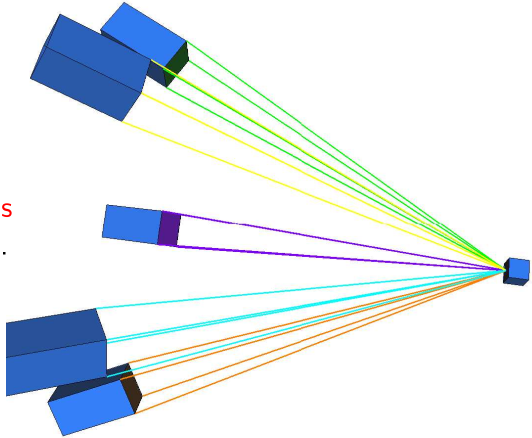


$$q = \frac{m{=}5}{n{=}1}$$

Laser focusing (ICF): Huygens-Fresnel Theorem.

$\simeq 30$ Laser Quads

$4(40cm \times 40cm)$.



Focal spot

$\simeq 1mm \times$

$$\boldsymbol{E}(t,\boldsymbol{x}) = \int_{\mathbb{R}^2} d\boldsymbol{\theta} \int_{\mathbb{R}} d\omega\, \hat{\mathcal{E}}(\boldsymbol{\theta},\omega)\ \exp\left[\imath\beta\boldsymbol{\theta}\cdot\boldsymbol{\xi}(\boldsymbol{x}) + k_0 n_0(\boldsymbol{x}) - \omega t\right]$$

Focusing Laser : Approximations and Computations.

$$\boldsymbol{E}(\boldsymbol{x},t) = \sum_{\ell=1}^{N_\ell}\sum_{m=1}^{N_w}\sum_{i=1}^{Nk_x}\sum_{j=1}^{Nk_y} \mathcal{E}_{\ell,i,j,m}\mathcal{G}_{\ell,h}(t,\boldsymbol{x},\omega_m,\boldsymbol{\theta}_{ij})e^{\imath\Phi_{l,i,j,m}}e^{\imath\omega_m \beta t - \frac{n_\ell(\boldsymbol{x})}{c}}$$

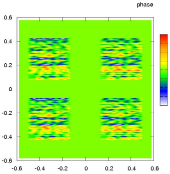
where $\quad \Phi_{l,i,j,m} = \boldsymbol{\beta}_\ell(\boldsymbol{x})\cdot\boldsymbol{k}_\ell(\boldsymbol{\theta}_{ij},\omega_m) + \dfrac{\omega_m n_\ell(\boldsymbol{x})}{c}$

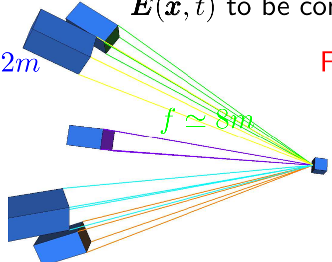
$$\boldsymbol{k}_\ell(\boldsymbol{\theta},\omega) = k_{0,\ell}(\boldsymbol{\theta},\omega)\boldsymbol{n}_{0,\ell} + \theta_{1,\ell}\boldsymbol{n}_{1,\ell} + \theta_{2,\ell}\boldsymbol{n}_{2,\ell}$$

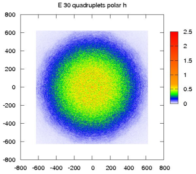
$$\boldsymbol{\beta}_\ell(\boldsymbol{x}) = \boldsymbol{x} - \boldsymbol{x}_\ell = n_\ell(\boldsymbol{x})\boldsymbol{n}_{0,\ell} + \xi_{1,\ell}(\boldsymbol{x})\boldsymbol{n}_{1,\ell} + \xi_{2,\ell}(\boldsymbol{x})\boldsymbol{n}_{2,\ell}$$

Given $\mathcal{E}_{\ell,i,j,m}$
$N_l = 30$ quads
Quad $\simeq 1.2m \times 1.2m$

$\boldsymbol{E}(\boldsymbol{x},t)$ to be computed with the resolution :
$Nf_x \times Nf_y \times Nf_t$
Focal spot : $1.2mm \times 1.2mm$







Focusing Laser Beams : CPU Consuming

Mega Joule Laser (Bordeaux) : Input and Operations

$$N_w \simeq 10^3, \quad Nk_x \simeq Nk_y \simeq 10^3 \quad N_\ell \simeq 30\text{quads}$$

$$\text{Total of Floating Operations } = \text{Flop} \simeq 3 \times 10^{10} \times \mathcal{R}_f$$

$$\mathcal{R}_f = Nf_x \times Nf_y \times Nf_t \texttt{ is the focal spot resolution}$$

CPU Time for $\mathcal{R}_f = 2048 \times 2048 \times 1000 \simeq 4 \times 10^9$

$$\text{Flop} \simeq 10^{20} = 10^8\text{Tera} \Longrightarrow \underline{28 \times 10^3 \text{ H with a Tera computer}} \equiv 3 \text{ yea}$$

Parallel Computing unavoidable !
Need of mathematical approximations!
Stationary phase approximation : asymptotic of oscillatory integrals

Focusing Laser Beams : Parallel strategy (simple)

