# Modelization and simulation of nearwell drying and alteration by CO2 injection in saline aquifers

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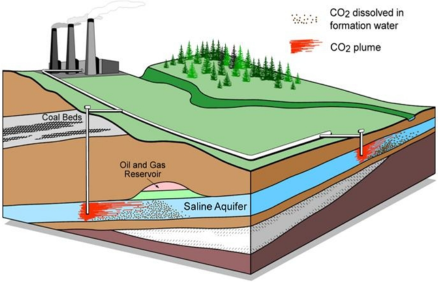
1. Multiphase Darcy flow models for $CO_2$ storage modelling

2. Discretization of Darcy fluxes on general meshes with vertex unknowns

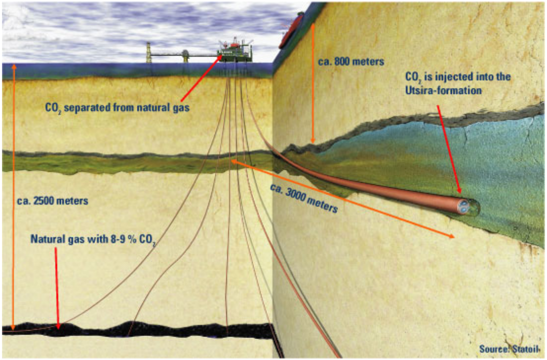
3. Near well drying and alteration: $CO_2$ injection in the Snohvit field

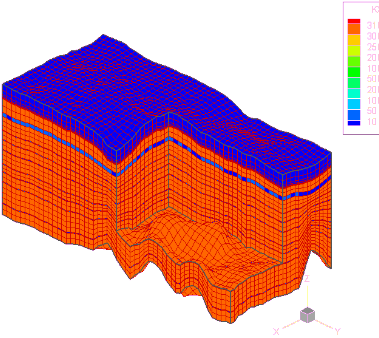
# CO2 geological storage in saline aquifers

- Objectives
  - Optimize the CO2 injection
  - Risk assessment (leakage)
- Model
  - Compositional multiphase Darcy flow

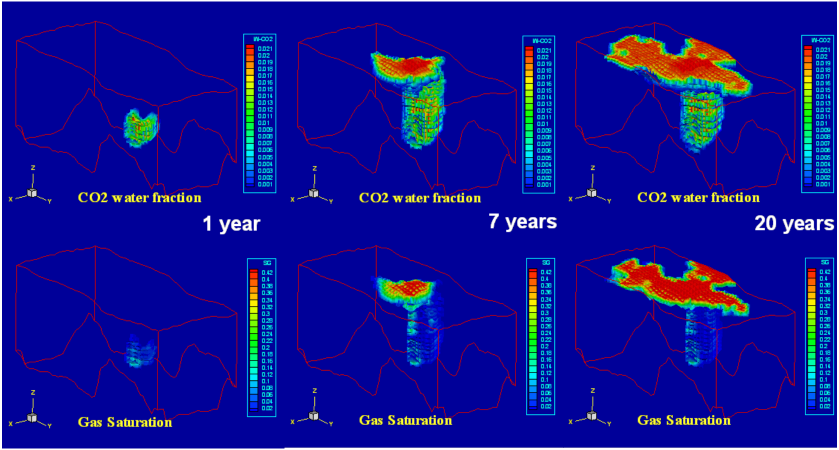


# Example of injection of CO2 in the Sleipner field, aquifer Utsira

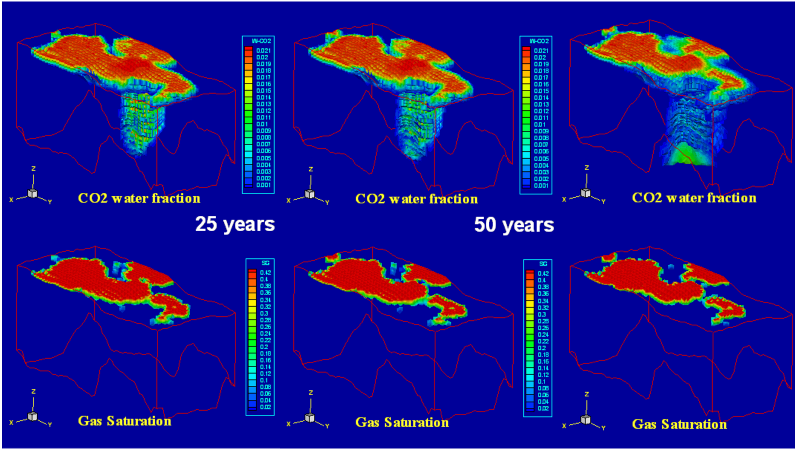


# Example of injection of CO2 in the Sleipner field, aquifer Utsira: injection during 20 years



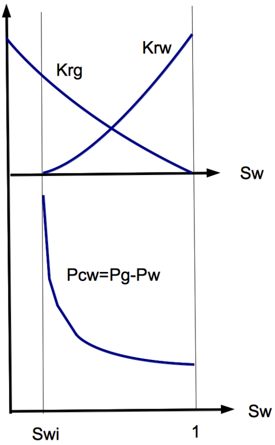
# Example of injection of CO2 in the Sleipner field, aquifer Utsira: storage during 1000 years



