

# **New interactions of Combinatorics and Probability**

**CIMPA – Summerschool**

**AUGUST 24 – SEPTEMBER 4, 2015**

**ICMC, Universidade de São Paulo, Brazil**

**PRELIMINARY SCIENTIFIC PROGRAM**

## **MINICOURSES**

### **Introductory Lecture**

**TBA**

(2.5 hrs) – The Introductory Lecture will open the School and give a broad overview of the history and current trends in the field of Interactions between Combinatorics and Probability.

### **Determinantal processes and combinatorics**

**Ph. Biane, CNRS - Univ. Paris Est, France**

(6 hrs) – The lecture will survey the theory of determinantal processes and give some examples coming from combinatorics and representation theory.

### **Algebraic Combinatorics**

**K. Ebrahimi-Fard, ICMAT-CSIC, Madrid, Spain**

**F. Patras, CNRS, Nice, France**

(3+3 hrs) – The seminal works of G.-C. Rota and his school, have transformed algebraic combinatorics into an important branch of mathematics with connections to a wide variety of subjects, among others, numerical methods for (partial/stochastic) differential equations; control theory; quantum field theory, (free) probability, number theory, discrete geometry, algebraic geometry. In these lectures we will introduce in a pedagogical way the basic notions and key results relevant in connection with applications in the main topics of this school.

### **Rough Paths Theory**

**David Kelly, Courant Institute, New York University, NY, USA**

(6 hrs) – We explain the basic ideas behind T. Lyons’ theory of rough paths. This theory provide a general framework in which it is possible to analyze the behavior of differential systems controlled by non-smooth signals, like for example, the paths of many interesting stochastic processes, such as Brownian motion and its fractional counterparts. The aim of the theory is to construct a solution map which sends an irregular signal  $x$  to the solution  $y$  of a differential equation of the form  $dy(t) = f(y(t))dx(t)$ . The key idea is that one must extract finer information from the signal  $x$  in the form of a more detailed object known as a ‘rough path’, this extra information is then used to construct the solution  $y$  in a systematic way. The space of rough paths itself contains rich algebraic and combinatorial structure akin to the Butcher series framework encountered in numerical analysis. If time permits, we will discuss very recent developments of rough path ideas and their algebraic counterparts in the realm of stochastic PDEs and renormalization.

## **Numerical integration, structure preservation and combinatorics**

**H. Munthe-Kaas, Univ. of Bergen, Norway**

(4 hrs) – Numerical integration of differential equations is a central topic in computational mathematics. A numerical integrator produces an approximation to the exact flow of the differential equation. Traditionally, order conditions and stability are the main design criteria of a numerical algorithm. More recently structure preservation has been recognized as an important qualitative feature. Much effort has been put into the design of algorithms which exactly preserve important geometric features such as first integrals, energy, impulse, continuous and discrete symmetries, symplectic and Hamiltonian structures as well as energy and volume preservation and intrinsic integration algorithms of systems evolving on manifolds. This gave rise to Geometric Integration as a research area in numerical analysis. For half a century B-series (after J. Butcher) has been one of the most important tools for analyzing numerical integration schemes. We have in the last two decades understood how various questions about structure preservation can be studied as algebraic conditions on the B-series expansion of the numerical integrator. More recently a comprehensive theory of integration of Lie groups and manifolds has been developed, and along with this a generalization of B-series to manifolds, so-called Lie-Butcher series. Whereas B-series is algebraically described in terms of pre-Lie algebras, the Lie-Butcher series are based on the recently defined concept of post-Lie algebras. These structures are closely related to invariant connections on manifolds.

We will in the talks give an extensive and self-contained introduction to the underlying algebraic structures and discuss how these relate to concepts in differential geometry and to various application areas.

## Free Probability

**R. Speicher, Univ. of Saarbrücken, Germany**

(6 hrs) – The theory of free probability is a relatively new branch of mathematics, which was started by D. Voiculescu in the early 1980s, when he realized that it could be useful to borrow techniques from probability theory to attack problems in the theory of operator algebras. This let him to define a new, non-commutative, probability, where the classical concept of independence is fruitfully replaced by the notion of freeness. In a nutshell, free probability is a non-commutative probability theory plus freeness. In this new context, most of the standard concepts of classical probability, such as the central limit theorem, entropy and Brownian motion, find their natural analogues. Free probability became not only useful in studying problems in operator algebra theory, but also as an independent subject, connected to various part of mathematics, for example, algebraic and enumerative combinatorics and random matrix theory.

## Stochastic Calculus

**A. Wiese, Heriot-Watt Univ., Edinburgh, Scotland**

(5 hrs) – This course will provide students with an introduction to the mathematical concepts of stochastic integral calculus and stochastic differential equations. It's integration by parts formula is then the starting point to investigate the structure of the algebra generated by iterated integrals. This algebraic structure has recently been explored to design new and efficient methods to solve stochastic systems numerically, and new results in this area will be presented in the course. The course will end with an outlook towards stochastic partial differential equations. The overall aim is to introduce students to stochastic calculus and to the structure of the algebra of multiple stochastic integrals, and to show how the algebraic structure facilitates the design and analysis of stochastic numerical integrators.

## ADVANCED LECTURES

### Peierls argument for Gibbs field with chess-board symmetric external field

**A. Iambartsev, IME-USP, Brazil**

(1.5 hrs) – We show the presence of a first-order phase transition for a ferromagnetic Ising model on integer lattice with a chess-board periodical external field. The external field takes two values  $h$  and  $-h$ . We prove that the phase transition takes place if  $h$  is small enough with known upper bound. The famous Peierls argument was used in order to prove the phase transition. We will discuss the relation of the model with antiferromagnetic model and also provide some generalizations.

### Decomposition of stochastic flows in dual foliated manifold: extensions of time

**P. R. C. Ruffino, IMECC-UNICAMP, Campinas, SP, Brazil**

(1.5 hrs) – Given two complementary foliations in a differentiable manifold say horizontal and vertical, consider the Lie subgroups of horizontal and vertical diffeomorphisms in  $M$  which preserve the corresponding leaves. There exists a factorization, up to a stopping time, of a stochastic flow of diffeomorphisms associated to an SDE on  $M$  into a diffusion in the horizontal subgroup composed with a process in the vertical subgroup. This decomposition has a natural geometric and dynamical interest e.g. on perturbation of a Hamiltonian system (constant energy leaves), coordinates of stochastic processes, horizontal lift of processes on fibre bundles, and others. (Catuogno, da Silva and Ruffino, Stoch. & Dyn. 2013). In this talk we are going to present extensions of time of this factorization. Essentially jumping diffeomorphisms which are not decomposable. The results open possibilities to study asymptotic properties. This is a joint work with Leandro Morgado.

### Universality in discrete copulas and Brownian bridges

**C. Tomei, Pontificia Univ. Católica do Rio de Janeiro, Brazil**

(1.5 hrs) – Copulas can be discretized in more than one way, yielding objects which for appropriate scaling limits give rise to Brownian bridges. Joint work with Juliana Freire and Nicolau Saldanha (both from PUC-Rio).