- Scatter the Flop:

$$\boldsymbol{E}(\boldsymbol{x}, t, \mathtt{me}) = \sum_{\ell} \sum_{m_1(\mathtt{me})}^{m_N(\mathtt{me})} \sum_{i_1(\mathtt{me})}^{iN(\mathtt{me})} \sum_{j_1(\mathtt{me})}^{j_N(\mathtt{me})} \mathcal{E}_{\ell,i,j,m} \mathcal{G}_{\ell,h} e^{\imath \Phi_{l,i,j,m}} e^{\imath \omega_m \tau}$$

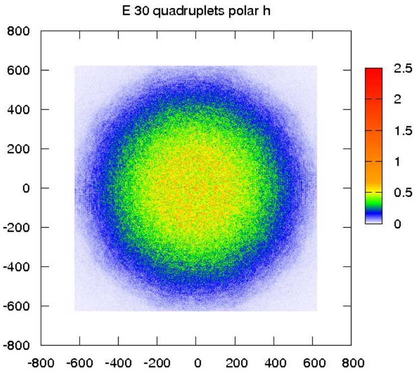
- Gather the solution :

$$\boldsymbol{E}(\boldsymbol{x}, t) = \sum_{\mathtt{me}=0}^{N_p-1}{}^{\oplus} \boldsymbol{E}(\boldsymbol{x}, t, \mathtt{me})$$

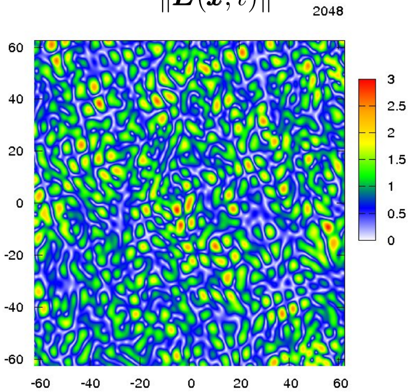
FFTW + MPI

Focusing Laser Beams : Computed focal spot and zoom.

$\|\boldsymbol{E}(\boldsymbol{x},t)\|$



$\|\boldsymbol{E}(\boldsymbol{x},t)\|$

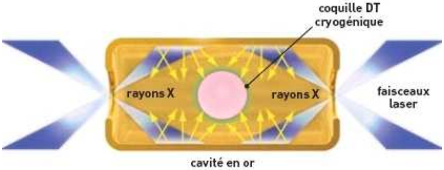


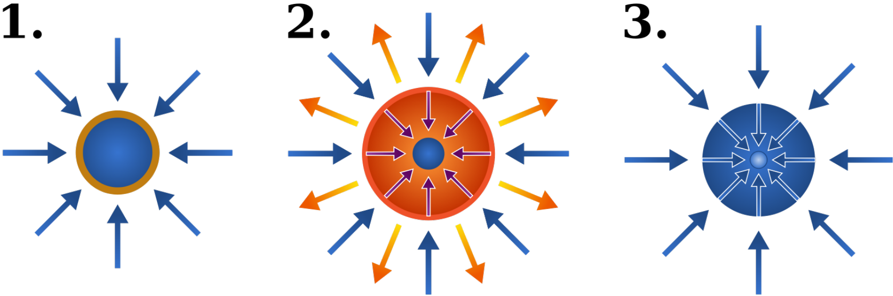
21H CPU with 1936 cores : 30 quads = 120 (40cm x 40cm)

Nodes : 4 IntelItanium II, Dual-Core, 1.6 Ghz, 128 Go

$N_w = Nf_t = 1$, $Nk_x = Nk_y = 2048$ and $Nf_x = Nf_y = 2048$

From the focal spot to Lagrangian Hydrodynamics





Weak formulation of Conservation Law

Integral form of in arbitrary coordinates

$$
\left\{
\begin{array}{lcl}
\dfrac{d}{dt}\displaystyle\int_{\mathcal{C}(t)} \rho \, d\boldsymbol{x} + \int_{\partial\mathcal{C}(t)} \rho\boldsymbol{\beta u} - \boldsymbol{\kappa}\cdot\boldsymbol{n}\, dS & = & 0, \\
\dfrac{d}{dt}\displaystyle\int_{\mathcal{C}(t)} \rho\boldsymbol{u} \, d\boldsymbol{x} + \int_{\partial\mathcal{C}(t)} \rho\boldsymbol{u\beta u} - \boldsymbol{\kappa}\cdot\boldsymbol{n}\, dS & = & -\displaystyle\int_{\partial\mathcal{C}(t)} p\boldsymbol{n}\, dS, \\
\dfrac{d}{dt}\displaystyle\int_{\mathcal{C}(t)} \rho\mathbf{e} \, d\boldsymbol{x} + \int_{\partial\mathcal{C}(t)} \rho\mathbf{e}\boldsymbol{\beta u} - \boldsymbol{\kappa}\cdot\boldsymbol{n}\, dS & = & -\displaystyle\int_{\partial\mathcal{C}(t)} p\boldsymbol{u}\cdot\boldsymbol{n}\, dS.
\end{array}
\right.
$$

Lagrangian controle volume : $\boldsymbol{\kappa} = \boldsymbol{u}$ on $\partial\mathcal{C}(t)$

- $\tilde{m}_C \dfrac{d}{dt}\tilde{\boldsymbol{u}}_C = -\displaystyle\int_{\partial\mathcal{C}_C(t)} p\boldsymbol{n}\, dS,$
- $\tilde{m}_C \dfrac{d}{dt}\tilde{e}_C = -\displaystyle\int_{\partial\mathcal{C}_C(t)} p\boldsymbol{\kappa}\cdot\boldsymbol{n}\, dS.$

where

- $\dfrac{d}{dt}\tilde{m}_C = 0,$

Coupling between averaged(mesh scale) and subscale states.