# Two Phase (water - gas) Darcy flow model

$S^w$ and $S^g$: volume fractions
$P^w$ and $P^g$: pressures

$$\begin{cases} \mathbf{U}^w = -\dfrac{k_{r,w}(S^w)}{\mu^w} K \left(\nabla P^w - \rho^w \mathbf{g}\right), \\ \mathbf{U}^g = -\dfrac{k_{r,g}(S^g)}{\mu^g} K \left(\nabla P^g - \rho^g \mathbf{g}\right), \\ P^g = P^w + P_{c,w}(S^w), \\ S^w + S^g = 1. \end{cases}$$



# Immiscible two phase (water-gas) Darcy flow

**2 phases :** water $(w)$, gas $(g)$ – **2 components :** $H2O$ and $CO2$

$$\mathcal{C}^w = \{H2O\} \quad \text{et} \quad \mathcal{C}^g = \{CO2\}$$

- Conservation equations

$$\begin{cases} \partial_t\Big(\phi\ \rho^g\ S^g\Big) + \operatorname{div}\left(\rho^g\ \mathbf{U}^g\right) = 0 \\ \\ \partial_t\Big(\phi\ \rho^w\ S^w\Big) + \operatorname{div}\left(\rho^w\ \mathbf{U}^w\right) = 0 \end{cases}$$

- Darcy velocities

$$\begin{cases} \mathbf{U}^g = -\dfrac{k_{r_\mathbf{g}}(S)}{\mu^g}\Big(K\ \nabla P^g - \rho^g \mathbf{g}\Big) \\ \mathbf{U}^w = -\dfrac{k_{r_\mathbf{w}}(S)}{\mu^w}\Big(K\ \nabla P^w - \rho^w \mathbf{g}\Big) \end{cases}$$

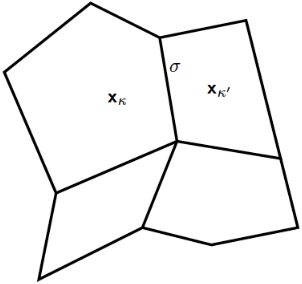
- Closure laws

$$\begin{cases} S^w + S^g = 1, \\ P^g = P^w + P_{c,w}(S^w). \end{cases}$$

# Cell centred finite volume discretization of Multiphase Darcy flows

- Discretization
- Cell centred unknowns: $P_\kappa^\alpha$, $S_\kappa^\alpha$, $\alpha = w, g$
- Discrete conservation laws on each cell $\kappa$:

$$\int_\kappa \Big( (\phi \ \rho^g \ S^g)^n - (\phi \ \rho^g \ S^g)^{n-1} \Big) d\boldsymbol{x} + \sum_{\kappa' \in \mathcal{M}_\kappa} \int_{\sigma = \kappa\kappa'} \int_{t^{n-1}}^{t^n} \rho^\alpha \ \mathbf{U}^\alpha \cdot \mathbf{n}_{\kappa\kappa'} d\sigma =$$




- Implicit Euler time integration
- Conservative approximation of the fluxes

$$\int_{\sigma = \kappa\kappa'} \rho^{\alpha,n} \ \mathbf{U}^{\alpha,n} \cdot \mathbf{n}_{\kappa\kappa'} d\sigma$$

# Cell centred finite volume discretization of Multiphase Darcy flows

$$\phi_\kappa \frac{\left(\rho_\kappa^{\alpha,n}\ S_\kappa^{\alpha,n} - \rho_\kappa^{\alpha,n-1} S_\kappa^{\alpha,n-1}\right)}{\Delta t} + \sum_{\kappa' \in \mathcal{M}_\kappa} \left(\frac{\rho^\alpha\ k_{r_\alpha}(S^\alpha)}{\mu^\alpha}\right)^n_{\kappa\kappa'_{\mathbf{up}_\alpha}} F_{\kappa\kappa'}^{\alpha,n} = 0$$

for all $\kappa \in \mathcal{M}$, $\alpha = w, g$, with $\phi_\kappa = \int_\kappa \phi(\boldsymbol{x}) d\boldsymbol{x}$ and

- the conservative discretization of the Darcy fluxes

$$F_{\kappa\kappa'}^\alpha = -F_{\kappa'\kappa}^\alpha \sim \int_{\kappa\kappa'} -K(\boldsymbol{x})\Big(\nabla P^\alpha + \rho_{\kappa\kappa'}^\alpha \nabla Z\Big) \cdot \mathbf{n}_{\kappa\kappa'} d\sigma$$

- the upwinding $\kappa\kappa'_{up_\alpha} = \begin{cases} \kappa & \text{if } F_{\kappa\kappa'}^\alpha \geq 0, \\ \kappa' & \text{if } F_{\kappa\kappa'}^\alpha < 0. \end{cases}$

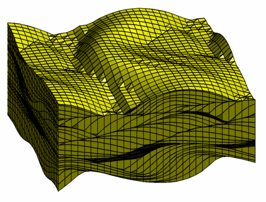
# Cell centered finite volume discretization of compositional darcy flows

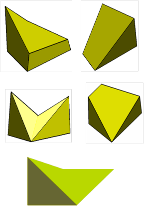
The local closure laws are coupled implicitely to the transport equations

$$\begin{cases} S_\kappa^{w,n} + S_\kappa^{g,n} = 1, \\ P^{g,n} = P^{w,n} + P_c(S^{w,n}). \end{cases}$$

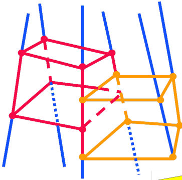
- The system is solved for the unknowns $P^w$,$P^g$, $S^w$, $S^g$ using a Newton type algorithm at each time step
- The local closure laws are locally eliminated from the linear system which reduces to two primary unknown (say $P^w$, $S^w$) and two conservation equations for each cell $\kappa$.

