5/6/2024

CNRS AND AIMS CAMEROON

WORKSHOP ON DIFFERENTIAL GEOMETRY

BOOK OF ABSTRACTS

CNRS and AIMS Cameroon

FROM MAY 08 TO 10, 2024

AIMS African Institute for Mathematical Sciences CAMEROON

AIMS – Cameroon
Limbe, Crystal Gardens, South-West Region
P.O. Box 608 Limbe, Cameroon

Objective: The objective of the workshop on Differential Geometry, organized by CNRS in partnership with Université Côte d'Azur, University of Buea, and hosted by AIMS Cameroon, is to provide participants with a comprehensive understanding of the fundamental concepts, techniques, and applications in the field of differential geometry.

WORKSHOP - TIMETABLE	
Wednesday May 08, 2024	
8:30-9:00	Opening
9:00-09:40	Alphonse Mba
09:45-10:15	Coffee Break
10:15-11:55	Salomomon Mbatakou
12:00-14:00	Lunch
14:00-14:40	Romain Nimpa
14:45-15:15	Coffee break
15:15-16:00	Theophile Kemajou
16:05-16:30	Presentation: Calvin Meli (Title: Null submanifolds and statistical connections in indefinite Kähler manifolds)
16:35-17: 00	Presentation: Marius Foka : (Title: The Prescribed Ricci curvature Problem)
Thursday May 09, 2024	
9:00-09:40	Calvin Tcheka
09:45-10:15	Coffee Break
10:15-11:55	Celestine Dor
12:00-14:00	Lunch
14:00-14:40	Ferdinand Ngakeu
14:45-15:15	Coffee break
15:15-15:55	Moussa Koivogui
16:00-16:30	Presentation: Etienne Djoukeng
16:30-17:30	Presentation: Ludovic + Table ronde
Friday May 10, 2024	
9:00-09:40	Ancille Ngedakumana
09:45-10:15	Coffee Break
10:15-11:55	Thomas Bouetou
12:00-14:00	Lunch
14:00-14:40	Michel Djiadeu
14:45-15:15	Coffee break
15:15-15:55	Hans Fotsing
16:00-16:30	Presentation : Pascal Guiffo (Title: Riemann's Surfaces)
16:30-17:00	Presentation : Frankel Mba (Title: Algebraic properties of a bounded killing fields on a Riemannian Lie group)

Day 1

Wednesday May 08, 2024

RESUME DE L'EXPOSE DU PROFESSEUR MBA ALPHONSE AU ERGDWORKSHOP AIM'S CAMEROUN DU 08 AU 10 mai 2024

Titre : Prolongement d'objets géométriques

L'exposé de ce jour est sur les prolongements des objets géométriques, associés aux champs de vecteurs et aux foncteurs de Weil, et, sur les variétés de Poisson.

Dans cet exposé, nous donnons une courte présentation, à base des catégories, des relèvements tangents des feuilletages et quelques propriétés géométriques des feuilletages relevés.

Soit Q une variété différentielle de dimension $m \geq 1$, on définit le relèvement vertical des multi-champs de vecteurs sur Q. Une application de cette notion est donnée dans la géométrie de Poisson, en particulier nous décrivons la structure du feuilletage singulier induite par le relèvement vertical des multi-champs de vecteurs sur une structure de Poisson. Etant donné un champ de vecteurs du second ordre sur Q, on définit aussi le relèvement horizontal des multi-champs de vecteurs sur Q et on donne certaines de leurs propriétés.

Nous présentons des relèvements, associés au foncteur fibré de jauge sur les fibrés vectoriels, des fibrés vectoriels doubles et des sections linéaires sur un fibré vectoriel double.

Nous donnons un théorème de classification des morphismes naturels et des applications de celui-ci sur les prolongements de champs de vecteurs de type $(1, s)$.

Soit (G, ω) un groupoïde symplectique, nous caractérisons la version infinitésimale du groupoïde symplectique tangent d'ordre supérieur $(T^r G, \omega^c)$ où $T^r G$ est le groupoïde tangent d'ordre r et ω^c le relèvement complet de la forme symplectique ω.

Nous caractérisons certaines transformations naturelles entre les foncteurs fibrés $T^A \circ T^*$ and $T^* \circ T^A$ on $\mathcal{M}f_m$, sur la catégorie $\mathcal{M}\mathrm{f_m}$. Dans le cas particulier où $A = j_0^r(\mathbb{R}, \mathbb{R})$, nous déterminons toutes

les transformations naturels entre ces foncteurs. Nous utilisons cette théorie pour étudier les relèvements des 1-formes différentielles d'une variété M sur $T^A M$.