Multiscale formulation and approximation

Mesh scale equations

$$\bullet \quad \tilde{m}_C \frac{d}{dt}\tilde{\boldsymbol{u}}_C = -\int_{\partial \mathcal{C}_C(t)} p\boldsymbol{n}\, dS, \qquad \text{where} \quad \bullet \quad \frac{d}{dt}\tilde{m}_C = 0,$$

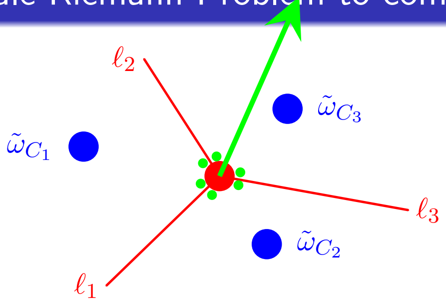
$$\bullet \quad \tilde{m}_C \frac{d}{dt}\tilde{e}_C = -\int_{\partial \mathcal{C}_C(t)} p\boldsymbol{\kappa}\cdot\boldsymbol{n}\, dS.$$

sub-scale $\mathcal{K}^{\epsilon}_{j,C} \subset \mathcal{C}_C$ and $\mathcal{K}^{\epsilon}_{j,C} \subset \mathcal{K}^{\epsilon}_{j}$ :: $\epsilon \mapsto 0$

$$\frac{d}{dt}\int_{\mathcal{K}^{\epsilon}_{j}(t)} \rho\boldsymbol{\kappa}\, d\boldsymbol{x} = -\int_{\partial\mathcal{K}^{\epsilon}_{j}(t)} p\boldsymbol{n}\, dS, \qquad |\mathcal{K}^{\epsilon}_{j}| \simeq O(\epsilon)$$

$$\frac{d}{dt}\int_{\mathcal{K}^{\epsilon}_{j,C}(t)} \rho\boldsymbol{\kappa}\, d\boldsymbol{x} = -\int_{\partial\mathcal{K}^{\epsilon}_{j,C}(t)} p\boldsymbol{n}\, dS, \qquad |\mathcal{K}^{\epsilon}_{j,C}| \simeq O(\epsilon)$$

Subscale Riemann Problem to compute $p^*_{C,\ell,j}$ and $\boldsymbol{\kappa}_j$



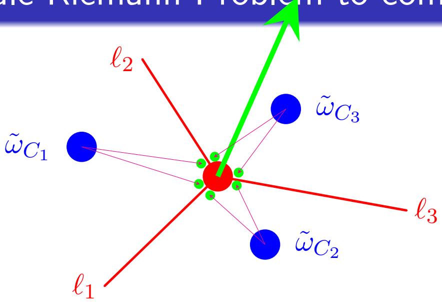
Half Riemann Solver for $\boldsymbol{\kappa}_j$ fixed: Godunov-type method

$$p^*_{C,\ell,j} = p^*_\ell \boldsymbol{\beta}\tilde{\omega}_C, \boldsymbol{\kappa}_j = p_C + Z_{C,j}\boldsymbol{\beta}\tilde{\boldsymbol{u}}_C - \boldsymbol{\kappa}_j \cdot \frac{\boldsymbol{m}_{C,\ell,j}}{\|\boldsymbol{m}_{C,\ell,j}\|}$$

Subscale compatibility: The local system can be nonlinear: $\hookrightarrow \boldsymbol{\kappa}_j$

$$\sum_{\ell \in \vartheta(j)} r_\ell (p^*_{C_1,\ell,j} - p^*_{C_2,\ell,j}) n_{C_1,\ell} = 0$$

Subscale Riemann Problem to compute $p^*_{C,\ell,j}$ and $\boldsymbol{\kappa}_j$



Half Riemann Solver for $\boldsymbol{\kappa}_j$ fixed: Godunov-type method

$$p^*_{C,\ell,j} = p^*_\ell \boldsymbol{\beta}\tilde{\omega}_C, \boldsymbol{\kappa}_j = p_C + Z_{C,j}\boldsymbol{\beta}\tilde{\boldsymbol{u}}_C - \boldsymbol{\kappa}_j \cdot \frac{\boldsymbol{m}_{C,\ell,j}}{\|\boldsymbol{m}_{C,\ell,j}\|}$$

Subscale compatibility: The local system can be nonlinear: $\hookrightarrow \boldsymbol{\kappa}_j$

$$\sum_{\ell \in \vartheta(j)} r_\ell (p^*_{C_1,\ell,j} - p^*_{C_2,\ell,j}) n_{C_1,\ell} = 0$$

Numerical Scheme : Linear mapping for $\psi_{\ell,j}(\xi)$

$$p^*_\ell \boldsymbol{\beta} \tilde{\omega}_C, \boldsymbol{\kappa}_j = p_C + Z_{C,j} \boldsymbol{\beta} \tilde{\boldsymbol{u}}_C - \boldsymbol{\kappa}_j \cdot \boldsymbol{m}_{C,\ell,j}$$

