

Examen, corrigé du sujet A

Durée : 1h. Documents, calculatrices et téléphones interdits.

1. On calcule : $C_4^2(1/2)^2(1 - 1/2)^{4-2} = \frac{4 \times 3}{2} \times \frac{1}{2^4} = \frac{3}{2^3} = \frac{3}{8}$.
2.
 - (a) La variable Z peut prendre les valeurs : 0, -1, 1.
 - (b) Calculons : $\mathbb{P}(Z = 0) = \mathbb{P}(\{X = 0, Y = 0\} \cup \{X = 1, Y = -1\}) = \mathbb{P}(X = 0, Y = 0) + \mathbb{P}(X = 1, Y = -1) = \frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} = \frac{1}{6} + \frac{2}{6} = \frac{1}{2}$, $\mathbb{P}(Z = 1) = \mathbb{P}(X = 1, Y = 0) = (2/3) \times (1/2) = 1/3$, $\mathbb{P}(Z = -1) = \mathbb{P}(X = 0, Y = -1) = (1/3) \times (1/2) = 1/6$.
3.
 - (a) $\mathbb{E}(U_1^2) = \int_0^2 \frac{1}{2} \times u^2 du = [\frac{u^3}{6}]_0^2 = \frac{8}{6} = \frac{4}{3}$.
 - (b) Par la loi des grands nombres, cette limite est $\mathbb{E}(U_1^2) = \frac{4}{3}$.
4. Soit $Y = Z/3$. Nous avons $Y \sim \mathcal{N}(0, 1)$. Donc $\mathbb{P}(Z < 0, 71) = \mathbb{P}(3Y < 0, 71) \approx \mathbb{P}(Y < 0, 24) = 0, 5948$.
5. On dérive, et on trouve : 0 si $x < 0$, e^{-x} si $x > 0$.
6. Calculons : $\mathbb{P}(\{X = 0\} \cup \{X = 1\}) = \mathbb{P}(X = 0) + \mathbb{P}(X = 1) = C_4^0(2/3)^4 + C_4^1(1/3)(2/3)^3 = \frac{2^4}{3^4} + 4 \times \frac{2^3}{3^4} = \frac{6 \times 2^3}{3^4} = \frac{2^4}{3^3} = \frac{16}{27}$.
7.
 - (a) $\mathbb{E}(X) = 0 \times (1/4) + 1 \times (1/4) + 2 \times (1/2) = 5/4$
 - (b) $\mathbb{E}(X^2) = 0^2 \times (1/4) + 1^2 \times (1/4) + 2^2 \times (1/2) = \frac{1}{4} + \frac{8}{4} = \frac{9}{4}$
 - (c) $\text{Var}(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = \frac{9}{4} - \frac{25}{16} = \frac{11}{16}$