Conformal change of Weyl-type curvature in Finsler geometry.

Salomon Joseph Mbatakou
Email: salomon-joseph.mbatakou@facsciences-uy1.cm or mbatakou@gmail.com
University of Yaounde 1, Faculty of Science, Department of Mathematics, P.O. Box 812, Yaounde, Republic of Cameroon.

Abstract

In this talk we give a global axiomatic formalism, to study the behavior of the Weyl-type curvature tensor in Finsler geometry, under the conformal change of the Finsler metrics and some related geometry objects: Like Chern connection and associated curvatures tensors. Then we obtain some characterization of the conformally flat Finsler manifold. Although our treatment is entirely global, the local expressions of the obtained results, when calculated, coincide with the existing classical local results.

The Geometry of tangent bundle of a Lie group.

Nimpa Pefoukeu Romain
Email: romain.nimpa@facsciences-uy1.cm or nimpapefoukeu24@gmail.com
University of Yaounde 1, Faculty of Science, Department of Mathematics, P.O. Box 812, Yaounde, Republic of Cameroon.

Abstract

The geometry of tangent bundle of Riemannian manifols (M, g) is study using vertical, horizontal and complete lift of geometric tools on the base Riemannian manifold (M, g). For the particular case of Lie groups (G, μ) of dimension n, the multiplication μ induce on the tangent bundle TG a Lie group structure of dimension $2n$ and gives us a frame to study the geometry of some Lies groups of hight dimensions.

In this talk, we present those geometrics tools, and find the best conditions to study the geometry of tangent Lie group as a Lie group in general.

References

[1] F. Asgary, H. R. Salimi Moghaddam *On the Riemannian geometry of tangent Lie group*, Rend. Circ. Mat. Palermo, II. Ser(2018) 67:185-195, https://doi.org/10.1007/s12215-017-0304-z 2004.

[2] Davood Seifipour, Esmaeil Peyghan, *Some properties of Riemannian geometry of tangent bundle of Lie groups*, Tur J Math, (2019) 43:2842-2864, TUBITAK, doi: 10.3906/mat-1812-94

[3] Yano K., Ishihara S., *Tangent and Cotangent bundle*, vol 16 of Pure and Applied Mathematics Marcel Dekker, Inc. New York City(1973)

Théophile KEMAJOU MBIAKOP

University of Maroua

K-Normalized null hypersurfaces of para-Sasakian manifolds

Abstract:

In this work, we study the geometry of a null hypersurface M of a para-Sasakian manifold $(\overline{M}, \overline{\phi}, K, \overline{\eta}, \overline{g})$ transversal to the structure vector field K. The latter is a rigging ζ for M, and M is called K normalized null hypersurface. We characterize the geometry of such a null hypersurface and prove under some conditions that there exist leaves of an integrable distribution of the screen distribution admitting an almost para complex structure. Also, we obtain some non-existence results and discuss some properties of semi-symmetric(resp. locally symmetric) K-normalized null hypersurfaces of para-Sasakian manifolds. We show that a K-normalized null hypersurface is totally geodesic if and only if it is locally symmetric and that any para-Sasakian manifold admitting a semi-symmetric totally geodesic K-normalized null hypersurface is of constant negative curvature along the null hypersurface.

This is a joint project with Pr Ferdinand Ngakeu, (University of Douala, Cameroon).

Null submanifolds and statistical connections in indefinite Kähler manifolds

MELI CALVIN. B[1]

[1] *Department of Mathematics and Computer Science, University of Douala*
[1] *calvinbricemeli@gmail.com*

Abstract: A holomorphic statistical connection in a (indefinite) Kähler manifold induces in a natural way a statistical connection in a nondegenerate submanifold, but not if the submanifold is degenerate because of the degeneracy of the induced metric. In the case of a null hypersurface, we construct a new nondegenerate metric and we study conditions for the induced connection to be statistical with respect to it. We also study the case of codimension two invariant null submanifolds.

Keywords: Null submanifolds, statistical structures, dual connection, holomorphic connection, Kähler manifold.

References

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Day 2

Thursday May 09, 2024

Course Title: Operads and Relative Operads: Simplicial Connected Multiplicative Structure, Posets, Incidence Algebras, Cohomology Comparison theorem.

Course abstract

We will investigate in this course, the fine structure of a fundamental but little known theorem, the Gerstenhaber and Schack cohomology comparison theorem. The theorem classically asserts that there is a cochain equivalence between the usual singular cochain complex of a simplicial complex and the relative Hochschild complex of its incidence algebra, and a quasi-isomorphism with the standard Hochschild complex. Here, we will be mostly interested in its application to arbitrary posets (or, equivalently, finite topologies) and their incidence algebras. We will construct various structures, classical and new, on the above two complexes: cosimplicial, differential graded algebra, operadic and brace algebra structures and show that the comparison theorem preserves all of them. These results provide non standard insights on links between the theory of posets, incidence algebras, endomorphism operads and finite and combinatorial topology. By *non standard*, we refer here to the use of *relative* versions of Hochschild complexes and operads.