Explicit scheme : $\left[\mathcal{A}_j \boldsymbol{\beta} \tilde{\omega}^n_*,\ \boldsymbol{\kappa}^n_*\right] \boldsymbol{\kappa}^*_j = \boldsymbol{g}_j \boldsymbol{\beta} \tilde{\omega}^n_*,\ \boldsymbol{\kappa}^n_* \longrightarrow \boldsymbol{x}^{n+\theta}$

$$\boldsymbol{\kappa}_\ell(\boldsymbol{x}) = \sum \varphi_{\ell,j}(\mathbf{x}) \boldsymbol{\kappa}^*_j, \qquad p_{C,\ell}(\boldsymbol{x}) = \sum \psi_{\ell,j}(\mathbf{x}) p^*_\ell \boldsymbol{\beta} \tilde{\omega}^n_C, \boldsymbol{\kappa}^*_j$$

$$\tilde{m}_C \frac{d}{dt} \tilde{\boldsymbol{u}}_C = - \sum_{\ell \in \partial C} \int_\ell p_{C,\ell}(\boldsymbol{x}) \boldsymbol{n}(\boldsymbol{\kappa}^n_*)\, d\ell,$$

$$\tilde{m}_C \frac{d}{dt} \tilde{e}_C = - \sum_{\ell \in \partial C} \int_\ell p_{C,\ell}(\boldsymbol{x}) \boldsymbol{\kappa}_\ell(\boldsymbol{x}) \cdot \boldsymbol{n}(\boldsymbol{\kappa}^n_*)\, d\ell.$$

Orthogonality constraint for interpolation gives

$\psi_{\ell,j}(\xi) = (d+1)\varphi_{\ell,j}(\xi) - 1$

and

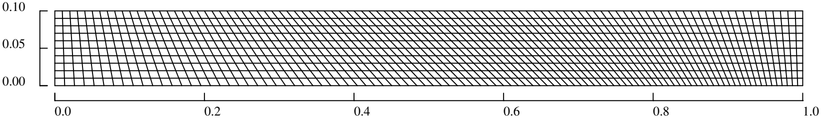
$$\int_\ell \psi_{\ell,i} \varphi_{\ell,j} d\ell = \frac{\|\ell\|}{d} \delta_{i,j}$$

Therefore we have analitical formula for the right hand side to

Saltzman problem

Computational domain $(x, y) \in [0, 1] \times [0, 0.1]$ with $(n_x, n_y) = (100, 10)$ skewed by the map

$$x_{\mathsf{sk}} = x + (0.1 - y)\sin(\pi x), \quad y_{\mathsf{sk}} = y.$$

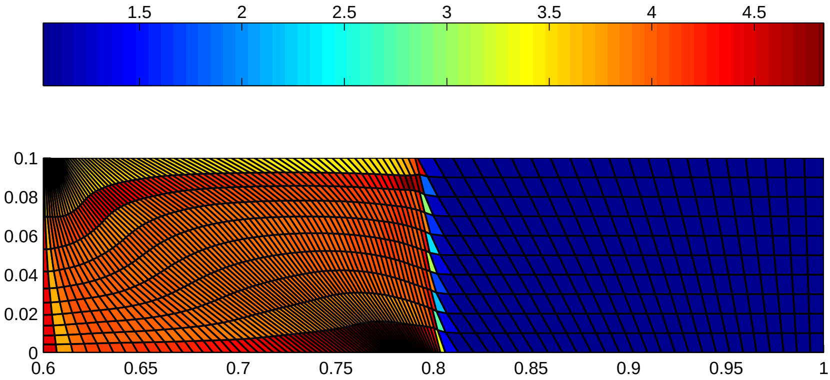


Initial conditions: $(\rho^0, P^0, \boldsymbol{U}^0) = (1, 10^{-6}, 0)$

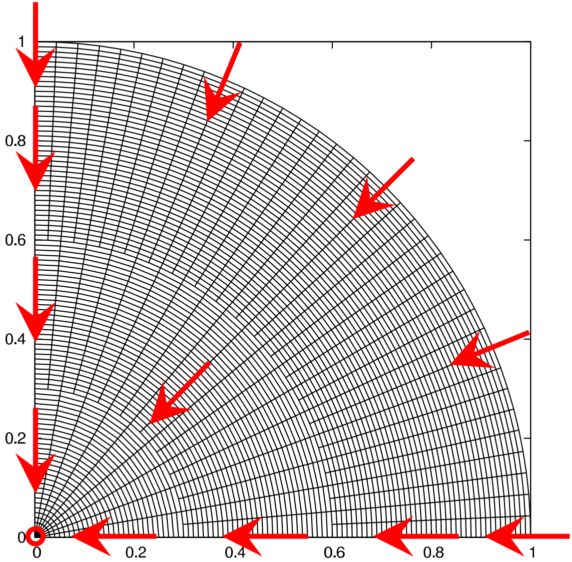
Materials: gamma law gas with $(\gamma = 5/3)$

Boundary conditions: inflow velocity $U^\star = 1$ at $x = 0$.

