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Efficient Optimal Transport on the Circle

(joint work with Julie Delon (Télécom Paris) and Julien Salomon (Université Paris-Dauphine/CEREMADE))

Abstract

Consider the problem of optimally matching two measures on the circle, or equivalently two periodic measures on $\mathbf{R}$, where the cost $c(x, y)$ of matching two points x, y satisfies the Monge condition: $c(p, q) + c(r, s) < c(p, s) + c(r, q)$ whenever $p < r$ and $q < s$. Motivated by the weak KAM theory, we introduce a notion of locally optimal transport plan and show that all locally optimal transport plans are conjugate to shifts. The theory is applied to a transportation problem arising in image processing: for two sets of point masses, both of which have the same total mass, find an optimal transport plan with respect to a given cost function that satisfies the Monge condition. For the case of N real-valued point masses we present an $O(N \log \epsilon)$ algorithm that approximates the optimal cost within ϵ; when all masses are integer multiples of $1/M$, the algorithm gives an exact solution in $O(N \log M)$ operations.