Implémentation des méthodes FETI-DP dans la résolution des problèmes direct et inverses en diffraction

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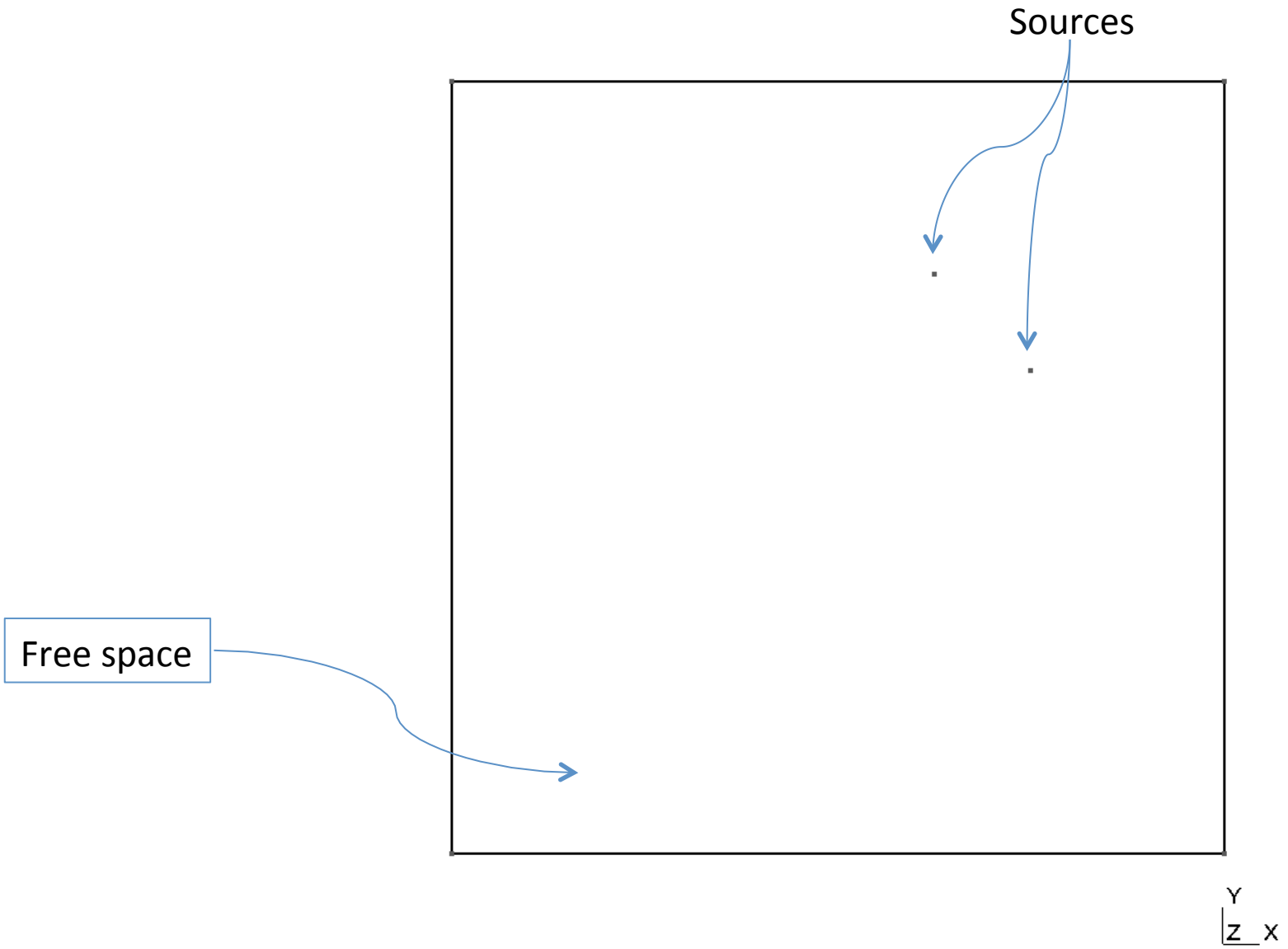
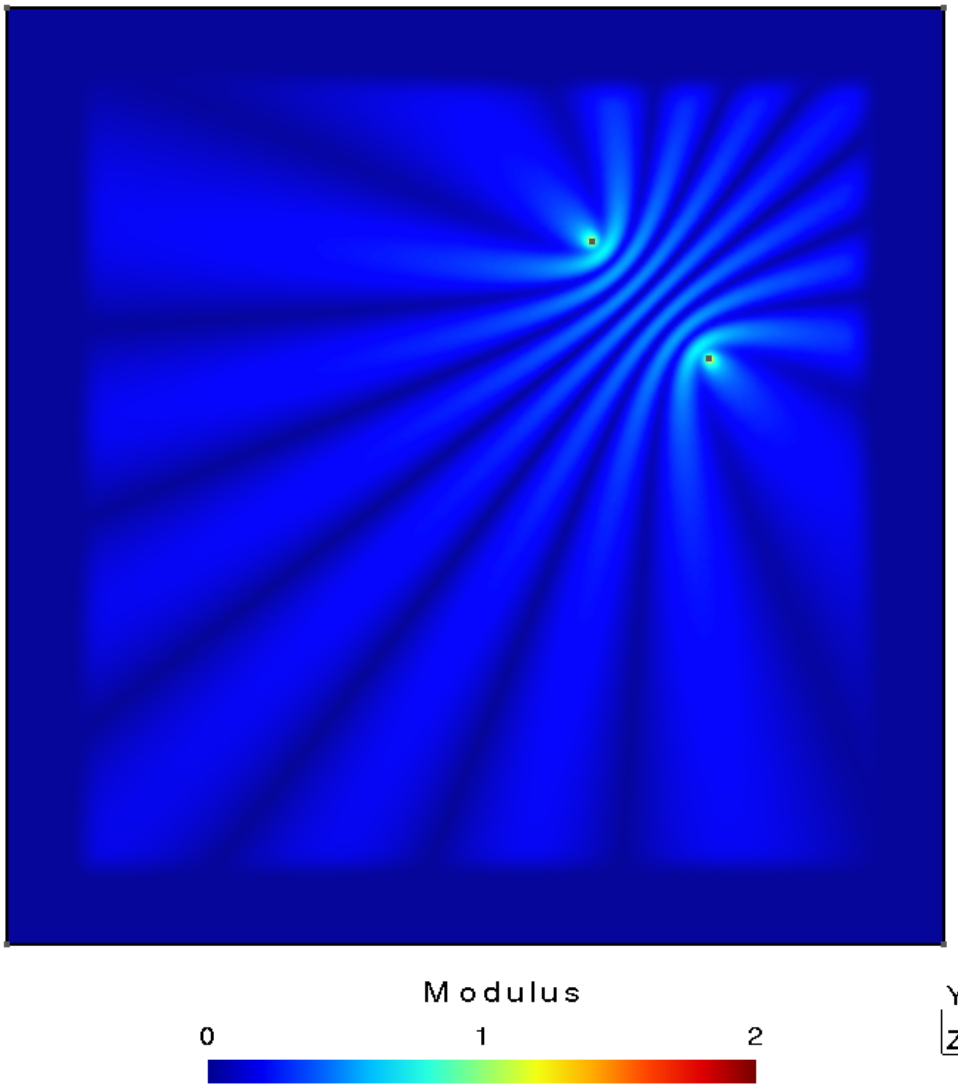
Vocabulary related to electromagnetic fields