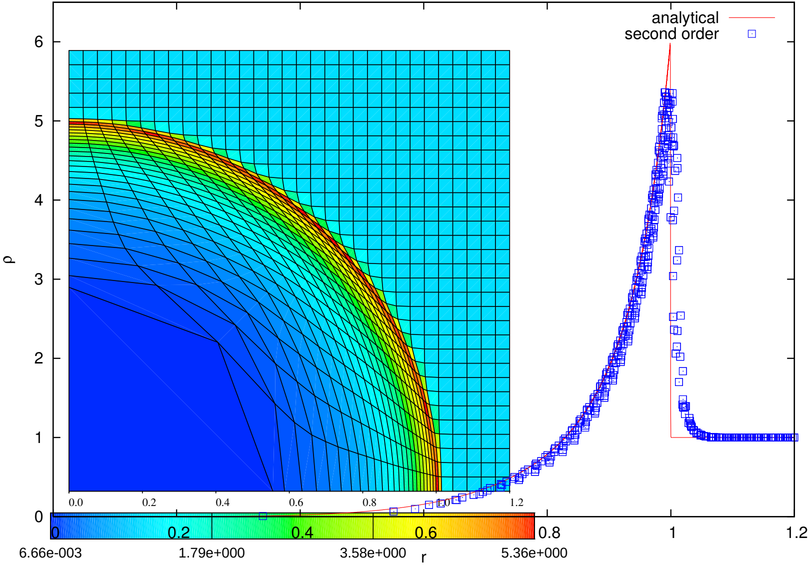
Saltzman problem : Density at $t = 0.6$



Cylindrical Noh Problem: non-conformal, 2250 cells

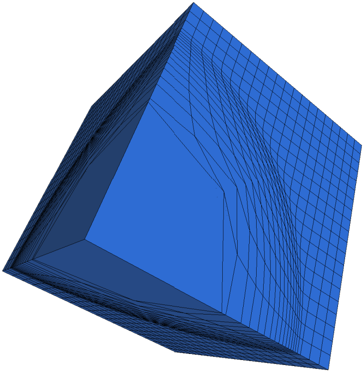


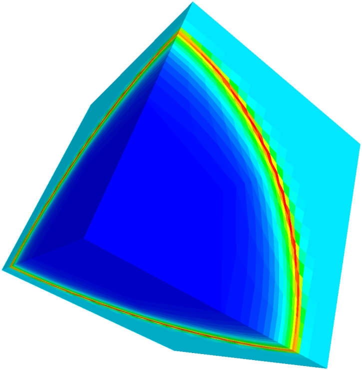
Sedov-Taylor blast wave: 31×31 Cartesian grid



Density map (left) and density in all the cells (right) at $t = 1$

Sedov-Taylor blast wave: 3D hexahedra mesh





- Initial high pressure in only one cell.
- Symmetry boundary conditions

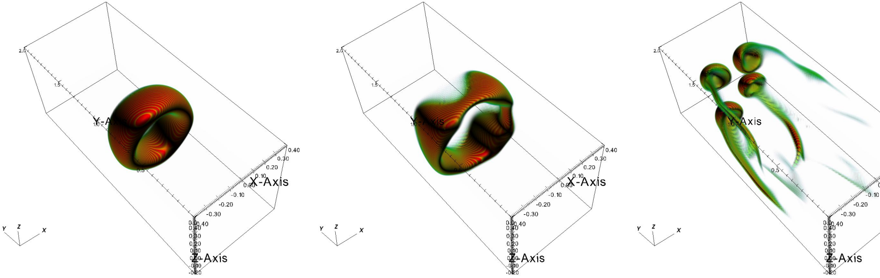
Multifluid Hydrodynamic 3D : 8×10^6 unknowns

$$\omega = (\alpha, \rho_1, \rho_2, \rho\boldsymbol{u}, E)^T$$

Implicit Low mach scheme, Surface tension, Gravity.

1.3×10^6 Cells, Time Steps $=$ 14 610, Final Physical Time $=$ 3.59s,

RAM used $=$ 1.35Go/Core $\simeq$ 11 Go, CPU $=$ 12 days$\times$ 8Pe



Pe : 2x3GHz Quad-core Intel Xeon

RAM: 16 Go 667MHz DDR2 FB-DIMM

Multifluid Hydrodynamic 3D : 8×10^6 unknowns

Explicit scheme : CFL 0.9

$$\frac{T_{phys}}{T_{cpu}} \simeq 1.7 \times 10^{-4}$$

$$\text{RAM used} = 0.45\text{Go}/\text{Core} \simeq 3.6\text{Go}$$

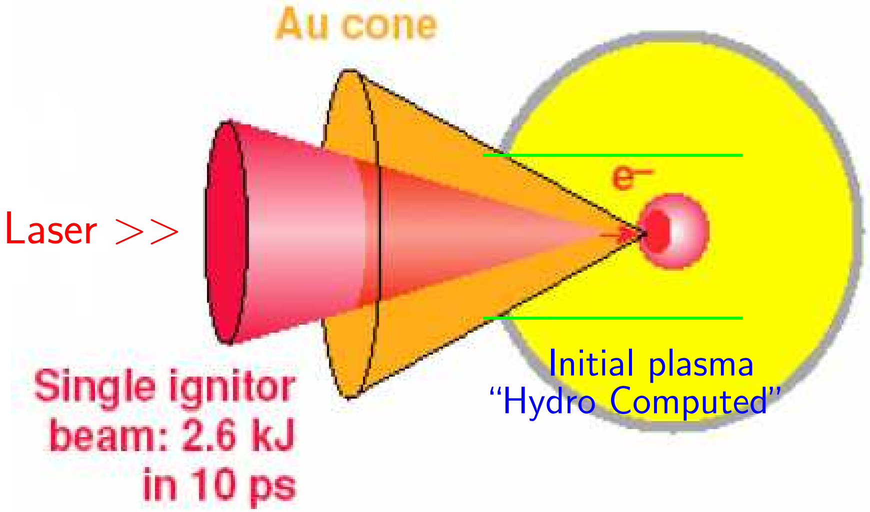
Implicit scheme : CFL 40 (GS Relaxations ≤ 25, $\varepsilon = 10^{-5}$)

$$\frac{T_{phys}}{T_{cpu}} \simeq 15 \times 10^{-4}$$

$$\text{RAM used} = 1.35\text{Go}/\text{Core} \simeq 11\ \text{Go}$$

$$\mathcal{E}_{cpu} = \frac{15}{1.7} \simeq 9 \qquad \mathcal{E}_{ram} = \frac{11}{3.6} \simeq 3$$