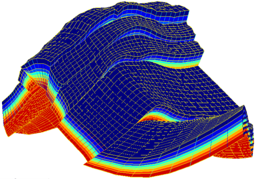
# Discretization of Darcy fluxes $\int_\sigma -K \nabla u \, ds$ on general meshes and heterogeneous anisotropic media

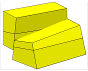




- Meshes
  - Polyhedral cells
  - Non planar faces
  - Faults
- Heterogeneous anisotropic media



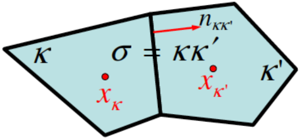




# Two Point Flux Approximation (TPFA): $-\Delta u = f$

## Finite Volume Discretization

- Finite volume mesh
  - Cells
  - Cell centers
  - Faces




- Degrees of freedom: $u_K$
- Discrete conservation law

$$\int_K -\Delta u dx = \sum_{\sigma = KK'} \int_\sigma -\nabla u . n_{KK'} ds = \int_K f dx$$

# TPFA: $-\Delta u = f$

## Two Point Flux Approximation (TPFA)

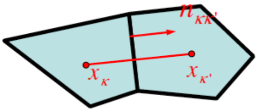
- TPFA $\int_\sigma -\nabla u . n_{\kappa\kappa'} ds \approx F_{\kappa\kappa'}(u_\kappa, u_{\kappa'})$

- Flux Conservativity

$$F_{\kappa\kappa'}(u_\kappa, u_{\kappa'}) + F_{\kappa'\kappa}(u_{\kappa'}, u_\kappa) = 0$$

- Flux Consistency

$$F_{\kappa\kappa'}(u_\kappa, u_{\kappa'}) = \frac{|\sigma|}{|x_\kappa x_{\kappa'}|}(u_\kappa - u_{\kappa'}) = \int_\sigma -\nabla u . n_{\kappa\kappa'} ds + O(|\sigma| h)$$



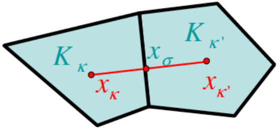

$x_{\kappa'} x_{\kappa'} \perp \kappa\kappa'$

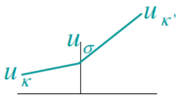
# TPFA: $\mathrm{div}\Big(-K(\boldsymbol{x})\nabla u\Big) = f$

## TPFA

### Isotropic Heterogeneous media

- FV scheme $\begin{cases} div(-K\nabla u) = f \ sur\ \Omega \\ u = g \ sur\ \partial\Omega \end{cases}$




$$F_{\kappa\kappa'} = K_\kappa \frac{|\kappa\kappa'|}{|x_\kappa x_\sigma|}(u_\kappa - u_\sigma) = K_{\kappa'} \frac{|\kappa\kappa'|}{|x_{\kappa'} x_\sigma|}(u_\sigma - u_{\kappa'}) = T_{\kappa\kappa'}(u_\kappa - u_{\kappa'})$$

Transmissibility: $\boxed{\frac{1}{T_{\kappa\kappa'}} = \frac{|x_\kappa x_\sigma|}{K_\kappa |\kappa\kappa'|} + \frac{|x_{\kappa'} x_\sigma|}{K_{\kappa'} |\kappa\kappa'|}}$

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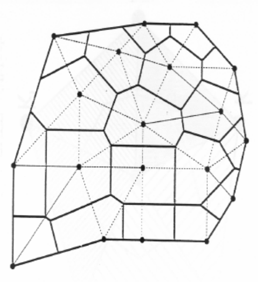
$$\int_{\kappa\kappa'} -K(\boldsymbol{x})\nabla u(\boldsymbol{x})\ d\boldsymbol{x} \sim F_{\kappa\kappa'}(u_\kappa, u'_\kappa) = T_{\kappa\kappa'}\Big(u_k - u_{\kappa'}\Big).$$

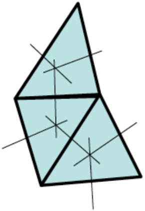
# TPFA: admissible meshes

## *Exemples of admissible meshes*

Voronoi



Triangles: angles $\leq \pi / 2$



Cartesian:

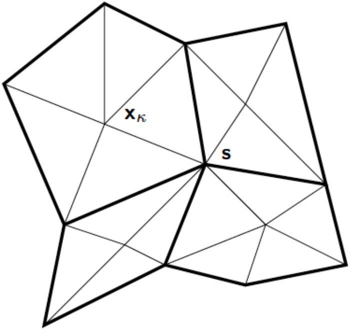
# Finite volume approximation of diffusive fluxes

- Centred but not symmetric schemes: conditionally coercive and convergent:
  - MPFA O [Aavatsmark et al., 1996], [Edwards and Rogers, 1994]
- Symmetric schemes but with with additional unknowns at faces:
  - VFH [Eymard and Herbin, 2007], [Eymard et al., 2010a],
  - MFD [Brezzi et al., 2005],
- Symmetric schemes but with with additional unknowns at vertices:
  - DDFV [Hermeline, Omnes, Boyer, Hubert, Coudière, ...]
  - VAG [Eymard et al., 2010b]