Incident field $\mathcal{E}^{\text{inc}}$



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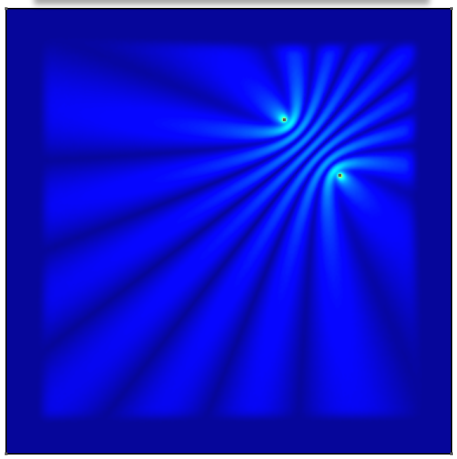
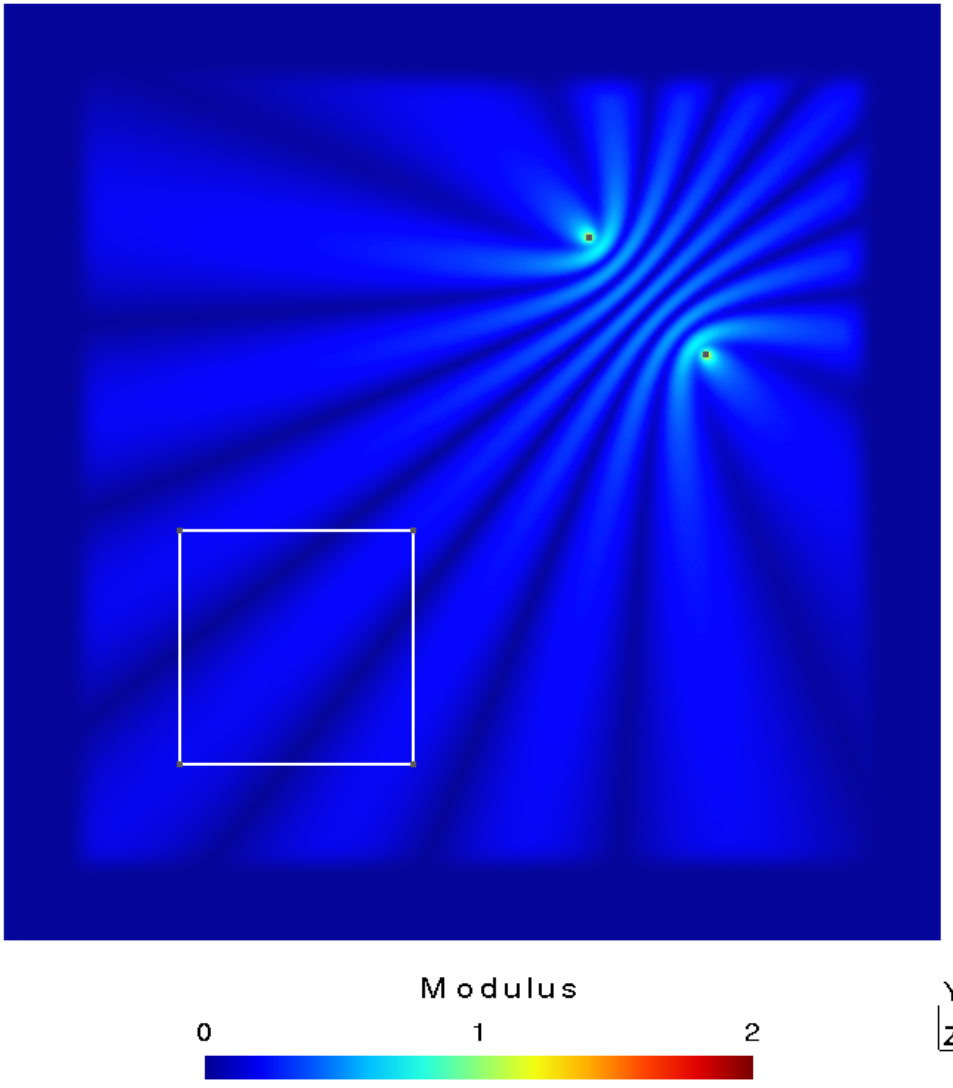
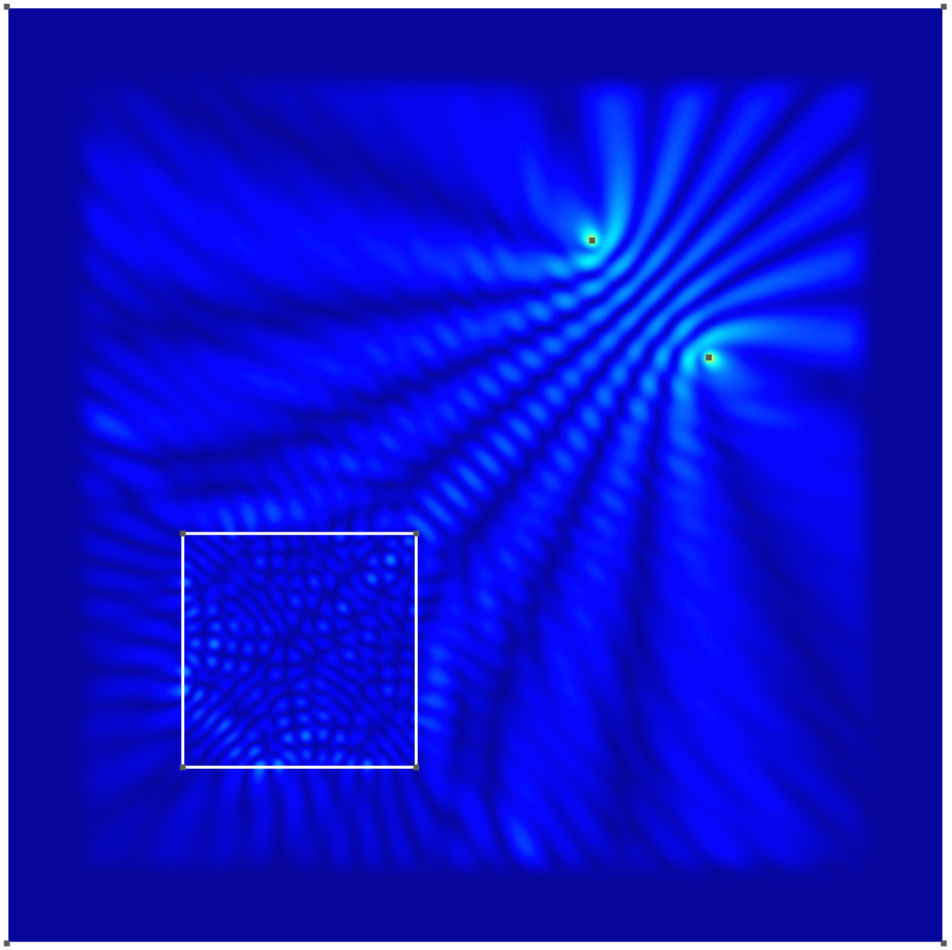




Vocabulary related to electromagnetic fields

Total field $\mathcal{E}^{\text{tot}}$



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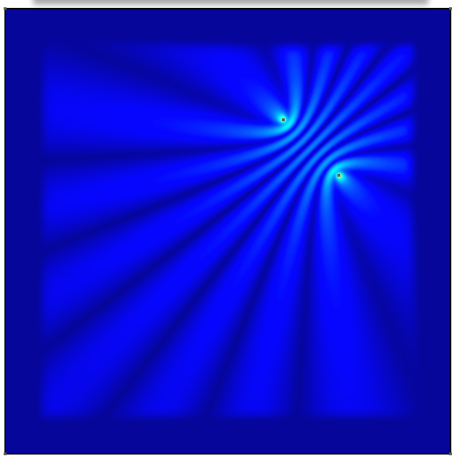

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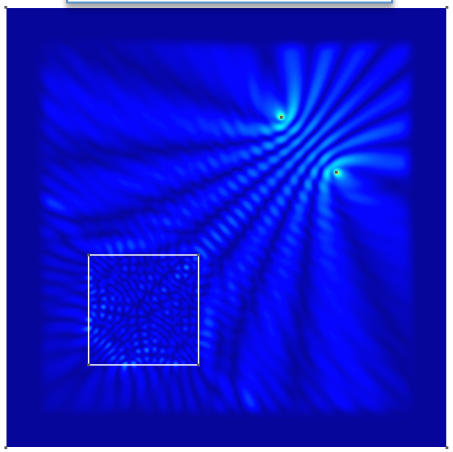

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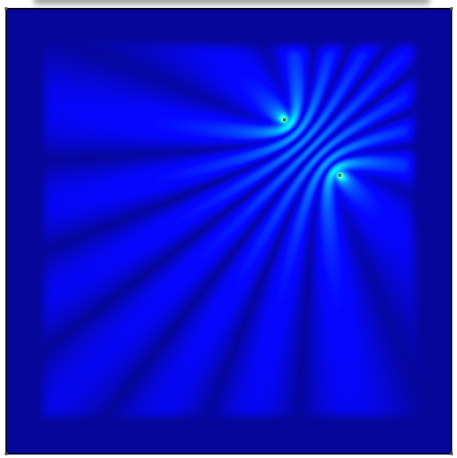

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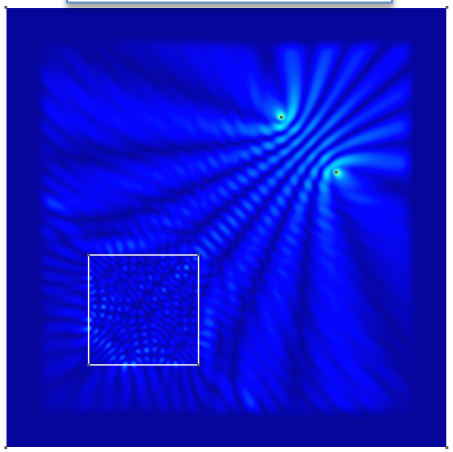