From Lagrangian Hydrodynamics to Nonlinear Laser/plasmas interaction



Laser/plasma interaction : Physical Model

$$
\begin{aligned}
\partial_t \rho + \mathbf{u} \cdot \nabla \rho &= -\rho \nabla \cdot \mathbf{u} \\
\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} &= -\frac{1}{\rho} \nabla p \\
\partial_t T_e + \mathbf{u} \cdot \nabla T_e &= -(\gamma_e - 1) T_e \nabla \cdot \mathbf{u} \\
\partial_t T_i + \mathbf{u} \cdot \nabla T_i &= -(\gamma_i - 1) T_i \nabla \cdot \mathbf{u}
\end{aligned}
$$

with $\rho = \rho_e + \rho_i$, $p = p_e + p_i$, $n_e = Z n_i$, $\rho_e = m_e n_e$, $\rho_i = m_i n_i$, $p_e = n_e T_e$ and $p_i = n_i T_i$. Therefore, $\frac{\rho_e}{m_e} = Z \frac{\rho_i}{m_i}$

$$
p_e = n_e T_e = \frac{\rho_e}{m_e} T_e = \frac{\rho T_e}{m_e \beta 1 + \frac{m_i}{m_e Z}} = \frac{\rho Z T_e}{m_i + Z m_e}
$$

$$
p_i = n_i T_i = \frac{\rho_i}{m_i} T_i = \frac{\rho T_i}{m_i \beta Z \frac{m_e}{m_i} + 1} = \frac{\rho T_i}{m_i + Z m_e}
$$

Laser/plasma interaction : Physical Model

- Paraxial approximation:

$$\frac{2\imath\omega_0}{c^2}\partial_t \mathbf{E} + 2\imath k_0 \partial_z \mathbf{E} + \boldsymbol{\beta}\imath\partial_z k_0 - \frac{\omega_0^2}{c^2}\frac{n_e - n_{e0}}{n_c} + \imath\frac{\nu_{ei}\omega_0}{c^2}\frac{n_{e0}}{n_c}\mathbf{E}$$
$$+\boldsymbol{\beta}\frac{2\nabla^2}{1+\sqrt{1+\nabla^2/k_0^2}}\mathbf{E} = 0.$$

Nonlinear nonlocal laser/plasma interaction

$$\begin{aligned}
\partial_t \rho + \mathbf{u}\cdot\nabla\rho &= -\rho\nabla\cdot\mathbf{u} & & ,\\
\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} &= -\frac{1}{\rho}\nabla p & -\frac{Z\nabla\boldsymbol{\beta}\|\mathbf{E}\|^2}{2cn_c m_i} & +\frac{1}{\rho}\nabla\cdot(\eta\nabla\mathbf{u}),\\
\partial_t T_e + \mathbf{u}\cdot\nabla T_e &= -\frac{p_e}{\rho}\nabla\cdot\mathbf{u} & +\frac{\nu_{ei}\Gamma_e}{n_c c}\|\mathbf{E}\|^2 & +\frac{\Gamma_e}{n_e}\nabla\cdot(\kappa_e\nabla T_e),\\
\partial_t T_i + \mathbf{u}\cdot\nabla T_i &= -\frac{p_i}{\rho}\nabla\cdot\mathbf{u} & & +\frac{\Gamma_i}{n_i}\nabla\cdot(\kappa_i\nabla T_i).\\
\beta_z\partial_z\mathbf{E} &= -\beta_t\partial_t\mathbf{E} & -\mathcal{S}(\rho)\mathbf{E} & -\mathcal{L}\boldsymbol{\beta}\nabla\mathbf{E}
\end{aligned}$$

Numerical Approximation : $\mathbf{W} = \boldsymbol{\beta}\rho, \mathbf{u}, T_e, T_i{}^T$

$$\begin{cases} \partial_t \mathbf{W} + \mathcal{A}(\mathbf{W})\nabla_{x,y}\mathbf{W} & = & \mathcal{P}\boldsymbol{\beta}\mathbf{W}, \mathbf{E} & +\mathcal{N}\boldsymbol{\beta}\mathbf{W}, \\ \beta_z \partial_z \mathbf{E} & = & -\mathcal{S}(\mathbf{W})\mathbf{E} & -\mathcal{L}\boldsymbol{\beta}\nabla_{x,y}\mathbf{E} \end{cases}$$

Given $\underline{\mathbf{E}^n(z=0),\ \mathbf{W}^{n-\frac{1}{2}}\boldsymbol{\beta}z + \frac{\delta z}{2}}$ and set $\mathcal{M}^{n-\frac{1}{2}}_{z+\frac{\delta z}{2}} = \mathcal{S}^{n-\frac{1}{2}}_{z+\frac{\delta z}{2}} + \mathcal{L}\boldsymbol{\beta}\nabla_{x,y}$

