The code $mono3\_wh$

Can be used to compute the Milnor fiber monodromy of any reduced plane curve C, and the pole order filtration on the cohomology of the Milnor fiber when C has only weighted homogeneous singularities

Let $f \in S = \mathbb{C}[x, y, z]$ be a homogeneous polynomial of degree d, such that the corresponding plane curve $C : f = 0$ in $\mathbb{P}^2$ is reduced. Then the program $mono3\_wh$ computes certain terms of the second term of the spectral sequences $E_*^{s,t}(f)_k$ for $k = 1, 2, ..., d$, following the procedure described in [2, 1]. The program computes first the global Milnor number $\mu(C)$ and the global Tjurina number $\tau(C)$.

Warning. Make sure that the coordinates x, y, z are chosen such that the line $L : z = 0$ contains no singularities of the curve C. This is achieved by making a linear substitution $z = ax + by + cz$ with a, b, c general enough. Otherwise the computation of $\mu(C)$ gives a wrong value.

Two cases are possible.

Case 1. If $\mu(C) = \tau(C)$, then $E_2^{s,t}(f)_k = E_\infty^{s,t}(f)_k$, since in this case C has only weighted homogeneous singularities, see [3], quoted in [2, Thm.2.1]. Hence in this case we get *the monodromy action and the pole order filtration* on $H^*(F)$, where $F : f = 1$ is the corresponding Milnor fiber in $\mathbb{C}^3$, see [2]. In Table 1 there are several invariants computed along the way as explained in [2, Section 4], but for applications only the data in Table 2 are necessary.

The sequence H_q^1 is defined as follows

$$H_q^1 = \dim E_\infty^{1,0}(f)_q \text{ for } q \in [3, d],$$

$$H_q^1 = \dim E_\infty^{0,1}(f)_{q-d} \text{ for } q \in [d + 1, 2d],$$

and H_q^1 is trivial for $q > 2d$. To compute the monodromy, e.g. the Alexander polynomial of C, it is enough to compute H_q^1 for $q \in [3, d]$, see [2, Remark 4.4]. The sequence H_q^2 is defined as follows

$$H_q^2 = \dim E_\infty^{2,0}(f)_q \text{ for } q \in [3, d],$$

$$H_q^2 = \dim E_\infty^{1,1}(f)_{q-d} \text{ for } q \in [d + 1, 2d],$$

$$H_q^2 = \dim E_\infty^{0,2}(f)_{q-2d} \text{ for } q \in [2d + 1, 3d],$$

and H_q^2 is trivial for $q > 3d$. This sequence is useful only if we are interested in the pole order filtration or the roots of the BS-polynomials, see [1, 2].

Case 2. If $\mu(C) > \tau(C)$, then at least one singularity of C is not weighted homogeneous. Then we compute only the terms

$$H_q^1 = \dim E_2^{1,0}(f)_q \text{ for } q \in [3, d],$$

since this is all we need to compute *the monodromy action* on $H^*(F)$, see [2, Remark 4.4]. This computation is very efficient, so we can treat curves with a high degree d. The computation of *the pole order filtration* takes much longer time and is done by another program *mono*3_*gen* on our web page.

References

[1] A. Dimca, *Hyperplane Arrangements: An Introduction*, Universitext, Springer, 2017.

[2] A. Dimca, G. Sticlaru, Computing the monodromy and pole order filtration on Milnor fiber cohomology of plane curves, arXiv: 1609.06818.

[3] M. Saito, Bernstein-Sato polynomials and graded Milnor algebras for projective hypersurfaces with weighted homogeneous isolated singularities, arXiv:1609.04801.