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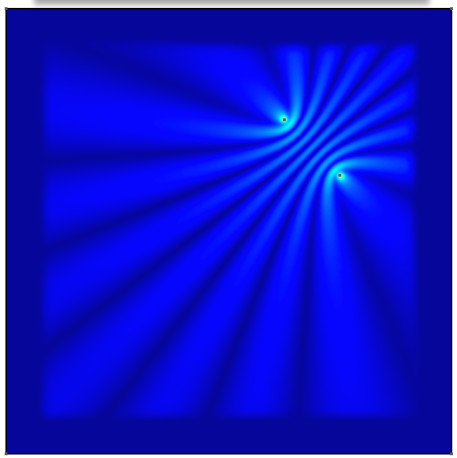
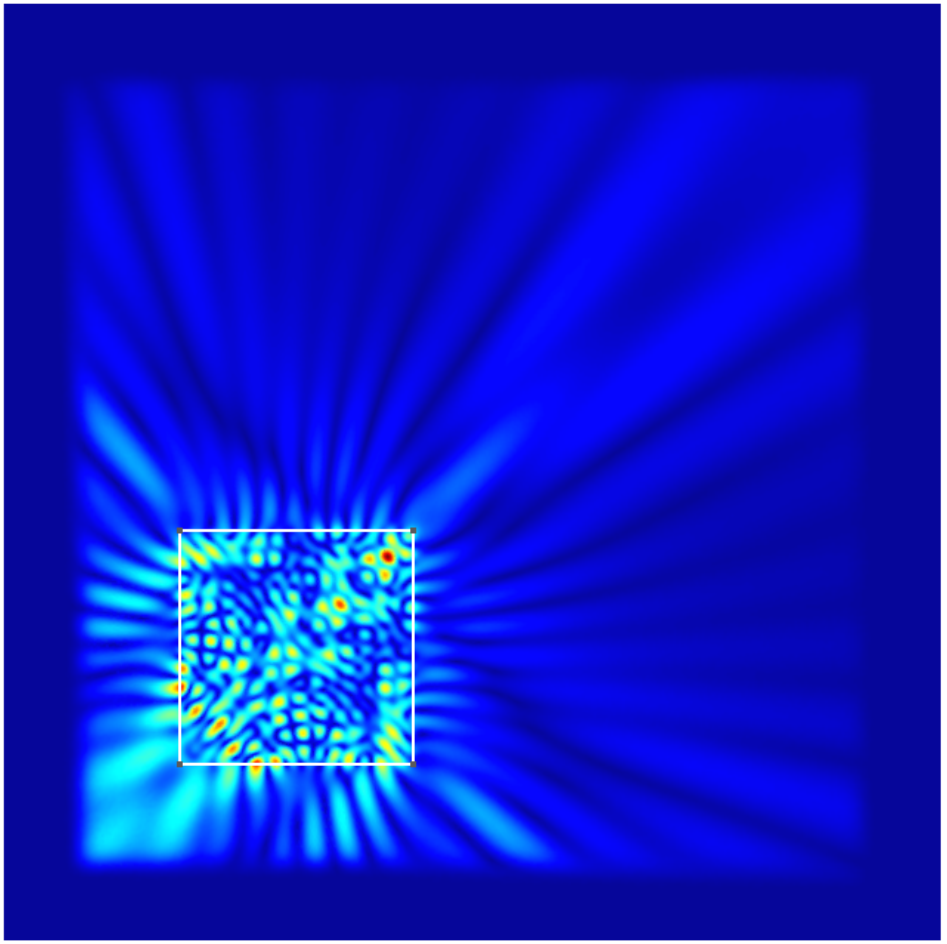
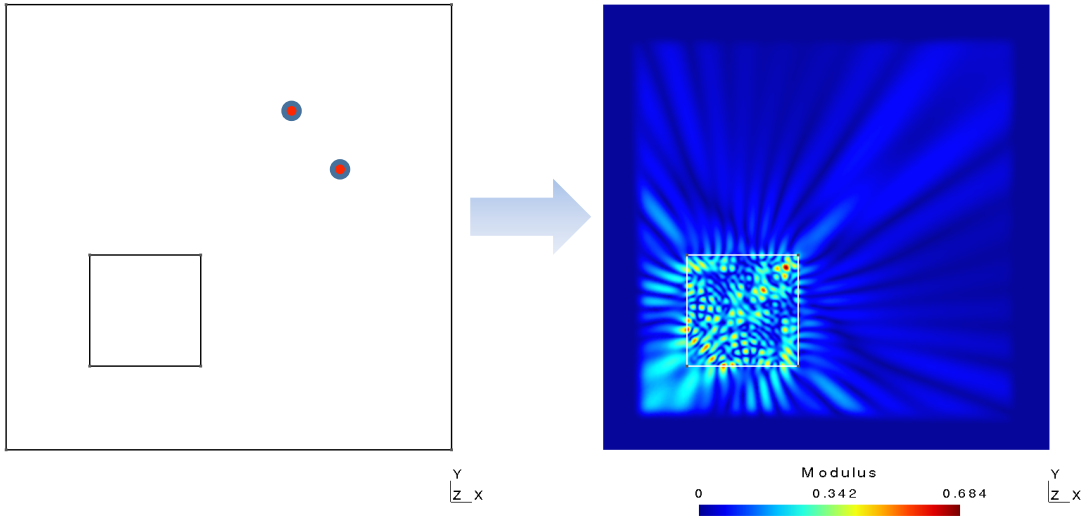


$=$

Scattered field $\mathcal{E}^{\text{sc}} = \mathcal{E}^{\text{tot}} - \mathcal{E}^{\text{inc}}$



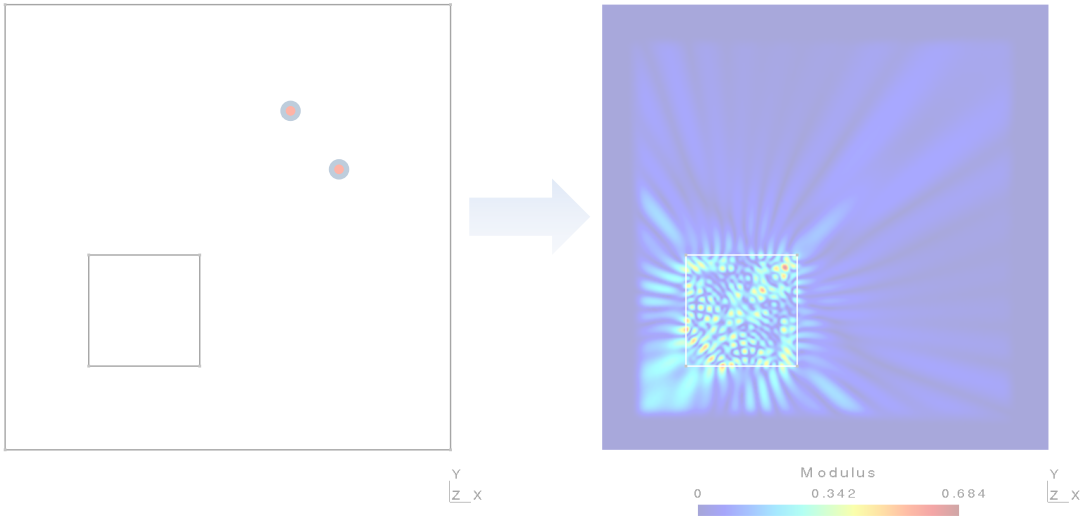
Direct	
Know	❑ Sources ❑ Objects
To find	❑ ***Scattered field***



Direct and Inverse problems. Definition

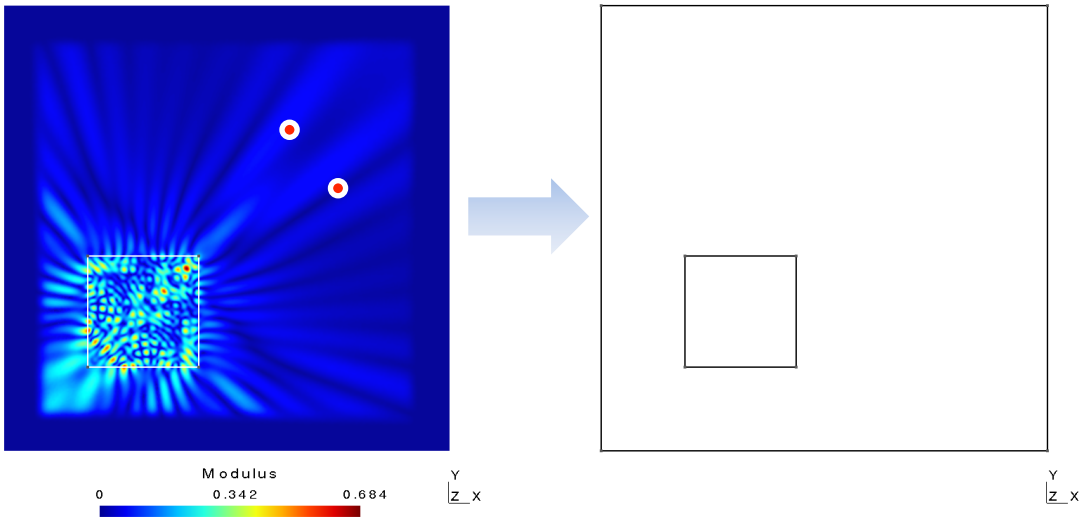
Direct

Know	❑ Sources ❑ Objects
To find	❑ ***Scattered field***



Inverse

Know	❑ Sources ❑ Scattered field
To find	❑ ***Objects***



2D Helmholtz equation

$$\operatorname{div}\left(\frac{1}{\mu_r^{\text{tot}}}\operatorname{grad}\mathcal{E}^{\text{sc}}\right)+k_0^2\varepsilon_r^{\text{tot}}\mathcal{E}^{\text{sc}}=\mathcal{J}^{\text{sc}}\text{ in }\Omega$$

where:

$$\mathcal{J}^{\text{sc}}=-\operatorname{div}\left(\left[\frac{1}{\mu_r^{\text{tot}}}-\frac{1}{\mu_r^{\text{inc}}}\right]\operatorname{grad}\mathcal{E}^{\text{inc}}\right)-k_0^2\left[\varepsilon_r^{\text{tot}}-\varepsilon_r^{\text{inc}}\right]\mathcal{E}^{\text{inc}}$$

+

Boundary conditions

3D Helmholtz equation

$$\nabla\times\left(\frac{1}{\mu_r^{\text{tot}}}\nabla\times\mathcal{E}^{\text{sc}}\right)-k_0^2\varepsilon_r^{\text{tot}}\mathcal{E}^{\text{sc}}=\mathcal{J}^{\text{sc}}\text{ in }\Omega$$

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+

Boundary conditions

Finite Element Method

Pros
✓ Well known
✓ Different media possible
Anisotropic
Inhomogeneous
✓ Arbitrary shaped objects

Finite Element Method

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Anisotropic
Inhomogeneous
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Cons
○ Time
○ Memory
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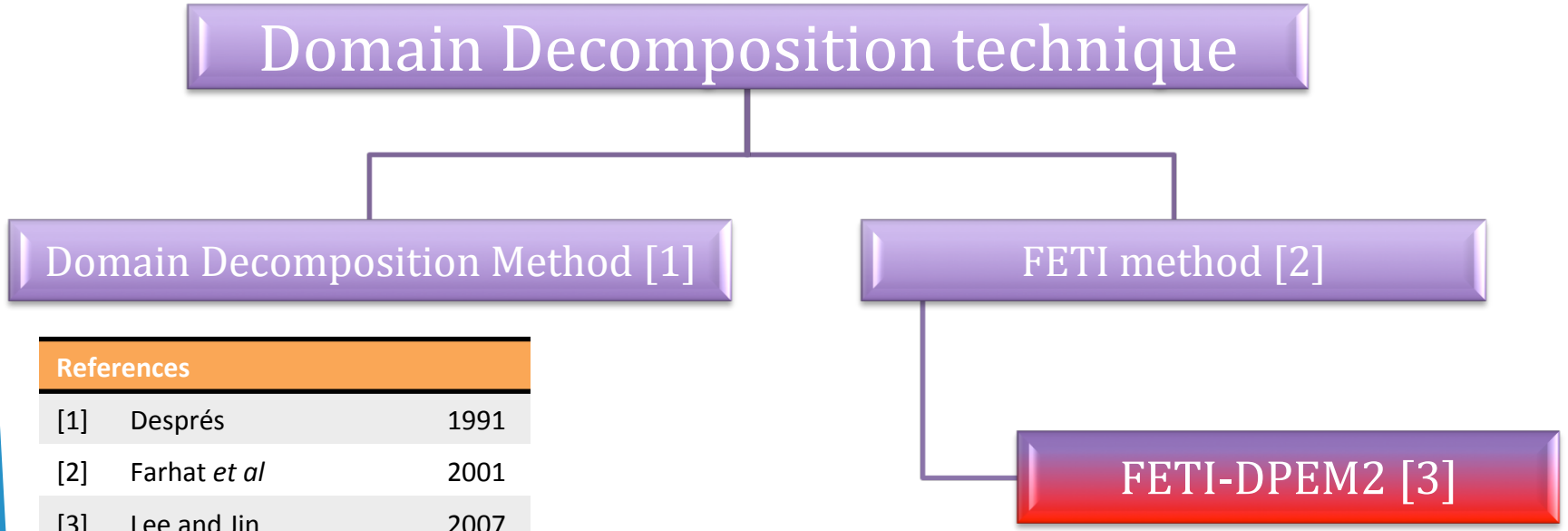