# Vertex Approximate Gradient (VAG) scheme in 2D

- Triangular submesh $\mathcal{T}$ of the mesh $\mathcal{M}$
- Cell centre and vertex unknowns $u_\kappa$, $u_{\mathbf{s}}$
- $\mathbb{P}_1$ finite element discretization: nodal basis $\eta_\kappa, \eta_{\mathbf{s}}$, $\mathbf{s} \in \mathcal{V}_\kappa, \kappa \in \mathcal{M}$
- $u_\mathcal{T}$: $\mathbb{P}_1$ finite element interpolation from the cell centre and nodal values $u_\kappa$ and $u_{\mathbf{s}}$:

$$u_\mathcal{T} = \sum_{\kappa \in \mathcal{M}} u_\kappa \eta_\kappa + \sum_{\mathbf{s} \in \mathcal{V}} u_{\mathbf{s}} \eta_{\mathbf{s}}$$




$$a(u_\mathcal{T}, v_\mathcal{T}) = \int_\Omega K(\boldsymbol{x}) \nabla u_\mathcal{T}(\boldsymbol{x}) \cdot \nabla v_\mathcal{T}(\boldsymbol{x}) \; d\boldsymbol{x} = \int_\Omega f(\boldsymbol{x}) \; v_\mathcal{T}(\boldsymbol{x}) \; d\boldsymbol{x}$$

# Vertex Approximate Gradient (VAG) scheme in 2D

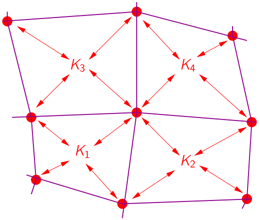
From $\nabla \eta_\kappa = -\sum_{\mathbf{s}\in\mathcal{V}_\kappa} \nabla \eta_\mathbf{s}|_\kappa$, we deduce that

$$a(u_\mathcal{T}, v_\mathcal{T}) = \sum_{\kappa\in\mathcal{M}} \sum_{s\in\mathcal{V}_\kappa} \Big( \int_\kappa -K(\boldsymbol{x})\nabla u_\mathcal{T}(\boldsymbol{x}) \cdot \nabla \eta_\mathbf{s}(\boldsymbol{x}) d\boldsymbol{x} \Big) \Big( v_\kappa - v_\mathbf{s} \Big),$$
$$= \sum_{\kappa\in\mathcal{M}} \sum_{s\in\mathcal{V}_\kappa} F_{\kappa,\mathbf{s}}(u_\mathcal{T}) \Big( v_\kappa - v_\mathbf{s} \Big)$$

with the fluxes $F_{\kappa,\mathbf{s}}(u_\mathcal{T}) = -F_{\mathbf{s},\kappa}(u_\mathcal{T}) = \int_\kappa -K(\boldsymbol{x})\nabla u_\mathcal{T} \cdot \nabla \eta_\mathbf{s}(x) d\boldsymbol{x}$.

- Discrete conservation laws:

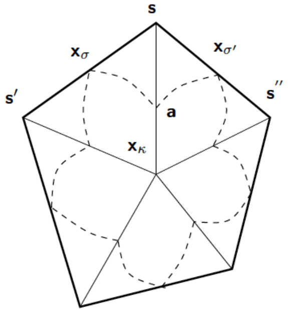
$$\begin{cases} \displaystyle\sum_{s\in\mathcal{V}_\kappa} F_{\kappa,\mathbf{s}}(u_\mathcal{T}) = \int_\kappa f(\boldsymbol{x})\nabla \eta_\kappa(\boldsymbol{x})\, d\boldsymbol{x} \text{ for all } \kappa \in \mathcal{M}, \\ \displaystyle\sum_{\kappa\in\mathcal{M}_\mathbf{s}} F_{\mathbf{s},\kappa}(u_\mathcal{T}) = \int_\Omega f(\boldsymbol{x})\nabla \eta_\mathbf{s}(\boldsymbol{x})\, d\boldsymbol{x} \text{ for all } \mathbf{s} \in \mathcal{V} \setminus \partial\Omega \end{cases}$$



# Vertex Approximate Gradient (VAG) scheme in 2D

- Control Volume Finite Element (CVFE) interpretation of the VAG Fluxes in 2D

$$
\begin{aligned}
F_{\kappa,\mathbf{s}}(u_{\mathcal{T}}) &= \int_{\kappa} -K(\boldsymbol{x})\nabla u_{\mathcal{T}}(\boldsymbol{x}) \cdot \nabla \eta_{\mathbf{s}}(\boldsymbol{x})\, d\boldsymbol{x}, \\
&= \int_{\boldsymbol{x}_\sigma \frown \mathbf{a} \,\cup\, \boldsymbol{x}_{\sigma'} \frown \mathbf{a}} -K(x)\nabla u_{\mathcal{T}}(\boldsymbol{x}) \cdot \mathbf{n}_\kappa d\sigma, \\
&= \sum_{\mathbf{s}' \in \mathcal{V}_\kappa} A_\kappa^{\mathbf{s},\mathbf{s}'} \left( u_\kappa - u_{\mathbf{s}'} \right),
\end{aligned}
$$



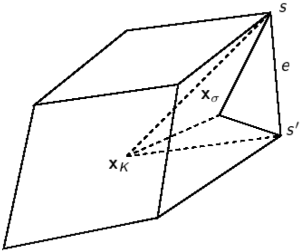

with

$$
A_\kappa^{\mathbf{s},\mathbf{s}'} = \int_{\kappa} K(\boldsymbol{x})\nabla \eta_{\mathbf{s}}(\boldsymbol{x}) \cdot \nabla \eta_{\mathbf{s}'}(\boldsymbol{x})\, d\boldsymbol{x}.
$$

