Some talk titles and abstracts
Carlos Simpson

—My 4 talks at the 2011 Talbot Workshop, Draper, Utah, May 1-7, 2011:
(i) “Harmonic bundles”, May 2nd, 2011
(ii) “Moduli of representations”, May 3rd, 2011
(iii) “Higher nonabelian Hodge theory”, May 5th, 2011
(iv) “Local systems on noncompact varieties”, May 6th, 2011

—Groupe de travail “Algèbre et Topologie Homotopiques”, Chevaleret, Institut de Mathématiques de Jussieu:
C. Simpson, “Catégories de modèles pour les $n$-catégories de Segal”, March 10th, 2010

—Conference “Cohomology of Algebraic Varieties, Hodge Theory, Algebraic Cycles, Motives”, IHP, Paris, April 26-30, 2010:
C. Simpson, “Mixed Hodge theory for the local structure of representation spaces”, April 28th, 2010
Abstract: Let $R = R(\pi_1(X), G)$ be the space of representations of $\pi_1(X)$ in a reductive group $G$. If $X$ is smooth complex projective and $\rho$ is the monodromy representation of a variation of Hodge structures, then the formal local ring of $R$ at $\rho$ has a MHS. We explore some possible extensions to higher homotopy.

—Conference “Géométrie algébrique à la dérive”, Montpellier, May 24-30, 2011:
C. Simpson, “Nonabelian cohomology as a shape theory for higher homotopy types”, May 24th, 2010

—Rencontre à l’ENS, ANR “Sediga”, Paris, September 21-22, 2010:
Abstract: We look at a natural family of Lagrangian subspaces of the moduli space of regular connections, in particular in the case of the projective line minus four points corresponding to Painlev VI. Iwasaki-Inaba-Saito as well as Szabo have given explicit constructions, which we compare with the abstract definition using the nonabelian Hodge filtration. This allows us to see that the subspaces form a foliation in this case, the quotient of which has appeared in work by Arinkin on the Langlands program. For a related result, see D. Arinkin and R. Fedorov, An example of Langlands correspondence for irregular rank two connections on $\mathbb{P}1$, on arXiv: 1003.6112.

—Algebraic geometry seminar, Institut de Mathématiques de Jussieu:

—Rencontre à l’ENS, ANR “Sediga”, Paris, February 8th, 2011:
C. Simpson, “Higher cohomology stacks over the twistor space”, February 8th, 2011
Most aspects of the theory of moduli of local systems on smooth projective varieties extend to smooth (or even just normal) proper DM-stacks, via a little covering lemma. For other singularities or simplicial varieties we meet a phenomenon of weight filtration. The DM-stack case also allows us to approach the question of open varieties while avoiding the more difficult technical aspects there, and it provides a convenient formalism for finite group actions. In the example of a root stack over the projective line, the moduli space can have components containing no representations into a compact group.

The first nonabelian cohomology of a variety is the moduli space of representations of its fundamental group. There are several different algebraic varieties corresponding to this space, and these have various interesting structures. We’ll discuss these structures, their relationships, and how some of them might be generalized to higher nonabelian cohomology.

This reports on joint work with Frank Loray and Masa-Hiko Saito. Given a connection with parabolic structure, one can look at the limit as \( t \to 0 \) in Hitchin’s twistor space. The limit is a \( \mathbb{C}^* \)-fixed Higgs bundle. Breaking up the moduli space according to the isomorphism class of the limit leads to a decomposition in locally closed subvarieties. In the case of rank 2 connections on \( \mathbb{P}^1 - \{ t_1, t_2, t_3, t_4 \} \) we are able to show that the subvarieties are closed. They are the fibers of fibrations, depending on the parabolic weights, which are already known: appearing for example in work of Arinkin and Lysenko, and of Iwasaki, Inaba, Saito. Katz’s middle convolution is one of Okamoto’s symmetries exchanging the different types of fibrations.

On expliquera le cadre de la cohomologie nonabélienne a coefficients dans un n-champ, et comment obtenir des éléments de la théorie de Hodge pour celle-ci. Cette approche se compare avec d’autres approches telle que l’homotopie schématique, les complexes parfaits, et la théorie tannakienne.