References		
[1]	Després	1991
[2]	Farhat *et al*	2001
[3]	Lee and Jin	2007

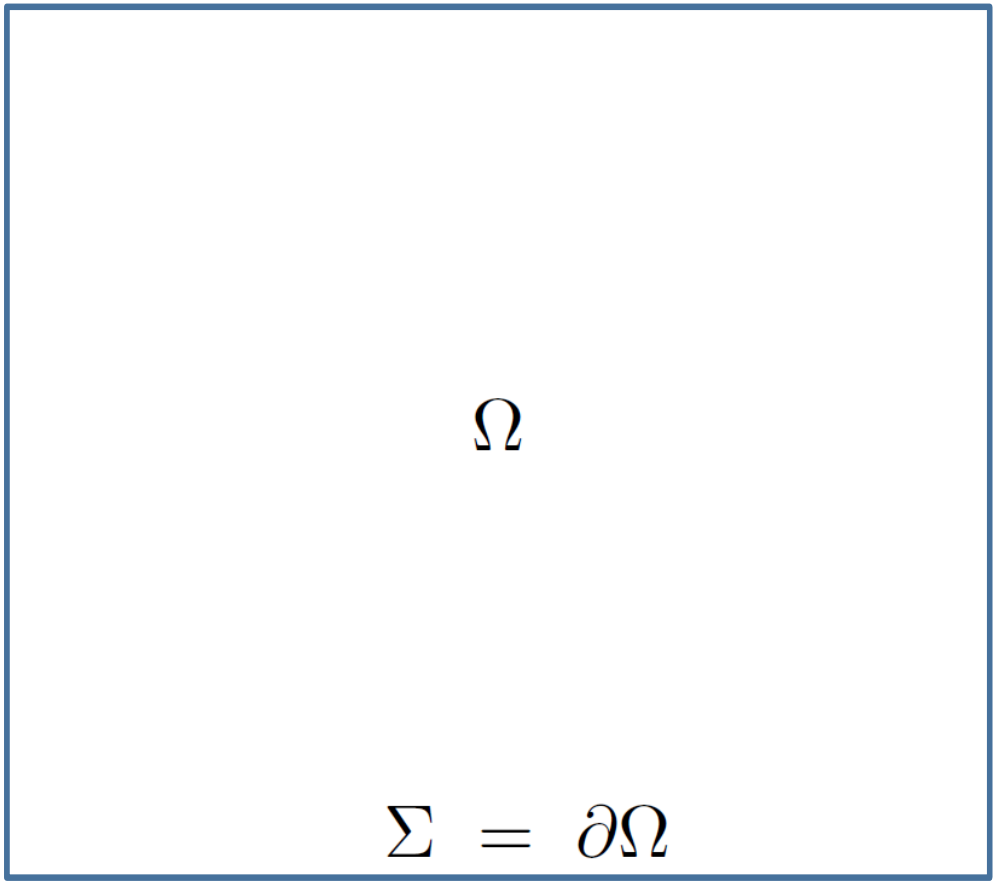
FEM statement in 2D case

Helmholtz equation in 2D case

$$-\mathrm{div}\left(\frac{1}{\mu_r}\mathrm{grad}\,\mathcal{E}\right) - k_0^2\varepsilon_r\mathcal{E} = jk_0Z_0\mathcal{J} \quad \text{in } \Omega$$

Radiation boundary condition

$$\frac{1}{\mu_r}\frac{\partial\mathcal{E}}{\partial n} - jk_0\mathcal{E} = 0 \quad \text{on } \Sigma$$



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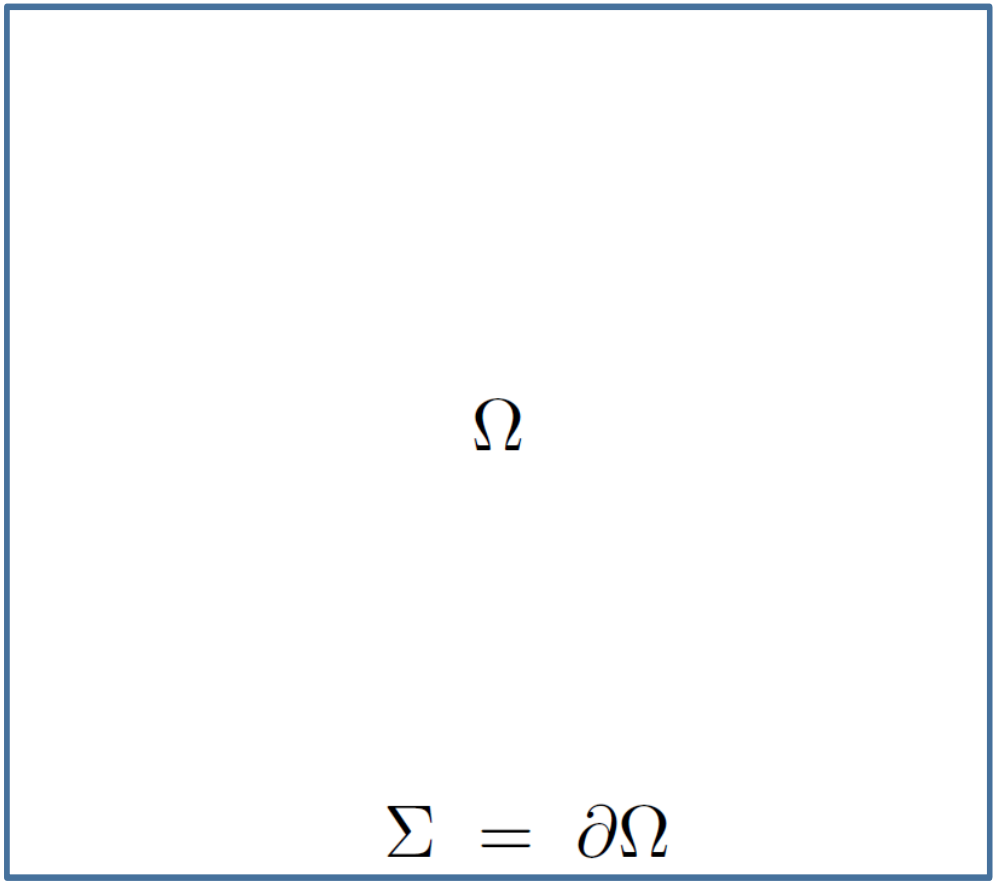
Finite-element discretization

$$\mathbf{KE} = \mathbf{f}$$



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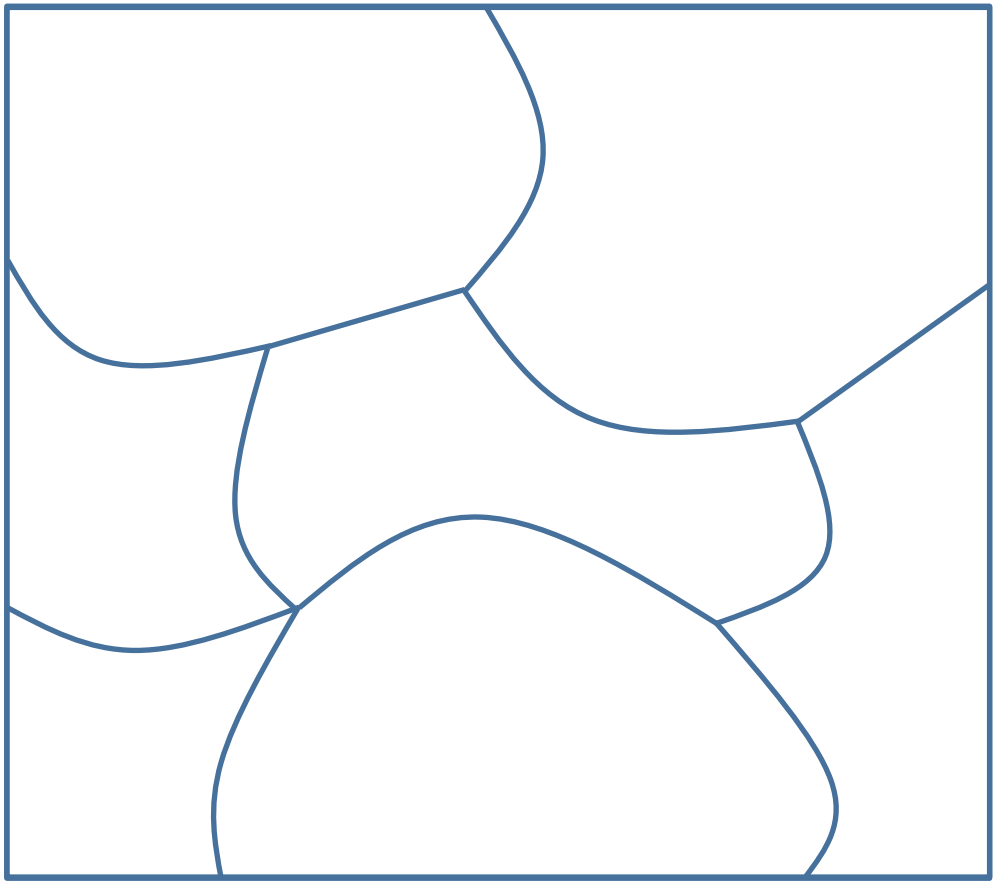