# VAG scheme: extension to 3D [Eymard et al., 2010b]

- Tetrahedral submesh $\mathcal{T}$
- Interpolation at the face centres $\boldsymbol{x}_\sigma$ using the face nodal values
- $\mathbb{P}_1$ finite element discretization on $\mathcal{T}$ with interpolation at the face centres
- Nodal basis: $\eta_\kappa, \eta_{\mathbf{s}}$, $\mathbf{s} \in \mathcal{V}_\kappa, \kappa \in \mathcal{M}$

$$\mathbf{x}_\sigma = \sum_{s \in \mathcal{V}_\sigma} \frac{1}{\mathrm{Card}\mathcal{V}_\sigma} \mathbf{x}_{\mathbf{s}}, \quad u_\sigma = \sum_{\mathbf{s} \in \mathcal{V}_\sigma} \frac{1}{\mathrm{Card}\mathcal{V}_\sigma} u_{\mathbf{s}}$$

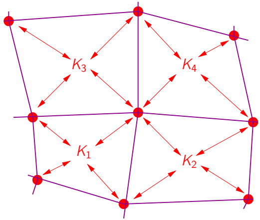


# Application to multiphase Darcy flows

- Write the discrete conservation equations on both cells $\kappa$ and vertices $\mathbf{s}$ using the conservative fluxes $F_{\kappa,\mathbf{s}}(u_\mathcal{T}) = -F_{\mathbf{s},\kappa}(u_\mathcal{T})$ between cell and vertices

$$\phi_X \frac{\left(\rho_X^{\alpha,n}\; S_X^{\alpha,n} - \rho_X^{\alpha,n-1} S_X^{\alpha,n-1}\right)}{\Delta t} + \sum_{Y \in \mathfrak{M}_X} \left(\frac{\rho^\alpha\; k_{r_\alpha}(S^\alpha)}{\mu^\alpha}\right)^n_{XY_{\mathbf{up}_\alpha}} F_{XY}^{\alpha,n} = 0$$

$$\text{for all } X \in \mathfrak{M} = \mathcal{M} \cup \mathcal{V}.$$




- Define the porous volumes $\phi_\mathbf{s}$, $\phi_\kappa$ at each vertex $\mathbf{s}$ and at each cell $\kappa$ such that

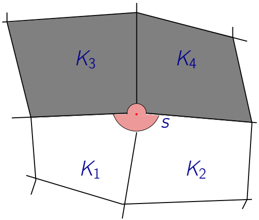
$$\sum_{\kappa \in \mathcal{M}} \phi_\kappa + \sum_{\mathbf{s} \in \mathcal{V}} \phi_\mathbf{s} = \int_\Omega \phi(\boldsymbol{x})\; d\boldsymbol{x}.$$

- Define a $k_r - P_c$ (ie a rocktype) at each vertex $\mathbf{s}$

# Definition of a porous volume and a rocktype to each vertex

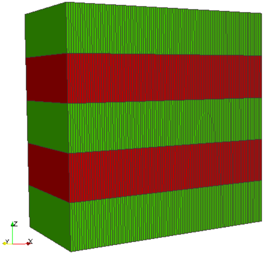
- Conservative redistribution of a fraction of neighbouring cells porous volume to vertices

- The porous volume is taken from the neighbouring cells
  - with the highest permeable rocktype
  - proportionally to the permeabilities of the neighbouring cells sharing this rocktype



# Highly heterogeneous test case on coarse grids

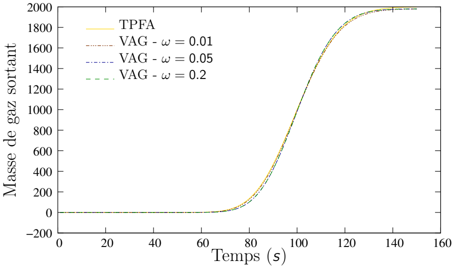
- Domain : $[0,100] \times [0,50] \times [0,100]$ $m^3$
- Isotropic heterogeneous media initially saturated with water
- Injection of gas at the left end $x = 0$
- Ratio of the drain and barrier permeabilities : $10^4$
- Cartesian mesh: $100 \times 1 \times 5$

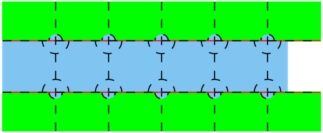


drains
barrières

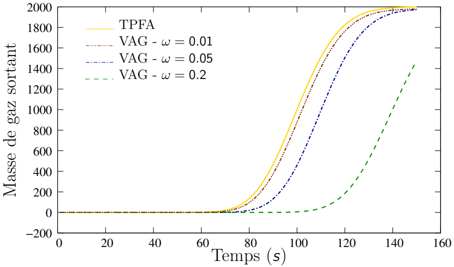
# Highly heterogeneous test case: $\phi_{\mathbf{s}} = \omega \sum_{\kappa \in \mathcal{M}_{\mathbf{s}}} \alpha_{\kappa,\mathbf{s}} \int_\kappa \phi(\mathbf{x}) d\mathbf{x}$

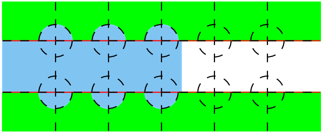
$\alpha_{\kappa,\mathbf{s}}$ proportional to $K_\kappa$