1. FFTW & θ-scheme + MPI $\Longrightarrow \mathbf{E}^n(z+\delta z)$

$$\left[\beta_z + \delta z \theta \mathcal{M}^{n-\frac{1}{2}}_{z+\frac{\delta z}{2}}\right] \mathbf{E}^n(z+\delta z) = \left[\beta_z - \delta z(1-\theta)\mathcal{M}^{n-\frac{1}{2}}_{z+\frac{\delta z}{2}}\right] \mathbf{E}^n(z)$$

2. FV Second order accurate + MPI $\Longrightarrow \mathbf{W}^{(1)}_{i,j}\boldsymbol{\beta}z + \frac{\delta z}{2}$

$$\mathbf{W}^{(1)}_{i,j} = \mathbf{W}^{n-\frac{1}{2}}_{i,j} + \frac{\delta t}{a_{i,j}}\left[-\Phi^{n-\frac{1}{2}}_{i,j} + a_{i,j}\mathcal{P}^{n-\frac{1}{2}}_{i,j}\boldsymbol{\beta}\frac{\mathbf{E}^n(z+\delta z) + \mathbf{E}^n(z)}{2}\right]$$

3. FFTW & θ-scheme + MPI $\Longrightarrow \mathbf{W}^{n+\frac{1}{2}}\boldsymbol{\beta}z + \frac{\delta z}{2}$

$$\mathbf{W}^{n+\frac{1}{2}}_{i,j} - \delta t \theta \mathcal{N}_{i,j}\boldsymbol{\beta}\mathbf{W}^{n+\frac{1}{2}} = \mathbf{W}^{(1)}_{i,j} + \delta t(1-\theta)\mathcal{N}_{i,j}\boldsymbol{\beta}\mathbf{W}^{(1)}$$

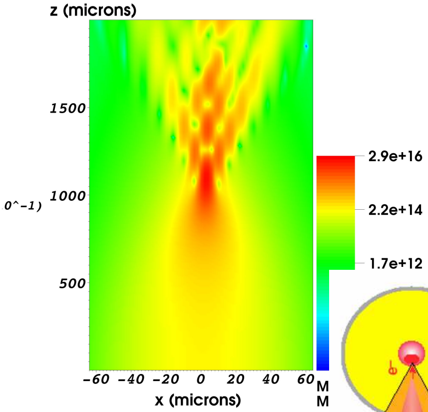
Validation by a proton diagnostic.

- Incoming Electric field : $I(t, \mathbf{r}) = I_{max} \exp \boldsymbol{\beta} - \frac{\mathbf{2r}}{W_0^2} - \frac{t^2}{t_0^2}.$
  $I_{max} = 3.7\ 10^{14} W.cm^2,\ W_0 = 60\mu m,\ t_0 = 400ps.$
  $\lambda_0 = 1.053\mu m.$
- Initial plasma (Helium: $Z = 2$) :
  $$n_0 = 0.014 n_c,\ T_{e0} = 100eV,\ T_{i0} = 30eV.$$
- Electron heat flux model:
  - Spitzer-Härm Conductivity, marginally valid for ICF : (non maxwellian electrons density functions.)
  - Brantov (98) nonlocal Conductivity.
    Based on a linearised theory of Fokker-Planck
    Valid for an "arbitrary" collisionality.
- Braginskii viscosity & ion Landau damping
  $--->$ ion heat conductivity. .

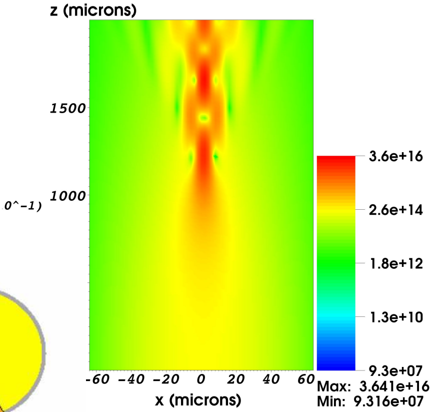
Energy distribution (on a cut plane) at $550ps$

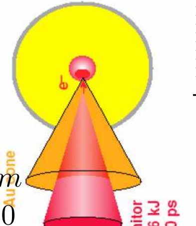
Initial Density is constant in space

Brantov (non local)



Spitzer-Härm





$\delta x = \delta y = 2\mu m \quad \delta z = 5\mu m$

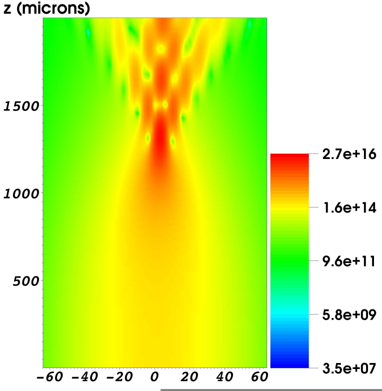
$\delta t = 5\ 10^{-2}$ps

$N_x = N_y = 256 \quad N_z = 400$

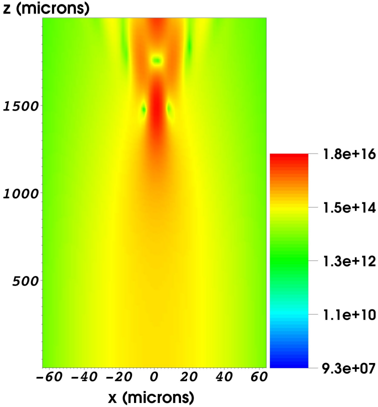
Energy after $550ps$:$\delta x = \delta y = 2\mu m,\ \delta z = 5\mu m$