Helmholtz equation in 2D case

$$-\mathrm{div}\left(\frac{1}{\mu_r}\mathrm{grad}\,\mathcal{E}^i\right) - k_0^2\varepsilon_r\mathcal{E}^i = jk_0Z_0\mathcal{J}^i \text{ in } \Omega^i$$

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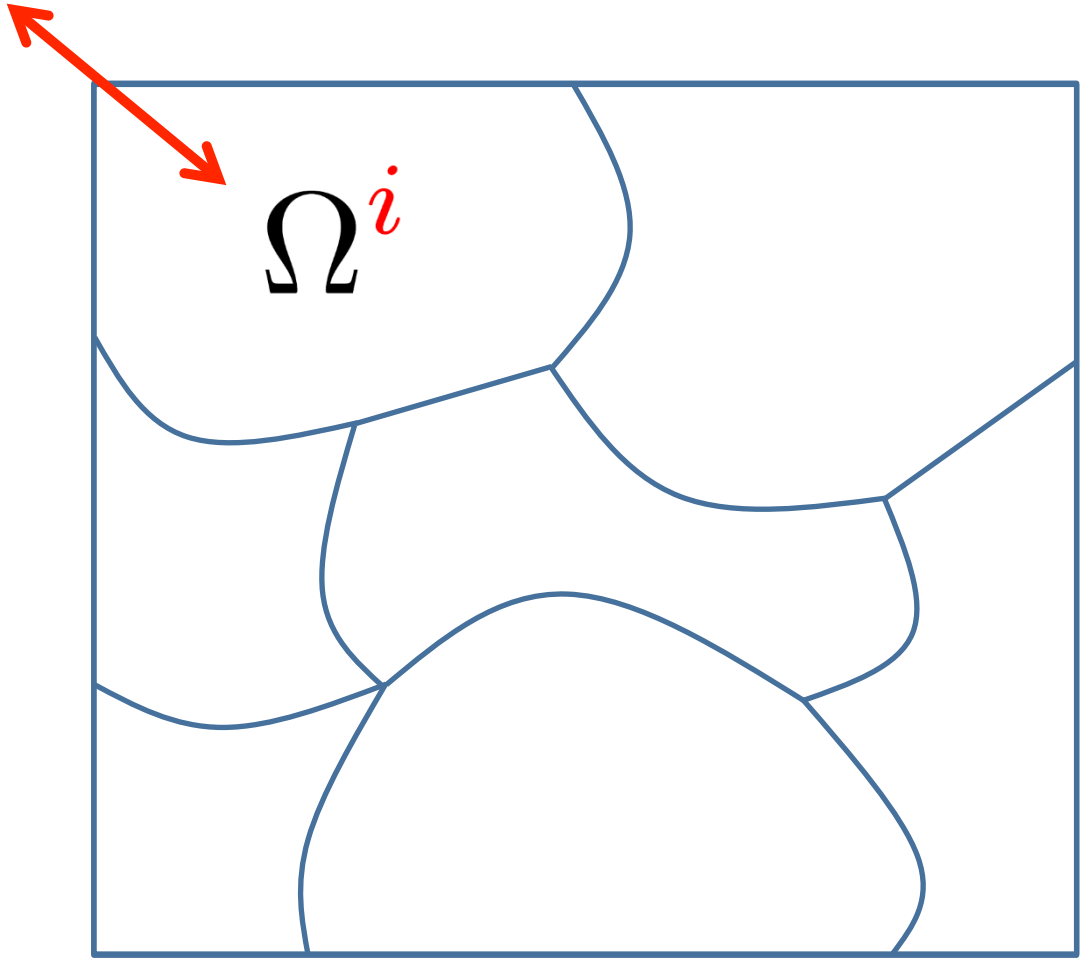

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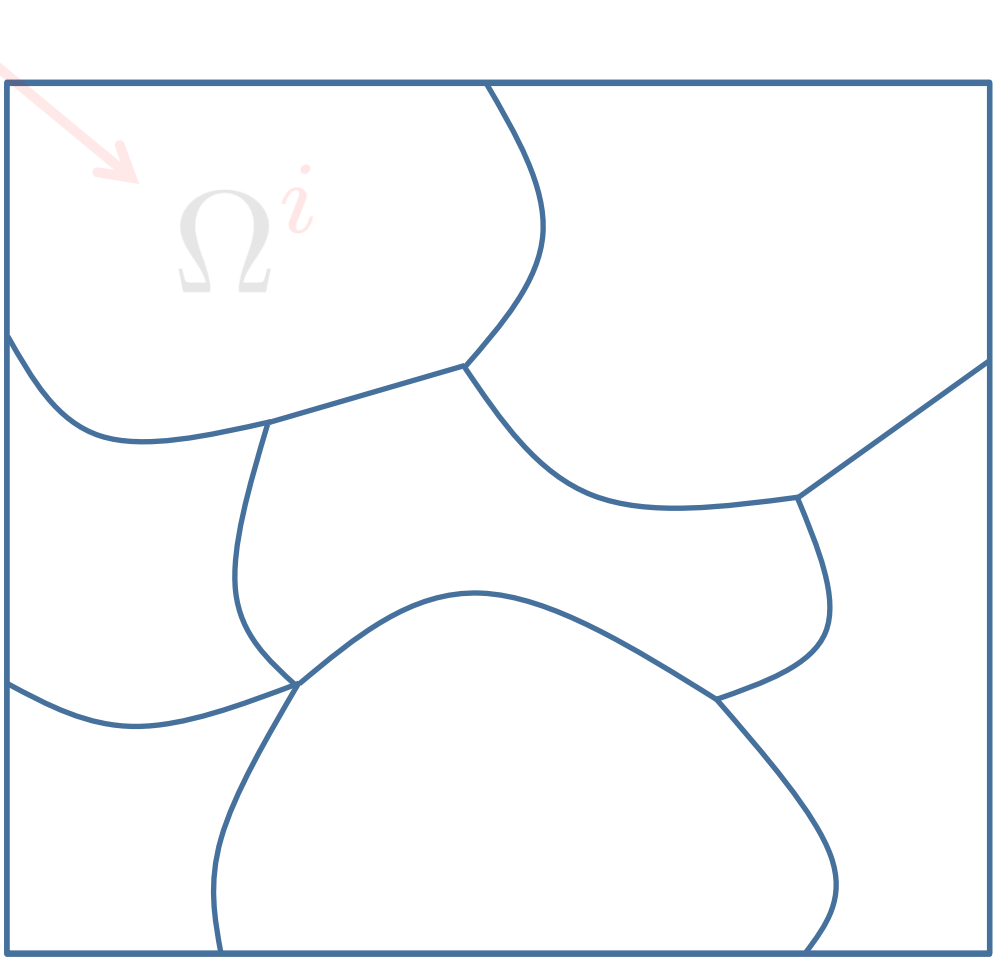

Finite-element discretization

$$\mathbf{KE} = \mathbf{f}$$

$$K^iE^i = f^i + \lambda^i$$

Notation of unknowns

Helmholtz equation in 2D case

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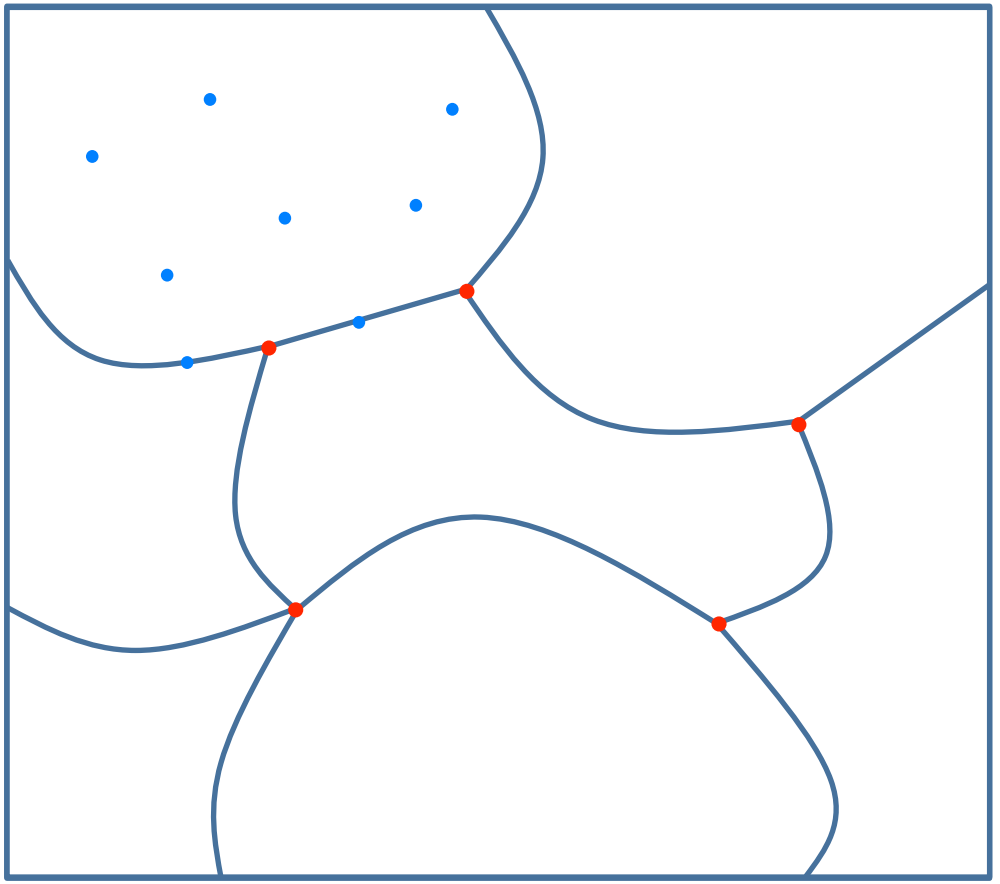
« c » - corner points in ***global*** numbering

