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The Dual Potential, the involution kernel and transport in Ergodic Optimization

(joint work with Elismar O. Oliveira and Ph. Thieullen)

Abstract

We address the question of transport from a maximizing probability to another. Consider the shift σ acting on the Bernoulli space $\Sigma = \{1, 2, ..., n\}^{\mathbb{N}}$. We denote $\hat{\Sigma} = \{1, 2, ..., n\}^{\mathbb{Z}}$. We analyze several properties of the maximizing probability $\mu_{\infty,A}$ of a Holder potential $A : \Sigma \to \mathbb{R}$. Associated to $A(x)$, via the involution kernel, $W : \hat{\Sigma} \to \mathbb{R}$, it is known that can we get the dual potential $A^*(y)$, where $(x, y) \in \hat{\Sigma}$. We assume here that the maximizing probability $\mu_{\infty,A}$ is unique. Consider μ_{∞,A^*} a maximizing probability for A^*. We also analyze the analogous problem for expanding transformations on the circle.

We would like to consider the transport problem from $\mu_{\infty,A}$ to μ_{∞,A^*}. In this case, it is natural to consider the cost function $c(x, y) = I(x) - W(x, y) + \gamma$, where I is the deviation function for $\mu_{\infty,A}$, as the limit of Gibbs probabilities $\mu_{\beta A}$ for the potential βA when $\beta \to \infty$. The value γ is a constant which depends on A. We could also take $c = -W$ above. We denote by $\mathcal{K} = \mathcal{K}(\mu_{\infty,A}, \mu_{\infty,A^*})$ the set of probabilities $\hat{\eta}(x, y)$ on $\hat{\Sigma}$, such that $\pi_x^*(\hat{\eta}) = \mu_{\infty,A}$, and $\pi_y^*(\hat{\eta}) = \mu_{\infty,A^*}$.

We have a dynamical characterization of the solution $\hat{\mu}$ of the Kantorovich Transport Problem, that is, the solution of

$$\inf_{\hat{\eta} \in \mathcal{K}} \int\int c(x, y)\, d\,\hat{\eta} = -\max_{\hat{\eta} \in \mathcal{K}} \int\int (W(x, y) - \gamma)\, d\,\hat{\eta}.$$

The pair of functions for the Kantorovich Transport dual Problem are given by $(-V, -V^*)$, where we denote the two calibrated sub-actions by V and V^*, respectively, for A and A^*. For a certain class of potentials A we show that the W kernel satisfies a twist condition and, finally, we analyze, in this case, if the support $\hat{\Sigma}$ of the probability $\hat{\mu}$ is a graph. We also analyze the question of finding an explicit expression for the function $f : \Sigma \to \mathbb{R}$ whose $c-$subderivative determines the graph.