

Partiel 1 - sujet B -corrigé

Durée : 1h. Documents, calculatrices et téléphones interdits. Toutes les réponses valent 3 points.

1. Formule du cours :

$$C_4^2 \left(\frac{1}{6}\right)^2 \left(1 - \frac{1}{6}\right)^{4-2} = \frac{4 \times 3}{2} \cdot \frac{1}{36} \times \frac{25}{36} = \frac{25}{6 \times 36} = \frac{25}{216}.$$

2. La variable $X + Y$ est à valeurs dans $\{-1, 0, 1, 2\}$. On utilise l'indépendance de X et Y :

$$\mathbb{P}(X + Y = -1) = \mathbb{P}(X = -1, Y = 0) = \mathbb{P}(X = -1)\mathbb{P}(Y = 0) = \frac{1}{12},$$

$$\begin{aligned} \mathbb{P}(X + Y = 0) &= \mathbb{P}(\{X = -1, Y = 1\} \cup \{X = 0, Y = 0\}) \\ &= \mathbb{P}(X = -1, Y = 1) + \mathbb{P}(X = 0, Y = 0) = \frac{1}{4} + \frac{1}{24} = \frac{7}{24}, \\ \mathbb{P}(X + Y = 1) &= \mathbb{P}(\{X = 0, Y = 1\} \cup \{X = 1, Y = 0\}) = \frac{3}{24} + \frac{1}{8} = \frac{3+3}{24} = \frac{6}{24} = \frac{1}{4}, \\ \mathbb{P}(X + Y = 2) &= \mathbb{P}(X = 1, Y = 1) = \frac{3}{8}. \end{aligned}$$

- 3.

$$\begin{aligned} \mathbb{E}(2^X) &= \sum_{k=0}^3 \mathbb{P}(X = k) 2^k \\ &= \sum_{k=0}^3 C_3^k \left(\frac{1}{3}\right)^k \left(1 - \frac{1}{3}\right)^{3-k} 2^k \\ &= \left(\frac{2}{3} + 1 - \frac{1}{3}\right)^3 = \frac{64}{27}. \end{aligned}$$

4. Soit $Y = Z/2$. Nous avons $Y \sim \mathcal{N}(0, 1)$. Et :

$$\mathbb{P}(Z \leq 1) = \mathbb{P}(2Y \leq 1) = \mathbb{P}(Y \leq 1/2) = 0,6915.$$

5. Intégrations par parties :

$$\begin{aligned} \int_{-\pi/2}^{\pi/2} x^2 \cos(x) dx &= [x^2 \sin(x)]_{-\pi/2}^{\pi/2} - \int_{-\pi/2}^{\pi/2} (2x) \sin(x) dx \\ &= \left(\frac{\pi^2}{4}\right) - \left(\frac{\pi^2}{4}\right)(-1) + [(2x) \cos(x)]_{-\pi/2}^{\pi/2} - \int_{-\pi/2}^{\pi/2} 2 \cos(x) dx \\ &= \frac{\pi^2}{2} - [2 \sin(x)]_{-\pi/2}^{\pi/2} \\ &= \frac{\pi^2}{2} - 4. \end{aligned}$$

6. La densité est :

$$x \in \mathbb{R} \mapsto \begin{cases} 0 & \text{si } x < 0 \\ 2x & \text{si } x \in]0; 1[\\ 0 & \text{si } x > 1. \end{cases}$$

7. $\mathbb{P}(Y \in \{0, 1, 2\}) = \mathbb{P}(Y = 0) + \mathbb{P}(Y = 1) + \mathbb{P}(Y = 2) = e^{-1} \left(1 + \frac{1^1}{1!} + \frac{1^2}{2!}\right) = e^{-1} \times \frac{5}{2}.$