« r » - interface points
- internal points



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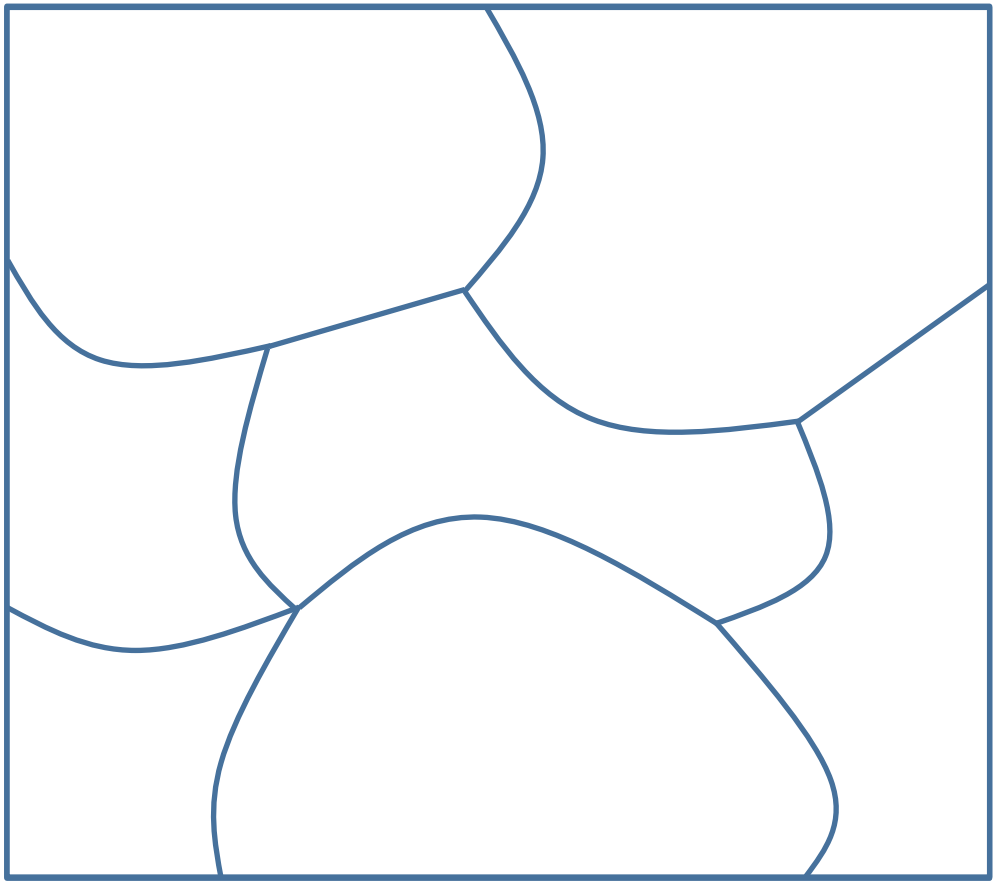
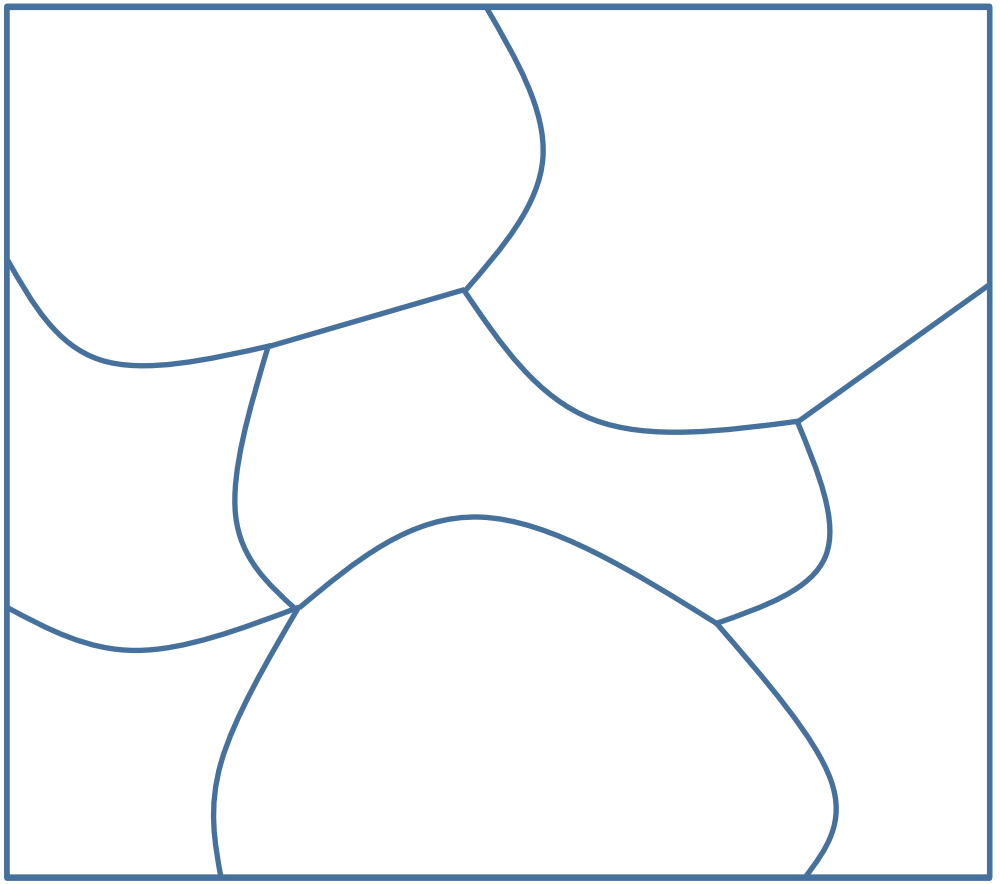
$$\begin{bmatrix} K^i_{rr} & K^i_{rc} \\ K^i_{cr} & K^i_{cc} \end{bmatrix} \begin{bmatrix} E^i_r \\ E^i_c \end{bmatrix} = \begin{bmatrix} f^i_r \\ f^i_c \end{bmatrix} + \begin{bmatrix} \lambda^i_r \\ \lambda^i_c \end{bmatrix}$$





