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Skew products of nearly integrable symplectic maps

Abstract

We discuss dynamics of compositions of a finite collection of nearly integrable twist maps f_k of the annulus $\mathbb{A}^n = \mathbb{T}^n \times \mathbb{R}^n$: $f_k(x,y) = (x + \rho_k(y), y) + O(\epsilon)$ with positive definite $\rho_k'(y)$. When ϵ is small, almost all $\mathbb{A}^n$ is foliated by invariant KAM tori of f_k and hyperbolicity for each individual map, if it exists, is exponentially small. However, for skew products of nearly integrable maps, it is easy to prove the existence of action minimizing orbits with prescribed drift of y and positive Lyapunov exponents of order $O(\sqrt{\epsilon})$. Motivation for this research comes from the study of almost collision orbits of the 3 body problem of celestial mechanics.