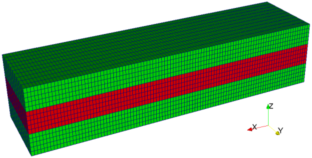
$\alpha_{\kappa,\mathbf{s}}$ uniform



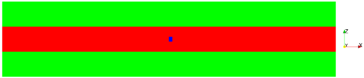



# Two phase flow with drain and barriers

- Injection of immiscible $CO_2$ in water
- Geometry : $[-100, 100] \times [0, 50] \times [0, 45]\ m^3$
- Large ratio of permeability between the drain and the barrier: $10^4$
- Cartesian mesh : $100 \times 10 \times 15$

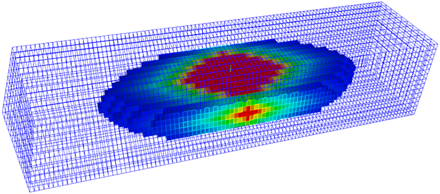


drain
barriers

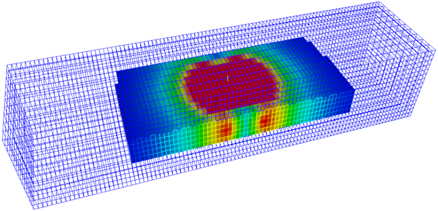


Injection well at the center
(one perforated cell)

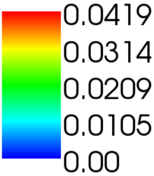
# Two phase flow in heterogeneous media
# Gaz saturation front



TPFA or MPFA O scheme



VAG scheme

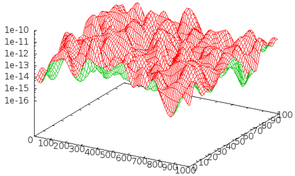



Saturation de gaz

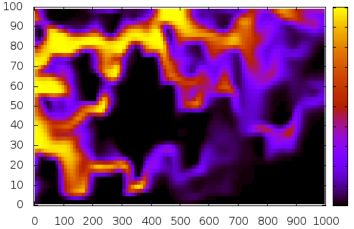
The VAG scheme is less sensitive to the grid orientation effect.

# Gas injection in an heterogeneous media saturated with water (log normal permeabiliy field)

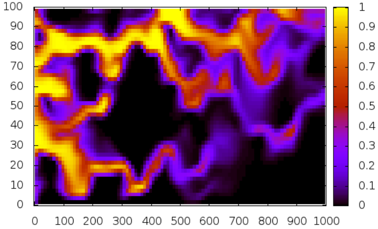
Permeability field




TPFA

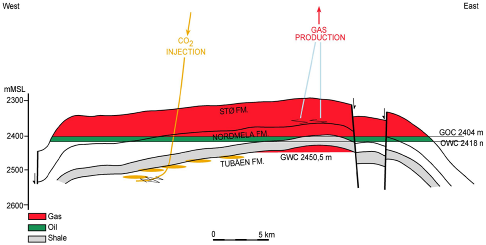


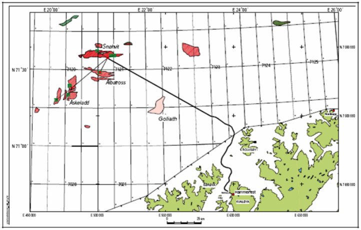

VAG

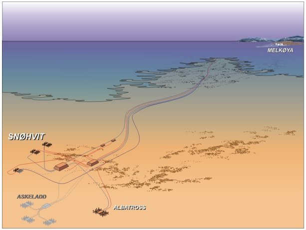


# Near well drying and alteration: CO2 injection in the Snohvit field

- Snohvit Gaz field contains 5 to 8 % of CO2
- Reinjection of CO2 in the saline aquifer Tubaen
- 700000 reinjected since 2008
- Unexplained periodic loss of injectivity



# Model with three phases *water, gaz and mineral*, and three components *H2O, CO2 and Salt*

$$\mathcal{C}^w = \{H2O, CO2, sel\}, \quad \mathcal{C}^g = \{H2O, CO2\}, \quad \mathcal{C}^m = \{sel\}$$

$$\begin{cases}
\partial_t \phi \left(\rho^w\ S^w\ C^w_{H2O} + \rho^g\ S^g\ C^g_{H20}\right) + \mathrm{div}\Big(C^w_{H2O}\ \rho^w\ \mathbf{U}^w + C^g_{H2O}\ \rho^g\ \mathbf{U}^g\Big) = 0, \\
\\
\partial_t \phi \left(\rho^w\ S^w\ C^w_{Salt} + \rho^m\ S^m\right) + \mathrm{div}\Big(C^w_{Salt}\ \rho^w\ \mathbf{U}^w\Big) = 0, \\
\\
\partial_t \phi \left(\rho^g\ S^g\ C^g_{CO2} + \rho^w\ S^w\ C^w_{CO2}\right) + \mathrm{div}\Big(C^g_{CO2}\ \rho^g\ \mathbf{U}^g + C^w_{CO2}\ \rho^w\ \mathbf{U}^w\Big) = 0, \\
\\
S^w + S^g + S^m = 1, \\
C^w_{H2O} + C^w_{CO2} + C^w_{Salt} = 1 \ \text{ if w present}, \\
C^g_{H20} + C^g_{CO2} = 1 \ \text{ if g present} \\
\\
\mathbf{U}^g = -\frac{k_{r_{\mathbf{g}}}(S)}{\mu^{\mathbf{g}}} K\Big(\nabla P^g - \rho^g \mathbf{g}\Big), \\
\mathbf{U}^w = -\frac{k_{r_{\mathbf{w}}}(S)}{\mu^{\mathbf{w}}} K\Big(\nabla\Big[P^g + P_{c,w}(S)\Big] - \rho^w \mathbf{g}\Big).
\end{cases}$$