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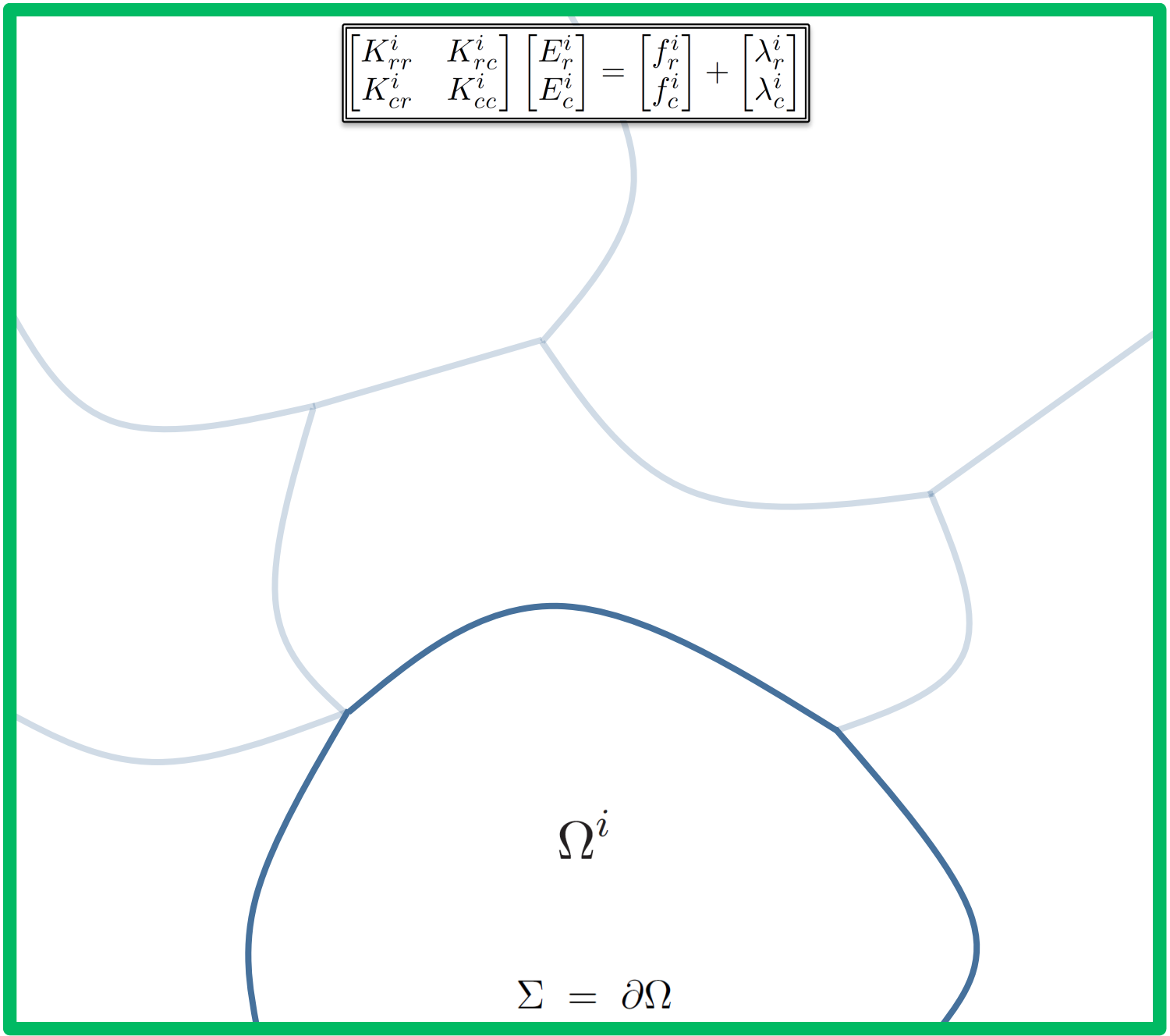
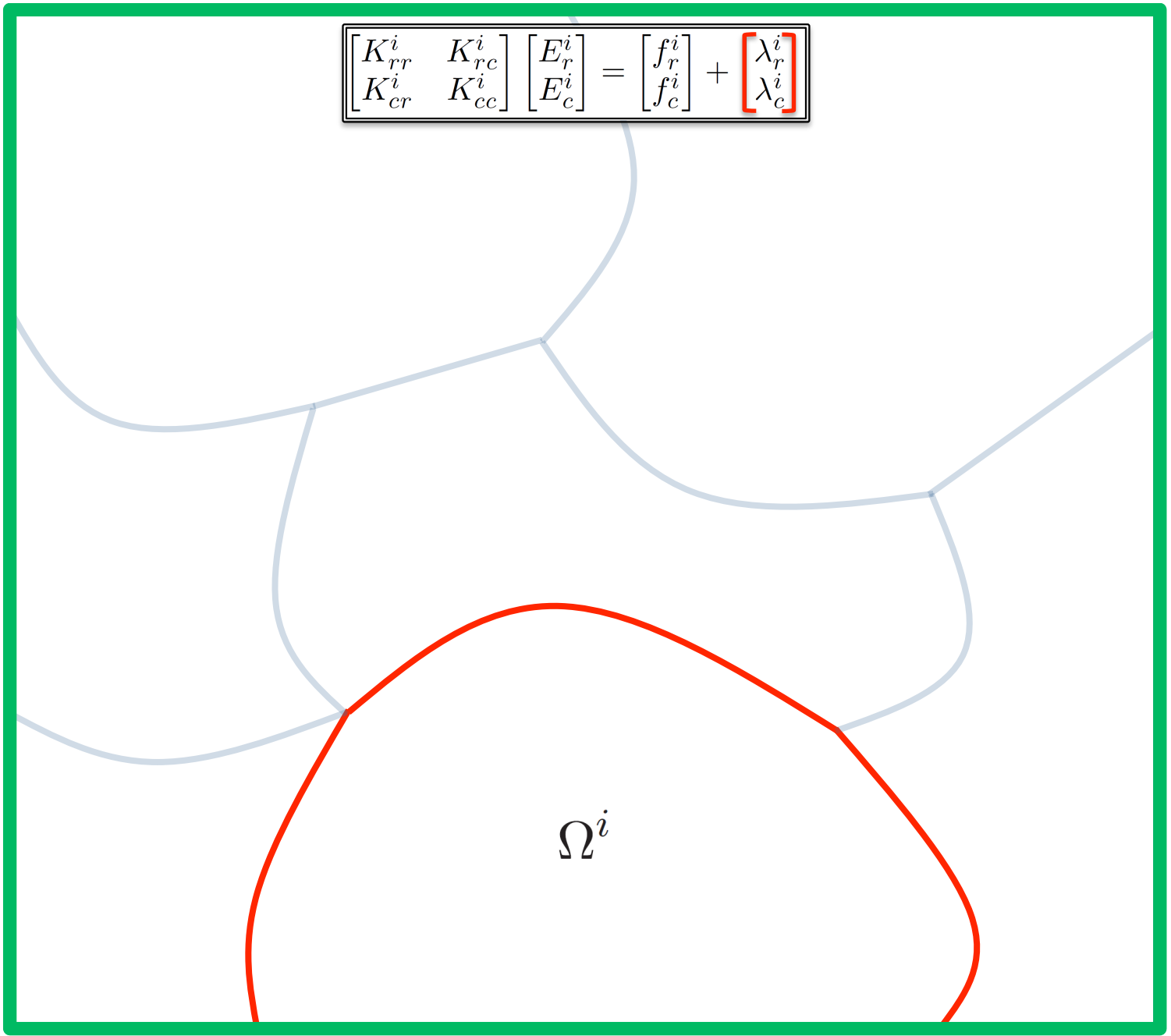


$$\begin{bmatrix} K_{rr}^i & K_{rc}^i \\ K_{cr}^i & K_{cc}^i \end{bmatrix} \begin{bmatrix} E_r^i \\ E_c^i \end{bmatrix} = \begin{bmatrix} f_r^i \\ f_c^i \end{bmatrix} + \begin{bmatrix} \lambda_r^i \\ \lambda_c^i \end{bmatrix}$$

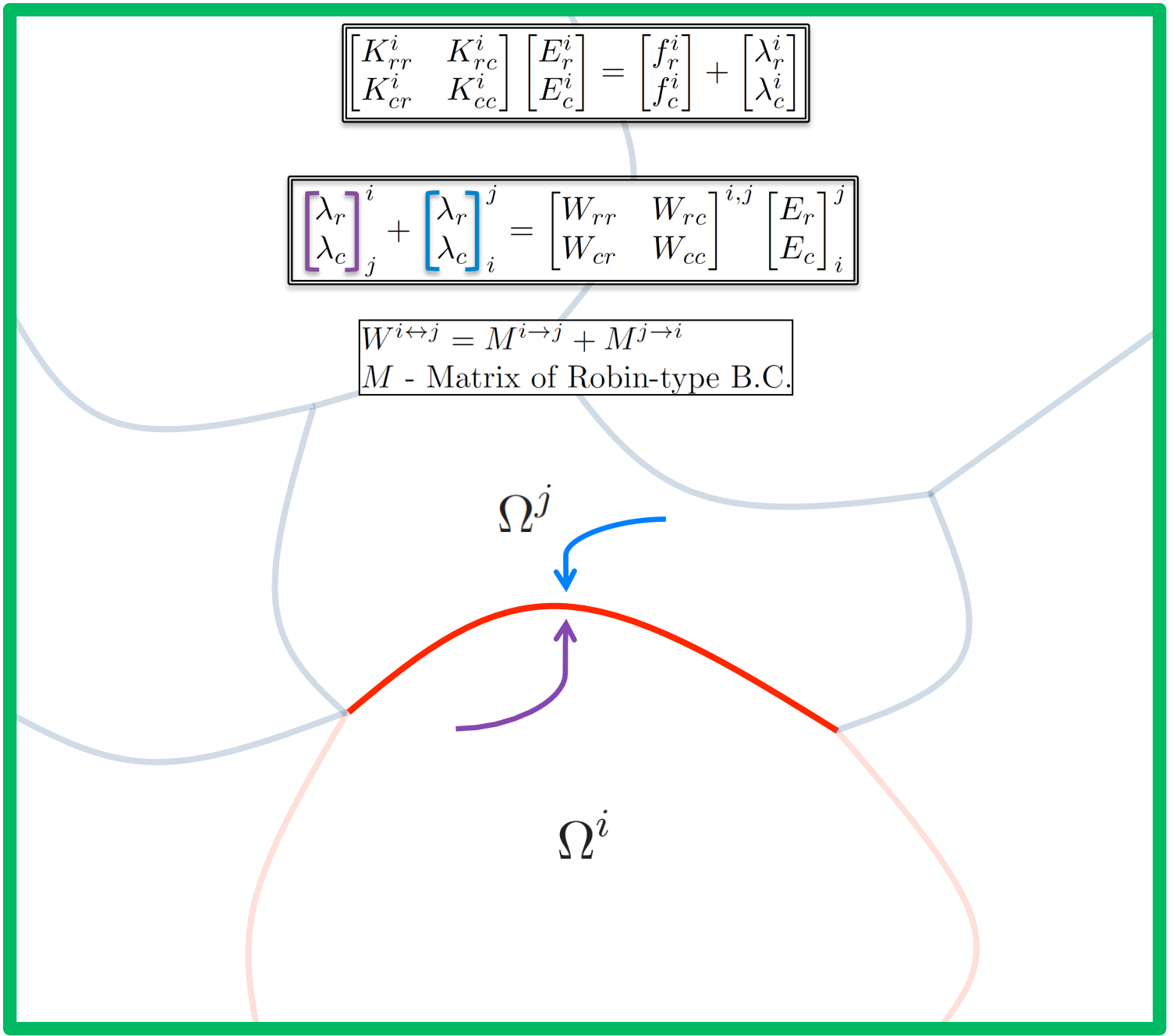


Connection between subdomains

$$\begin{bmatrix} K^i_{rr} & K^i_{rc} \\ K^i_{cr} & K^i_{cc} \end{bmatrix} \begin{bmatrix} E^i_r \\ E^i_c \end{bmatrix} = \begin{bmatrix} f^i_r \\ f^i_c \end{bmatrix} + \begin{bmatrix} \lambda^i_r \\ \lambda^i_c \end{bmatrix}$$

$$\begin{bmatrix} \lambda_r \\ \lambda_c \end{bmatrix}^i_j + \begin{bmatrix} \lambda_r \\ \lambda_c \end{bmatrix}^j_i = \begin{bmatrix} W_{rr} & W_{rc} \\ W_{cr} & W_{cc} \end{bmatrix}^{i,j} \begin{bmatrix} E_r \\ E_c \end{bmatrix}^j_i$$

$W^{i \leftrightarrow j} = M^{i \to j} + M^{j \to i}$
M - Matrix of Robin-type B.C.



