

Corrigé du partiel B - 30/03/05

1. (a) $\mathbb{P}(S = k|D = j) = \mathbb{P}(X_1 + \dots + X_j = k, D = j)/\mathbb{P}(D = j) = \mathbb{P}(X_1 + \dots + X_j = k)$ par indépendance.
(b) $\mathbb{E}(S|D = j) = \sum_{k \geq 1} k \mathbb{P}(S = k|D = j) = \sum_{k \geq 1} k \mathbb{P}(X_1 + \dots + X_j = k) = \mathbb{E}(X_1 + \dots + X_j) = j \mathbb{E}(X_1) = pj$.
(c) $\mathbb{E}(S|D) = pD$.
(d) $\mathbb{E}(S) = \mathbb{E}(\mathbb{E}(S|D)) = \mathbb{E}(pD) = 7p/2$.
2. (a) Pour tout n , $\{T = n\} = \{X_2 = X_1, \dots, X_{n-1} = X_1, X_n \neq X_1\} \in \mathcal{F}_n$.
(b) $\mathbb{P}(T = k|X_1 = i) = \mathbb{P}(X_2 = i, \dots, X_{k-1} = i, X_k \neq i) = (1/7)^{k-2}(6/7)$ par indépendance.
(c) T ne prend que des valeurs ≥ 2 . Pour $k \geq 2$, $\mathbb{P}(T = k) = \sum_{1 \leq i \leq 7} \mathbb{P}(X_1 = i) \mathbb{P}(T = k|X_1 = i) = \sum_{1 \leq i \leq 7} (1/7)(1/7)^{k-2}(6/7) = (1/7)^{k-2}(6/7)$.
(d) $\mathbb{E}(T) = \sum_{k \geq 2} k(1/7)^{k-2}(6/7) = 1 + \sum_{k \geq 2} (k-1)(1/7)^{k-2}(6/7) = 1 + 7/6 = 13/6$.