Keywords: Simplicial operad, Simplicial right brace algebra, Simplicial coalgebra, dot-product, odot-product, Hochschild cohomology, Shift operad, Poset, Finite topology, Nerve, simplicial set, Hochschild complex, cohomology.

HOMOLOGICAL CHARACTERIZATION OF u-S-INJECTIVE MODULES and u-S-FLAT MODULES

Dr Dor Celestine Kewir
Department of Mathematics, University of Buea

(WORKSHOP ON DIFFERENTIAL GEOMETRY 08-10 MAY 2024 AIMS LIMBE, CAMEROON)

ABSTRACT

Let R be a ring and S a multiplicative subset of R. That is, $1_R \in S$ and $\forall\ x, y \in S, \quad xy \in S$. Anderson and Dumitrescu in 2002 introduced the notion of S-finiteness. Since then several authors have invistigated various notions of S-finiteness. Using the notions of u-S-monomorphism, u-S- exact sequences and u-S-flat modules introduced by Xiaolei Zhang in 2022, we define u-S-injective modules. Some properties of these modules are studied. In particular, a Baer-like characterization of this class of modules is stated and u-S- injective modules are used to characterize u-S-flat modules via character module $M^\star = Hom_{\mathbb{Z}}(M, \mathbb{Q}/\mathbb{Z})$, in a manner similar to the characterization given by Cheatham and Stone in 1981.

Keywords: u-S-torsion module, u-S-monomorphism, u-S- exact sequences, u-S-injective modules, u-S-flat modules.

Semi-symmetric statistical manifolds

F. Ngakeu, N. Takam Fotsing and H. Fotsing Tetsing

Workshop in Differential Geometry,
Limbe, 8-10-2024

Abstract

In this work, we introduce and study semi-symmetric statitistical manifolds (3S-manifolds for short) as generalization of semi-Weyl manifolds. We prove that 3S-structures on a Riemannian manifold qre invqriqnt under the conformal changing of metrics. We show that every 3S-structure $(g, \omega, \omega^*, \nabla)$ on a Riemannian manifold (M, g) induces a statistical structure $(g, \widetilde{\nabla})$ on M and Vice-versa. In addition, the analog of the Statistical Curvature is defined for 3S-structures and its properties are investigated.

Keywords: Semi-symmetric connection; Semi-Weyl structure; Dual connection; Statistical manifold; 3S-manifold.

Dr KOIVOGUI MOUSSA

CONFERENCE DE DIFFERENTIELLE DE LIMBE
DU 8MAI AU 10 MAI 2024

WEIERSTRASS FORMULA FOR MINIMAL SURFACES IN SPECIAL THREE-DIMENSIONAL $\delta-$LORENTZIAN TRANS SASAKIAN MANIFOLD

ABSTRACT

Minimal surface, such as soap film, has zero curvature at every point. And it is well-known that the classical Weierstrass representation formula represents minimal surfaces in $\mathbb{R}^3$ via holomorphic functions. This reprentation of minimal surfaces is a very important notion in mathematics and physics for its applications. Mathematicians are attracted in studying of minimal surfaces that have certain properties, such as completedness and finite total curvature, while scientists are more inclined to periodic minimal surfaces observed in crystals or biosystems such as lipid bilayers.
For the first time Weierstrass representation for conformal immersion of surface into $\mathbb{R}^3$ appeared in the result of variational problem on search of minimal surface.

Weierstrass representation for minimal surfaces into Hyperbolic space have been obtained by kokubu ,Kenmotsu gave a type of weierstrass representation of prescribed mean curvature . Bonbenko have contributed to construct minimal surfaces and to understand thier propertie . The works of Konopelchenko, Taimanov,Land, Berdinski and Taimanov , Hofiman and Osserman , Uhlenbeck , and Osserman have helped to build many examples and understand many properties of mimal surfaces.

More recently, Mercuri, Montaldo and Piu have published A Weierstrass representation formula for minimal surfaces $\mathbb{H}_3$ and $\mathbb{H}^2 \times \mathbb{R}$ Later,Turhan and köpinar described minimal immersion in sol space, Koivogui and Todjihounde in give a Weierstrass representation for minimal immersions into Damek-Ricci spaces. And more recently Adriana, Francesco and Irene have worked on complex analysis and paracomplex analysis served to discuss a type of Weierstrass representation for minimal surfaces in Lorentzian Heisenberg group and Damek-Ricci spaces.

