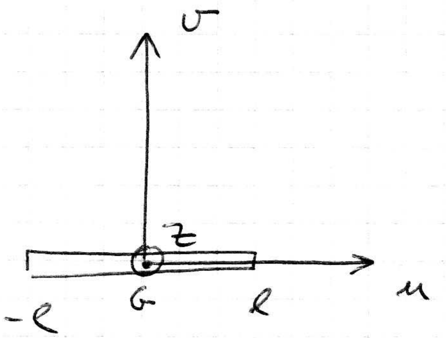
PENDULE ELLIPTIQUE

1 : Calcul des éléments d'inertie :



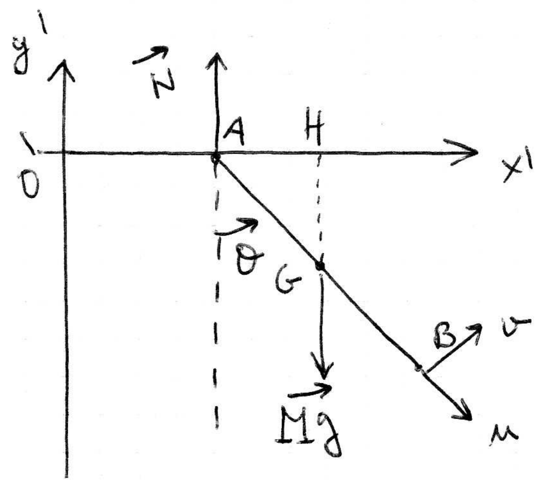
$$\lambda = \frac{M}{2l}$$

$$(I_G)_u = 0$$

$$(I_G)_v = (I_G)_z = \lambda \int_{-l}^{l} u^2 \, du = \lambda \frac{2l^3}{3} = \frac{Ml^3}{3}$$

$$\mathbb{I} = \frac{Ml^2}{3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2 : Degrés de libertés : 2 $\begin{cases} x_G \text{ ou } x_A \\ \theta \end{cases}$



$$\vec{\omega} = \dot{\theta} \, \hat{z}' = \dot{\theta} \, \hat{z} .$$

(Base référentiel mobile $\{u, v, z\}$.)

3) • $M\vec{a}_G = \vec{N} - M\vec{g}$. Suivant l'axe x : $\underline{m\ddot{x}_G = 0}$
(pas de forces en $\hat{x}$)

$\Rightarrow x_G(t) = C_1 t + C_2$. On choisit $\dot{x}_G(0) \Rightarrow x_G = C_2 = H$

Donc : $x_G(t) = H$; $y_G(t) = -l\cos\theta$.

Donc : $\vec{O'G} = H\hat{x} - l\cos\theta\,\hat{y}$; $\vec{v}_G = l\sin\theta\,\dot{\theta}\,\hat{y}$;

$\vec{a}_G = l\left(\cos\theta\,\dot{\theta}^2 + \sin\theta\,\ddot{\theta}\right)\hat{y}$.

• Relation Chasles : $\vec{O'B} = \vec{O'G} + \vec{GB}$. $\vec{GB} = -l\cos\theta\,\hat{y} + l\sin\theta\,\hat{x}$.

Donc : $\vec{O'B} = (H + l\sin\theta)\hat{x} - 2l\cos\theta\,\hat{y}$

Eq. ellipse : $x = x_0 + a\sin\theta$, $y = y_0 + b\cos\theta$ (centre (x_0, y_0), rayons a et b).

Donc : centre $(H, 0)$, rayons $(l, 2l)$.

• Eqs. mouvement. $E_c = \frac{1}{2}M|\vec{v}_G|^2 + \frac{1}{2}\vec{\omega}\cdot\underline{\underline{I}}\cdot\vec{\omega} =$

$= \frac{1}{2}Ml^2\sin^2\theta\,\dot{\theta}^2 + \frac{1}{6}Ml^2\dot{\theta}^2$

$U = -mgl\cos\theta + C$. $\|$ $E = E_c + U$.

$$\frac{dE}{dt} = 0 = \left[l^2\sin^2\theta + \frac{1}{3}l^2\right]\ddot{\theta} + l^2\sin\theta\cos\theta\,\dot{\theta}^2 + gl\sin\theta$$

Petites oscillations : $\theta \ll 1$ $\quad \dot{\theta}^2 \ll \ddot{\theta} \Rightarrow \ddot{\theta} + \underbrace{\frac{3g}{4l}}_{=\omega^2}\theta = 0$

• Eqs mouvement à partir dynamique.

$$\left.\begin{aligned} M\ddot{y}_G &= N - Mg = Ml\left(\cos\theta\,\dot{\theta}^2 + \sin\theta\,\ddot{\theta}\right) \\ (I_G)_z\ddot{\theta} &= \frac{Ml^2}{3}\ddot{\theta} = -l\sin\theta\,N \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \left[l^2\sin^2\theta + \frac{l^2}{3}\right]\ddot{\theta} + l^2\cos\theta\sin\theta\,\dot{\theta}^2 + l\sin\theta\,g = 0.$$