On the decomposition groups of non-rational plane curves

Jérémie Blanc (Nice)

Abstract. I will speak about the group $G$ of birational transformations of the plane that fix (each point of) a curve $C$ of positive genus. This group is called the inertia group of the curve in the Cremona group. Castelnuovo proved in 1892 that if the curve $C$ has genus at least 2, the order of an element of finite order of $G$ is 2,3 or 4. I will present a recent classification, that shows in particular that the group $G$ is always abelian, and is either isomorphic to $\mathbb{Z}/2\mathbb{Z}$, $\mathbb{Z}/3\mathbb{Z}$ or to an infinite group with only one torsion element of order 2. The case of curves of genus 1 is very different, I will try to describe it briefly.

A Barth-Lefschetz theorem for submanifolds of a product of projective spaces

Lucian Bădescu (Genoa)

Abstract. Let $X$ be a complex submanifold of dimension $d$ of $\mathbb{P}^m \times \mathbb{P}^n$ ($m \geq n \geq 2$) and denote by $\alpha : \text{Pic}(\mathbb{P}^m \times \mathbb{P}^n) \to \text{Pic}(X)$ the restriction map of Picard groups, by $N_{X|\mathbb{P}^m \times \mathbb{P}^n}$ the normal bundle of $X$ in $\mathbb{P}^m \times \mathbb{P}^n$. Set $t := \max\{\dim \pi_1(X), \dim \pi_2(X)\}$, where $\pi_1$ and $\pi_2$ are the two projections of $\mathbb{P}^m \times \mathbb{P}^n$. We prove a Barth-Lefschetz type result as follows:

Theorem. — If $d \geq \frac{m+n+t+1}{2}$ then $X$ is algebraically simply connected, the map $\alpha$ is injective and $\text{Coker}(\alpha)$ is torsion-free. Moreover $\alpha$ is an isomorphism if $d \geq \frac{m+n+t+2}{2}$, or if $d = \frac{m+n+t+1}{2}$ and $N_{X|\mathbb{P}^m \times \mathbb{P}^n}$ is decomposable.

Explicit examples show that these bounds are optimal. The main technical ingredients in the proof are: the Kodaira-Le Potier vanishing theorem in the generalized form of Sommese, the join construction and an algebraisation result of Faltings concerning small codimensional subvarieties in $\mathbb{P}^N$. This is a joint work with Flavia Repetto.
On varieties uniruled by lines
Carla Novelli (Genoa)

Abstract. It is well-known that an irreducible non-degenerate complex variety $X \subset \mathbb{P}^N$ of degree $d$ satisfies $d \geq N - \dim X + 1$. Varieties with $d$ “small” compared to $N$ have been intensively studied throughout the years, and one of their common features is that they are covered by rational curves, i.e. they are uniruled. More generally, given a pair $(X, H)$ where $X$ is an irreducible $n$-dimensional variety polarized by a nef and big line bundle $H$, we can study the degree of the covering curves with respect to $H$, and say that $X$ is uniruled of $H$-degree at most $m$ if all the covering curves $\Gamma$ satisfies $\Gamma \cdot H \leq m$. Our result is the following:

Let $(X, H)$ be a pair consisting of a reduced and irreducible variety $X$ of dimension $n \geq 3$ polarized by a globally generated big line bundle $H$. Set $d := H^n$ and $N := h^0(X, H) - 1$. If $d < 2(N - n) - 4$, then $X$ is uniruled of $H$-degree one, except if $(d, N) = (27, 19)$ and a $\sharp$-minimal model of $(X, H)$ is $(\mathbb{P}^3, O_{\mathbb{P}^3}(3))$. The result is optimal for $n = 3$. These results are contained in a joint paper with Andreas Knutsen and Alessandra Sarti.

Espaces de modules de connexions sur $\mathbb{P}^1$ et l’algorithme de Katz
Carlos Simpson (Nice)

Abstract. Letting $X = \mathbb{P}^1$, and $D = p_1 + \ldots + p_n \subset X$ be a divisor, put $U := X - D$. We can define a moduli space $M_B(U; C_1, \ldots, C_n)$ of representations of $\pi_1(U, u)$ into $GL(r, \mathbb{C})$, with fixed semisimple conjugacy classes for the images of loops around the $p_i$. We will explain a version of the method used by Kostov and Crawley-Boevey, based on Katz’s convolution operation, to study these moduli spaces. One can write a formula for the dimension in the form

$$\dim M_B(U; C_1, \ldots, C_n) = 2 + r\delta(C_1, \ldots, C_n) + \sum_{i=1}^{n} \sigma(C_i)$$

where $\delta(C_1, \ldots, C_n) := (n - 2)r - \sum_{i=1}^{n} \nu(C_i)$ with $\nu(C_i)$ being the largest multiplicity for an eigenvalue of $C_i$, and $\sigma(C_i) \geq 0$. Kostov observed that Katz’s algorithm (originating in the rigid case) allows us to reduce to the case $\delta \geq 0$. We then have the following result.

Theorem (Kostov, Crawley-Boevey).— If $C_1, \ldots, C_n$ are conjugacy classes such that $\delta > 0$, or else $\delta = 0$ and $\sigma > 0$, then $M_B(U; C_1, \ldots, C_n)$ is non-empty.

An amusing proof uses certain Higgs bundles called cyclotomic, which are like VHS’s except that the Kodaira-Spencer map can go all the way around in a circle. Also, the case of dimension 2 with $\delta = \sigma = 0$ is very interesting.
Lattice cohomology of normal surface singularities

Andras Némethi (Budapest)

Abstract. The topology of a normal surface singularity is identified by (one of) its resolution graphs, which is connected and negative definite. Starting from such a graph (or, from the corresponding intersection lattice) we construct a cohomology theory. It turns out that the theory has some remarkable properties: if the link is a rational homology sphere then its (normalized) Euler characteristic (conjecturally) is the Seiberg-Witten invariant of the link, and the cohomology modules itself (up to shift in grading) constitute the Heegaard-Floer homology of the link. Moreover, the 0-th module provides upper bound for the sheaf-cohomologies of line bundles defined on the resolution of the singularity, connecting the topology and analytic invariants. Finally, it provides new topological characterizations for rational and (weakly) elliptic singularities, and hopefully it is the right topological guiding object for further classifications.

Some applications of adjoint systems

Paltin Ionescu (Genoa)

Abstract. Adjoint systems combine the canonical divisor and the hyperplane class, thus giving an efficient tool for understanding the geometry of embedded manifolds. We illustrate this through a number of applications including: a generalization of a result by Van de Ven, characterizing linear subspaces by the splitting of the normal bundle sequence; a proof that manifolds “of small degree” are rationally connected.