# Thermodynamical equilibrium

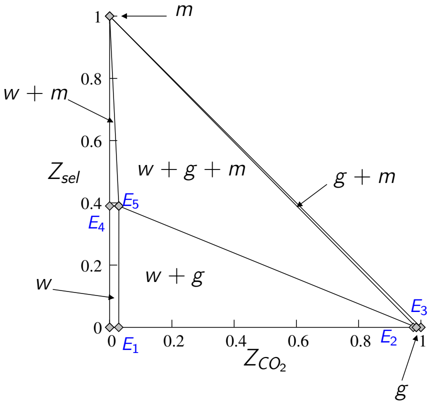
$$\left\{ \begin{array}{lll} C^w_{CO2} = K_{CO2}\ C^g_{CO2} & & \text{if w,g present} \\ C^g_{H2O} = K_{H20}\ C^w_{H2O} & & \text{if w,g present,} \\ C^w_{Salt} = K_{Salt} & & \text{if w,m present.} \end{array} \right.$$

The model is not closed since we don't know which phases are present:
7 possible set of present phases: w-g-m, w-g, w-m, g-m, w,g,m:

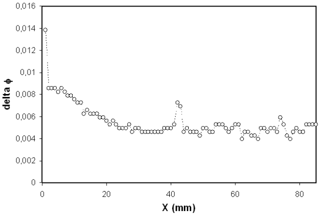
# Three phase Flash: phase diagram in compositional space $Z$ at fixed pressure $P$ (and $T$)

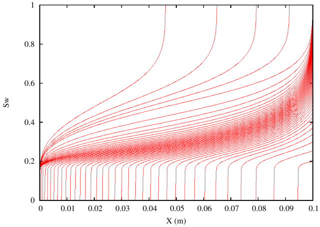
$Z_i$: total mass fractions of the components $i = CO_2, H_2O, Salt$

$$\left(Z_{CO2}, Z_{salt}\right):$$
$$Z_{CO2} \geq 0,$$
$$Z_{salt} \geq 0,$$
$$Z_{H2O} = 1 - Z_{CO2} - Z_{salt} \geq 0.$$

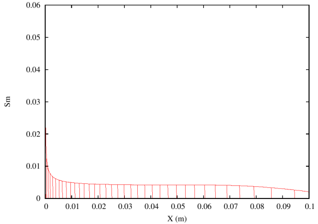


# Nearwell drying and alteration: laboratory experiment versus simulation



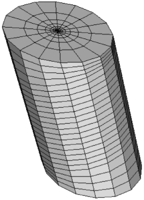




Water saturation

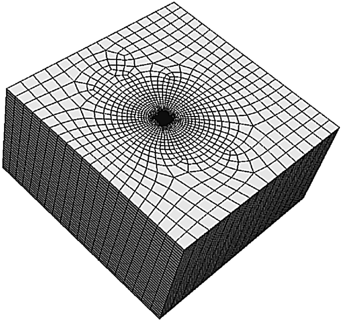



Mineral saturation

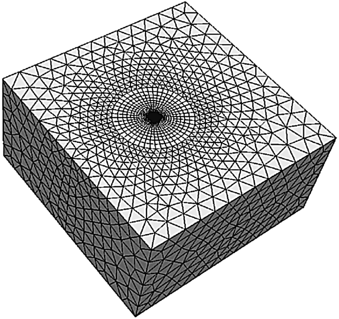
# 3D nearwell meshes for a deviated well



Radial mesh

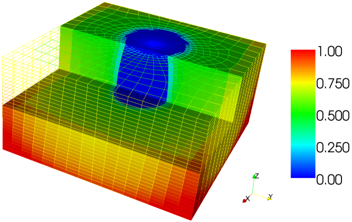


Hexahedral mesh

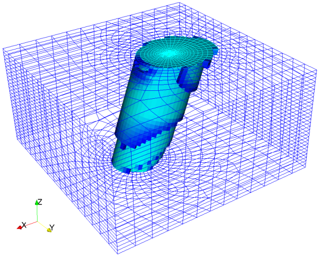


Hybrid mesh

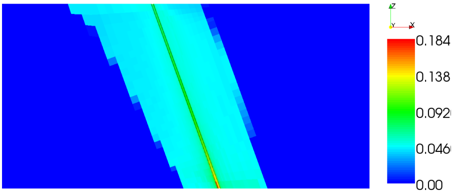
# Water and mineral saturations at final time



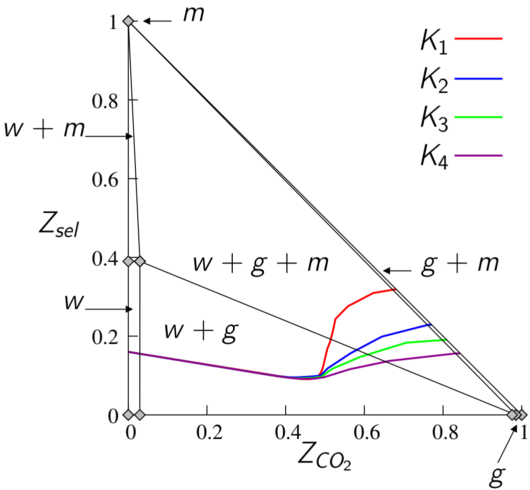

Water saturation




Mineral saturation $|S^m > 0.1\%$




2D $xz$ – mineral saturation $S^m$




Trajectory of four cells in the compositional space ($Z_{CO_2}$, $Z_{salt}$)

