PROPOSAL FOR A PHD THESIS

Context: The start-up company WEVER offers small-distance rideshare services, taking place either regularly or occasionally together with special events, for users registered in their system. The idea is to offer incentives for users willing to accept a detour to accept other users as passengers, in the form of vouchers or discounts for certain events. These vouchers or discounts are offered by companies that in turn pay a fixed amount y to WEVER for every rideshare that indeed takes place and that is sponsored by their vouchers.

Goal of this thesis: The goals of this thesis are twofold: on the one hand, a sound mathematical model (see below) describing the interactions between users and companies, and on the other hand also an implementation of the model.

The mathematical model: We suppose that each user x is represented by a point in a space $\mathcal{E}$ representing its source location s_x and destination d_x of the user as well as other sociological criteria of interest, its susceptibility to change his mind when offered discounts, etc., and we may assume the space is equipped with a distance function $d(\cdot, \cdot)$. Within a certain time window users may launch their requests for rideshares, and then an algorithm is proposed for matching users. We assume also that each user x has an initially unknown probability function $p(x, d, v)$ that represents the probability that user x accepts a detour, as a function of the distance d to the other user and also as a function of the vouchers v offered to her. Furthermore, each company also has a certain gain function $g(v)$, depending on the quantity of vouchers v offered to a user. We may assume that $g(v)$ is a convex continuous function in v, and we assume $g(v)$ is known to the company.

The different interactions can be explained in a game-theoretical way. On the one hand, consider the interactions between the users and a company. Again, we observe a Stackelberg game: The company acts as a leader in this game and first proposes a value v for the vouchers. The user x then takes the binary decision whether to accept the rideshare offer or not, according to her probability function $p(x, d, v)$. The company then chooses v^* as to maximize $\sum_x p(d, x, v)g(v)$. On the other hand, consider the interactions between WEVER and a company. Clearly, if $y > \max_v g(v)$, then the company will reject to pay y. Otherwise, independently of y, the company chooses v^* so that $\sum_x g(v)p(x, d, v)$ is maximized (coming from the first interaction). Thus, the interactions between WEVER and a company can be seen as a Stackelberg game: WEVER first chooses a value y each company should pay. The company then checks whether $y \leq \max_v g(v)$, and then chooses v^* that maximizes $\sum_x g(v)p(x,d,v)$. Given this value of v^*, WEVER then adapts their value of y again by maximizing the value of y while at the same time still satisfying $y < g(v^*)$. If all involved functions were known, we would find the desired maxima. Since we do not want to assume this, we use the following probabilistic tools to find the desired functions. More precisely, in order to so, we want to address the following goals:

Goal 1: Finding an efficient matching: Assume first that each user accepting a detour is offered the same voucher v. In the simple case of pairing up users, a first goal is to provide an efficient algorithm for a matching M of users with and users without cars, so that the expected total distance

$$\sum_{(x,y)\in M} \left(d(s_x,s_y)+d(s_y,d_y)+d(d_y,d_x)\right)p(d,x,v)+\left(d(s_x,d_x)+d(s_y,d_y)\right)\left(1-p(d,x,v)\right) \tag{1}$$

is minimized.
Subgoal 1: Finding a good estimation for $p(\cdot)$*:* Since $p(\cdot)$ is unknown, we might initially, when lacking previous data, assume $p(\cdot)$ to be 1 (or a simple indicator function depending on some threshold of distance), and find an algorithm for minimizing (1) with this value of $p(\cdot)$. Once sufficient past data are gathered, assuming continuity of $p(\cdot)$ we might use a sampling of previous realizations of rideshares to minimize the expectation of (1) with the sampled average of these realizations as the representation for $p(\cdot)$. Assuming a sufficiently big number of realizations, we hope to prove convergence of the calculated expectation to the actual value of (1).
Subgoal 2: Finding algorithmically the optimal matching: Having or not obtained enough data for provably well estimating $p(\cdot)$, the next goal is how to find algorithmically the desired matching. One reasonable starting point would be an attempt to modify the deterministic polynomial-time algorithm by [2] for finding a perfect matching in Euclidean space. In case the analysis of this algorithm adapted to this problem turns out to be infeasible, the solution obtained might still serve as a starting configuration for a simulated annealing algorithm. The latter then might proceed by iteratively switching one pair of proposed rideshares in the matching (x_1,x_2), (x_3,x_4) by (x_1,x_3), (x_2,x_4) (or by (x_1,x_4), (x_2,x_3), respectively). Again, we hope to obtain provable results about the correctness (in case the first attempt works) or about the convergence of the second algorithm.
Subgoal 3: Calculating the value of (1)*:* Assuming a certain (say uniform) distribution of the users in $\mathcal{E}$, the goal is to find for given concrete functions of $p(\cdot)$ (for example, $p(\cdot)$ being constant over $\mathcal{E}$, following a simple law depending on the distance), the value of (1). Ideally, an objective method approach such as used in Section 5 of [1] could be used. If not, classic tessellation arguments used in the

analysis of random geometric graphs might be used to find approximations of the value of (1).

Goal 2: Finding an optimal repartition of the vouchers: Orthogonal to the first goal, we may also the consider the following problem: suppose that the total number of discounts/vouchers can be split arbitrarily among users, and suppose that each user has a probability function $p(\cdot)$ being continuous and monotone as a function of discounts offered to her. Assume now that different discounts may be given to different users, depending on the detour they might accept, but assume that the total number of discounts is fixed. It would be interesting to find the optimal (or at least provably good) partition of the vouchers. Assuming as in the previous subgoal a certain distribution of the users in $\mathcal{E}$, we hope also to quantify the improvement of the value of (1) when using a non-uniform distribution of vouchers compared to a uniform distribution, and to quantify also the ratio of the expected value of a non-uniform distribution of vouchers and the theoretical global optimum, corresponding to the situation where all users accept the offered rideshares, and the total distance by all users is minimized. Clearly, once a given non-uniform distribution of vouchers is found, the same algorithmic questions as in Goal 1) can be asked.

Desired background: probability theory, analysis of algorithms, programming skills. Knowledge of French is not required. Desired starting date: as soon as possible (October 2016); there is some flexibility with the starting date though. Fixed 3-year-contract, net salary minimum 1500 Euro/month. The candidate will spend half of the time at Univ. Nice, Laboratoire Dieudonné in the probability theory group, half of the time at WEVER.

Deadline for applications: Sept 5, 2016. Send a CV, a short motivation letter and 2 names that can be contacted for reference letters to dmitsche@unice.fr. Informal inquiries welcome.

Contact address:
Dieter Mitsche
Univ. Nice Sophia Antipolis
email: dmitsche@unice.fr

References

[1] D. Aldous, M. Steele, *The objective method: probabilistic combinatorial optimization and local weak convergence*, Probability on discrete structures, Volume 110.

[2] K. Varadarajan, *A divide-and-conquer algorithm for min-cost perfect matching in the plane*, FOCS 1998.