Initial Density is profiled in space

Brantov (non local)



Spitzer-Härm

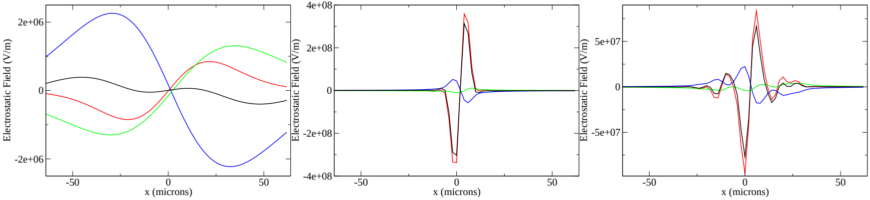


Parallel computing : Quad Itanium II, Dual-Core, 1.6 Ghz, 128 Go

$\simeq 150 \times 10^6$variables 10^4time steps : 40H CPU with 32 cores

Electrostatic fields at 500, 1300, 1700 μm

$$\mathbf{E} = k_1 \nabla I + k_2 \nabla T + k_3 \nabla n$$



Tokamak plasmas : KINETIC or/and FLUID ?

- What is the range of applicability of fluids modelizations for large Tokamaks plasmas ?
- Can we accurately take into account unresolved kinetic and/or particles orbits effects on large macroscopic scaled?

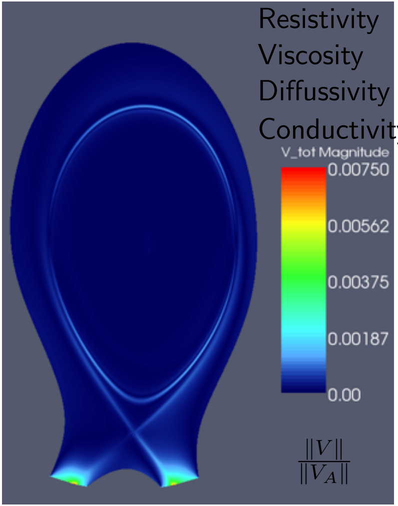
- What are characteristic behaviors of “Fluid like” modelizations, their stiffness and asymptotic?
- Can we design appropriate, stable, accurate, efficient and scalable numerical approximations that are able to simulate long time MHD instabilities for ITER and DEMO?

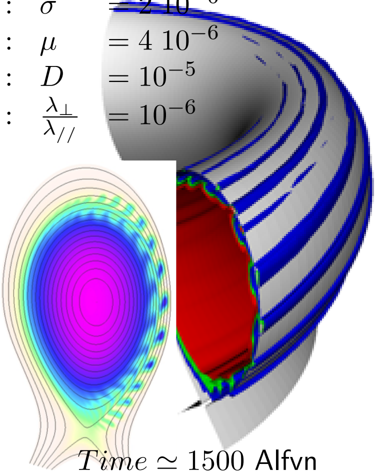
Ballooning instability : Reduced MHD (Jorek)

Equilibrium: Flow $\simeq 40km/s$

Edge localized modes

Resistivity	:	σ	$= 2\ 10^{-6}$
Viscosity	:	μ	$= 4\ 10^{-6}$
Diffussivity	:	D	$= 10^{-5}$
Conductivity	:	$\frac{\lambda_\perp}{\lambda_{//}}$	$= 10^{-6}$





$Time \simeq 1500$ Alfvn

Temperature $\simeq 10kev$, $\|V_A\| \simeq 5000km/s$,

18 Torodal modes used

Numerical Developments : ASTER (ANR-CIS.2006)

1. Refinable cubic-Bezier FEM (O. Czarny, G. Huysmans).
2. Direct/iterative parallel sparse matrix solver (P. Ramet, P. Henon, ...)
3. Optimized time-stepping algorithm (G. Huysmans, B. Nkonga, ...)
4. Stabilised FEM, RD schemes (R. Abgrall, R. Huart, B. Nkonga,...)
5. Extended MHD model (E. van der Plas, G. Huysmans, B. Nkonga,...)
6. Boundary conditions (M. Becoulet, G. Huysmans, ...)

ANR program Intensive Computing and Simulation (ANR-CIS.2006)
http://aster.gforge.inria.fr/index.html

Open Questions: fluid "like" models for Hot plasmas

1. Numerical schemes
   - Dynamic mesh aligned with magnetic flux surfaces.
   - High, i.e. realistic, (magnetic) Reynolds numbers
   - Resolution of boundary layers (open field line and curved boundary)
   - Long time integration: complete (internal disruption) ELM cycle (different ELM types).
2. Non-linear evolution of MHD models
   - Trigger of neoclassical transport (low collisionality, tearing modes).
   - Extended MHD (Ti-Te + Generalized Ohm's law).
   - Interaction with micromagnetic turbulence (anomalous transport).
3. Fast particles interaction with MHD modes (nonlocal transport).
4. Charge exchange neutrals, radiation, local heating, ...