# Conclusions

- Vertex Approximate Gradient (VAG) scheme for compositional multiphase Darcy flows
  - Unconditionally coercive for arbitrary permeability tensors and meshes
  - Easy to implement on general meshes and for complex models
  - More efficient than cell centred schemes on tetrahedral meshes

# References I


- Agelas L., D.A. Di Pietro, Eymard R., Masson R.: *An abstract analysis framework for nonconforming approximations of the single phase Darcy equation*. IJFV, 2010.
- Agelas L., C. Guichard, Masson R.: *Convergence of finite volume MPFA O type schemes for heterogeneous anisotropic diffusion problems on general meshes*, IJFV, december 2010.
- V. Gervais, R. Masson *Numerical simulation of a stratigraphic model at geological time scales*, Computational Geosciences, 12:163-179, 2008.
- L. Jeannin, M. Mainguy, R. Masson, S. Vidal-Gilbert *Accelerating the convergence of coupled geomechanical-reservoir simulations*, Int. Journal for Numerical and Analytical Methods in Geomechanics, vol. 31, issue 10, 2007.
- J. Blum, G. Dobranszky, R. Eymard, R. Masson *Identification of a stratigraphic model with seismic constraints*, Inverse Problems, 22, 1207-1225, 2006.
- R. Eymard, T. Gallouët, V. Gervais, R. Masson *Convergence of a numerical scheme for stratigraphic modeling*, SIAM Journal of Numerical Analysis, vol. 43, 2, pp. 474-501, 2005.
- V. Gervais, R. Masson *Mathematical and numerical analysis of a stratigraphic model*, M2AN, Vol. 38, 4, pp 585-612, july/august 2004.
- R. Eymard, T. Gallouët, D. Granjeon, R. Masson, Q. H. Tran *Multi-lithology stratigraphic models under maximum erosion rate constraint*, International Journal of Numerical Methods in Engineering, 60, pp 527-548, 2004.
- E. Flauraud, F. Nataf, I. Faille, R. Masson *Domain Decomposition for an asymptotic geological fault modeling*, Comptes Rendus à l'académie des Sciences, Mécanique, 331, pp 849-855, 2003.

# References II


- R. Scheichl, R. Masson, J. Wendebourg *Decoupling and block preconditioning for sedimentary basin simulations*, Computational Geosciences 7, pp 295-318, 2003.
- R. Masson, M.C. Cacas, G. Dobranszky *Méthode pour simuler le dépôt d'une séquence sédimentaire dans un bassin*, brevet d'invention, 2003.
- R. Masson, M.C. Cacas, G. Dobranszky *Méthode pour simuler le dépôt d'une séquence sédimentaire dans un bassin respectant les épaisseurs des séquences sédimentaires*, brevet d'invention, 2004.
- R. Eymard, S. Lemaire, R. Masson: *Hybrid finite volume discretization of linear elasticity models on general meshes*. In proceedings of FVCA VI, Prague, accepted 2011.
- R. Eymard, R. Herbin, C. Guichard, R. Masson: *Multiphase flow in porous media using the VAG scheme*. In proceedings of FVCA VI, Prague, accepted 2011.
- R. Eymard, C. Guichard, R. Masson: *Grid orientation effect and multiPoint flux approximation*. In proceedings of FVCA VI, Prague, accepted 2011.
- Agelas L., Di Pietro D., Masson R. *A symmetric finite volume scheme for multiphase porous media flow problems with applications in the oil industry*, in the Proceedings of the FVCA5 conference, june 2008.
- Agelas L., D.A. Di Pietro, I. Kapyrin, R. Masson: *Generalized L-scheme for the discretization of diffusion fluxes on general meshes*, in the proceedings of ECMOR XI, Bergen, september 2008.
- Aavatsmark, I., Barkve, T., Boe, O., and Mannseth, T. (1996).
  Discretization on non-othogonal, quadrilateral grids for inhomogeneous, anisotropic media.
  *J. Comput. Phys.*, 127 (1):2–14.

# References III

Brezzi, F., Lipnikov, K., and Simoncini, V. (2005).
A family of mimetic finite difference methods on polygonal and polyhedral meshes.
*Math. Mod. Meths. Appli. Sci. (M3AS)*, 15:1533–1553.

Edwards, M. and Rogers, C. (1994).
A flux continuous scheme for the full tensor pressure equation.
In *Prov. of the 4th European Conf. on the Mathematics of Oil Recovery*, volume D, Røros, Norway.

Eymard, R., Gallouet, T., and Herbin, R. (2010a).
Discretisation of heterogeneous and anisotropic diffusion problems on general non-conforming meshes, sushi: a scheme using stabilisation and hybrid interfaces.
*IMA J. Numer. Anal.*, 30 (4):1009–1043.

Eymard, R., Guichard, C., and Herbin, R. (2010b).
Small-stencil 3d schemes for diffusive flows in porous media.
*http://hal.archives-ouvertes.fr*, submitted.

Eymard, R. and Herbin, R. (2007).
A new colocated finite volume scheme for the icompressible Navier-Stokes equations on general non matching grids.
*C. R. Math. Acad. Sci.*, 344(10):659–662.