LINEAR STABILITY OF TWO- AND THREE-LAYER POISEUILLE FLOWS. APPLICATION TO THE COEXTRUSION OF POLYMERS.

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Introduction

The stability of multilayer Poiseuille flow has been recently studied by numerous authors [1,2] due to the growing industrial importance of multilayer products in polymer processing. For example, food packaging commonly uses films formed of three, five or more layers in order to put together polymers having different optical, mechanical and barrier (to air or water) properties. In the industrial process, the polymers are molten in screw extruders, and then pushed through a feedblock and a flat die. This process is limited by the onset in some conditions of a wavy instability deteriorating optical qualities of the film.

In the sequel, temporal stability analysis, experiments and time dependent finite element simulations are presented. The computations are restricted to streamwise periodic perturbations and Olroyd-B fluids with constant viscosities and relaxation times. Moreover in this process, the Reynolds number and surface tension are assumed to be very small. Therefore, the thickness, viscosity and elasticity ratios are the more discriminant parameters.

Temporal stability analysis

As usual two different methods are used [1,2]. First the longwave analysis allows us to separate the kinetic equation from equations of motion (so the instability can be referred as interfacial). This way provides asymptotic stable regions in the plane defined by the thickness and the viscosity ratios. However, this asymptotic analysis has to be completed by a moderate wavelength analysis in order to insure the full stability.

Unlike purely Newtonian fluids, it is found that for viscoelastic fluids and vanishing Reynolds number, the longwave analysis allows to forecast the stability with respect to moderate wave number. This property is found for two-layer [3] and three-layer flows [4]. In particular, we show that the configuration is stable if the Poiseuille velocity profile is both convex and asymptotically stable. The full stability of the other asymptotically stable regions has to be checked by means of the moderate wavelength analysis. This result is illustrated in the Fig. 1 for two-layer flows with elasticity ratios smaller than 1. The area below the line $m = 1$ is stable whereas the domain above this line is asymptotically stable but can be unstable with respect to moderate wavelength.

Experiments

The very comprehensive experiments made by Wilson and Khomani [5] have shown that the theoretical approach based on Olroyd-B approximation can give rather qualitative and quantitative good results. Experiments was performed with a two-layer Poiseuille flow (Pe/PS). Our apparatus consists of a flat die connected by a feedblock to two different extruders. The flow channel has a length and a width of 10 cm and a thickness of 1.5 mm. Experiments are made for various flow rate ratios and temperatures in order to change respectively the thickness ratios and rheological parameters of each fluid (and therefore the viscosity and the elasticity ratios). Wavy interfaces with a spatial periodicity about 2-5 mm are detected for specific parameter values. As these polymers follow a Carreau-Yasuda law, the thickness ratio ϵ and

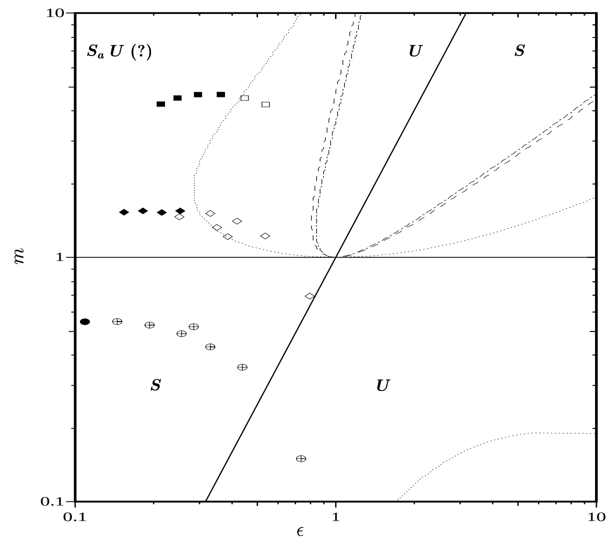

Figure 1: Neutral curves in the plane (ϵ, m) for two-layer flows. ϵ and m are respectively the thickness and viscosity ratios. Theoretical results: the full lines (–) delimit the area in which the Poiseuille velocity profile is convex; the dashed lines (– –), the asymptotically stable areas for various elasticity ratios. The symbols U and S mean that the Poiseuille profile is respectively unstable or stable; the symbols S_a $U(?)$ mean that the full stability is given by moderate wavelength analysis. Experimental data for various temperatures: T= 180 (□,■), T = 200 (◇,♦) and T= 220 (⊕, •); disturbed interfaces (•,♦,■)

viscosity ratio at the interface m_i are determined from the experimental flow rates by a numerical prodecure. Theses values collected in Fig. 1 show that there is rather good agreement between theoretical results and experimental observations. In fact, the interface becomes more often unstable as the data go across the line $m = 1$. Then, the interface can be stabilized or destabilized by changing the melt temperature. However, there are several points which are theoretically unstable but experimentally stable. The discrepancy can come from our simple theoretical model or the nonlinear behavior of unstable modes.

Finite element computations

We have restricted our computations to Newtonian fluid. The aim is to describe the nonlinear behavior of disturbances when the Poiseuille profile is unstable. By using a finite element method based on the pseudoconcentration function [6], we find that these interfacial instabilities are convective [7]. As such instabilities are essentially noise amplifiers, the temporal instability does not always involve an obvious deterioration in the interface. In fact, the growth of unstable modes is given by the spatial amplification along flow motion and needs a spatial stability analysis. In this way, it is also possible to identify the most amplified frequencies.

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