In this wprk, we applied the general setting on $\delta-$Lorentzian trans sasakian manifold and described a method to derive Weierstrass-type representation formulas for simply connected minimal surfaces into three-dimensional special $\delta-$Lorantzian trans sasakian manifold $\mathbb{T}^3$

On Volume preserving homeomorphisms and applications to finite-energy maps

Djoukeng Etienne
University of Buea, Cameroon
e-mail: fiss300@gmail.com

ERGDWorkshop AIMS-Cameroon, –10/05/2024, Limbe, Cameroon

Keywords: Homeomorphism, Flux, Geodesics , Homotopy, Finite Energy.

Abstract

In this work, we study the extension of the flux homomorphism defined on the identity component of the C^0-closure of the classical group of volume preserving diffeomorphisms [1] on any closed oriented manifold (M,Ω). We generalize the group $\mathrm{FHomeo}(M,\Omega)$ of finite-energy Hamiltonian homeomorphisms [2] by defining the group of finite-energy symplectic homeomorphisms. This is a topological group which contains $\mathrm{FHomeo}(M,\Omega)$ as a normal subgroup, and the kernel of the corresponding flux homomorphism contains $\mathrm{FHomeo}(M,\Omega)$.

References

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Day 3

Friday May 10, 2024

LEVI-MALCEV THEOREM FOR HOMOGENEOUS BOL ALGEBRAS AND REPRESENTATION OF BOL ALGEBRAS

THOMAS BOUETOU BOUETOU

May 6, 2024

Abstrat

Bol algebras are a broad generalization of Lie algebras that include Lie algebras, Malcev algebras and Lie triple systems as very particular examples. We present the concept of solvable ideals, nilpotent ideals in the category of Bol algebras, the existence of radical and nilradical is proven. The Levi-Malcev theorem is established for homogeneous Bol algebras. Finally we introduce the notion of representation of Bol algebras. We prove an analogue of the classical Engel's theorem for Bol algebras. An extension of Ado-Iwasawa theorem to Bol Algebras is also proved.

Course Title: Operads and Relative Operads: Simplicial Connected Multiplicative Structure, Posets, Incidence Algebras, Cohomology Comparison theorem.

Course abstract

We will investigate in this course, the fine structure of a fundamental but little known theorem, the Gerstenhaber and Schack cohomology comparison theorem. The theorem classically asserts that there is a cochain equivalence between the usual singular cochain complex of a simplicial complex and the relative Hochschild complex of its incidence algebra, and a quasi-isomorphism with the standard Hochschild complex. Here, we will be mostly interested in its application to arbitrary posets (or, equivalently, finite topologies) and their incidence algebras. We will construct various structures, classical and new, on the above two complexes: cosimplicial, differential graded algebra, operadic and brace algebra structures and show that the comparison theorem preserves all of them. These results provide non standard insights on links between the theory of posets, incidence algebras, endomorphism operads and finite and combinatorial topology. By *non standard*, we refer here to the use of *relative* versions of Hochschild complexes and operads.

Keywords: Simplicial operad, Simplicial right brace algebra, Simplicial coalgebra, dot-product, odot-product, Hochschild cohomology, Shift operad, Poset, Finite topology, Nerve, simplicial set, Hochschild complex, cohomology.

Conformal Geometry of Statistical Manifolds

DJIADEU NGAHA,
e-mail: michel.djiadeu@facsciences-uy1.cm
University of Yaounde 1, Faculty of Science, Department of Mathematics,
P.O. Box 812, Yaounde, Republic of Cameroon.

Abstract

Observing the statistical spaces from the doorway of differential geometry makes it convenient to study the statistical behaviors profoundly. For decades, information geometry as a combinaition of statistics and differential geometry has an effective role in science.

In this lecture structured on [[1],[2]], i will introduce the conformal vector fields on a statistical manifold, present necessary and sufficient conditions for a vector field on a statistical manifold to be conformal and give somes examples.

keywords: Statistical manifold, Information geometry, Conformal vector fields.

References

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Symplectic Realization of a class of the three-dimensional groups

April 26, 2024

Abstract

We realize symplectically classical dynamical systems of Lie groups within the framework of coadjoint orbit method. Precisely, we start with model with symmetry groups and construct the associated maximal coadjoint orbit. The latter are symplectic manifolds equipped with modified symplectic 2-forms. Through these constructions, we realize that the corresponding dynamical system can be physically interpreted An example of these constructions is given in detail.

THANK YOU FOR YOUR PARTICIPATION