Analysis of PDEs. The Cauchy problem in collisionless kinetic theory (N. Besse).

The aim of this lecture is to present the state of art of the Cauchy problem for the collisionless kinetic equations such as the Vlasov–Poisson and Vlasov–Maxwell systems. Collisionless kinetic equations, which are Hamiltonian systems, appear among others in plasma physics and astrophysics. In plasma physics, these models describe accurately the wave-particle interaction which plays a crucial role in turbulent plasmas such as magnetic fusion plasmas (ITER project). In astrophysics theses models allow to describe the large scale structure of the universe such as clusters of galaxies and the dark matter. Here, we present the theory of weak solutions and classical regular solutions. Existence theory of classical solutions is based on natural a priori estimates like among others the conservation of energy, on the theory of characteristics and on the control of the velocity support of the distribution function. Uniqueness follows from regularity properties of classical solutions. Existence theory of weak solutions relies on a priori functional estimates and on compactness results such as standard compact embeddings in Sobolev spaces for the Vlasov–Poisson equations, or averaging lemmas for the Vlasov–Maxwell system. Uniqueness of weak solutions is a more tricky task and sometimes it is still an open issue.

Contents:

1) Introduction: theory of characteristics; formal properties of the Vlasov–Poisson and Vlasov–Maxwell systems; conservation laws; basic a priori estimates.
2) Weak and classical solutions for the Vlasov–Poisson system.
3) Weak and classical solutions for the Vlasov–Maxwell system.