Connection between subdomains

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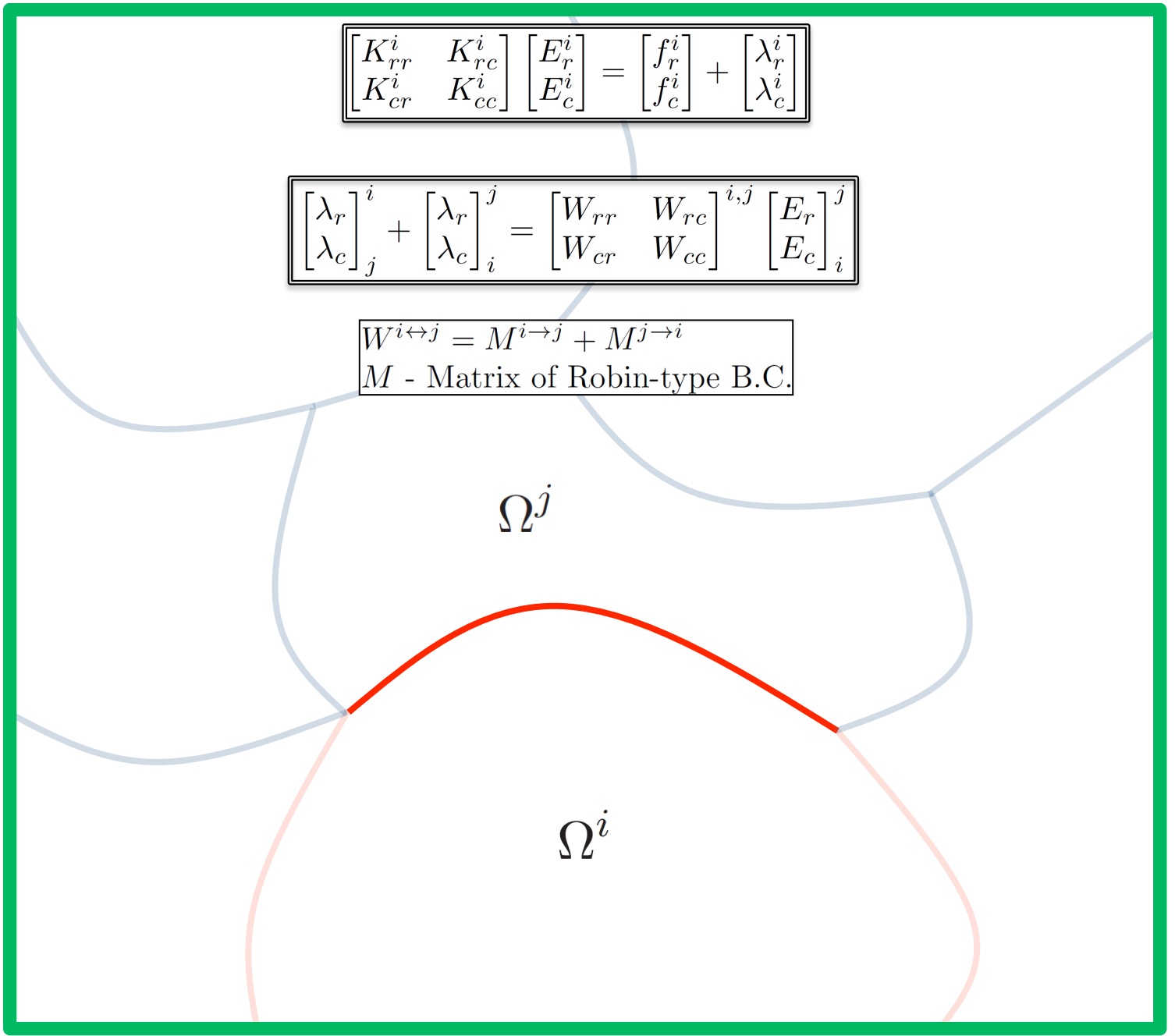


Interface problem of the proposed approach

$$\begin{bmatrix} K^i_{rr} & K^i_{rc} \\ K^i_{cr} & K^i_{cc} \end{bmatrix} \begin{bmatrix} E^i_r \\ E^i_c \end{bmatrix} = \begin{bmatrix} f^i_r \\ f^i_c \end{bmatrix} + \begin{bmatrix} \lambda^i_r \\ \lambda^i_c \end{bmatrix}$$

$$\begin{bmatrix} \lambda_r \\ \lambda_c \end{bmatrix}^i_j + \begin{bmatrix} \lambda_r \\ \lambda_c \end{bmatrix}^j_i = \begin{bmatrix} W_{rr} & W_{rc} \\ W_{cr} & W_{cc} \end{bmatrix}^{i,j} \begin{bmatrix} E_r \\ E_c \end{bmatrix}^j_i$$

(a) [2]

"r" – Robin type
"c" – Neumann type

$$\begin{bmatrix} W_{rr} & 0 \\ 0 & 0 \end{bmatrix}^{i,j}$$

(b) [5]

"r" – Robin type
"c" – Neumann type

$$\begin{bmatrix} W_{rr} & W_{rc} \\ 0 & 0 \end{bmatrix}^{i,j}$$

[3,4]

"r" – Robin type
"c" – Robin type

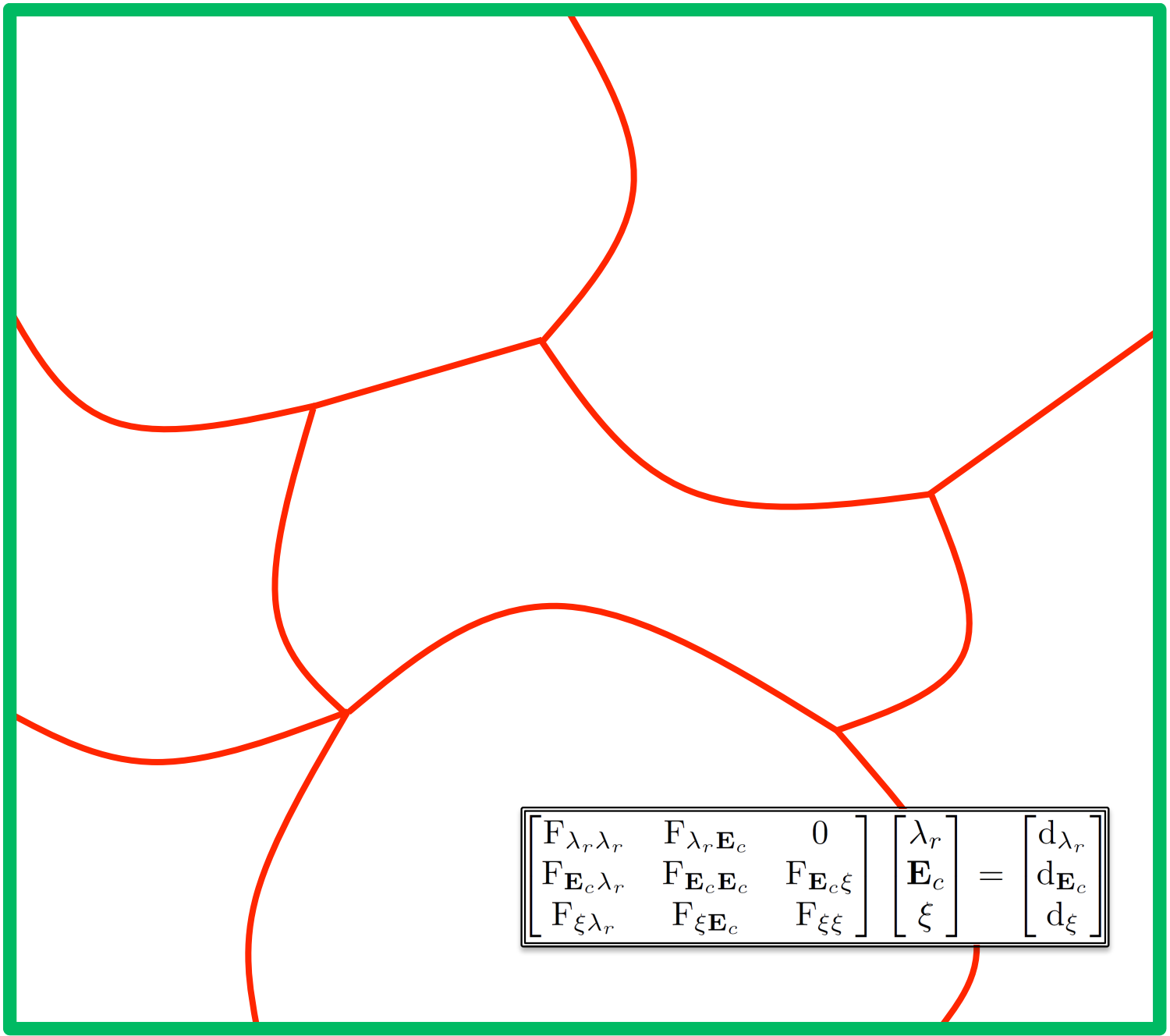
$$\begin{bmatrix} W_{rr} & W_{rc} \\ W_{cr} & W_{cc} \end{bmatrix}^{i,j}$$

$$\begin{bmatrix} \mathrm{F}_{\lambda_r\lambda_r} & \mathrm{F}_{\lambda_r\mathbf{E}_c} \\ \mathrm{F}_{\mathbf{E}_c\lambda_r} & \mathrm{F}_{\mathbf{E}_c\mathbf{E}_c} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \end{bmatrix} = \begin{bmatrix} \mathrm{d}_{\lambda_r} \\ \mathrm{d}_{\mathbf{E}_c} \end{bmatrix}$$

$$\begin{bmatrix} \mathrm{F}_{\lambda_r\lambda_r} & \mathrm{F}_{\lambda_r\mathbf{E}_c} & 0 \\ \mathrm{F}_{\mathbf{E}_c\lambda_r} & \mathrm{F}_{\mathbf{E}_c\mathbf{E}_c} & \mathrm{F}_{\mathbf{E}_c\xi} \\ \mathrm{F}_{\xi\lambda_r} & \mathrm{F}_{\xi\mathbf{E}_c} & \mathrm{F}_{\xi\xi} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \\ \xi \end{bmatrix} = \begin{bmatrix} \mathrm{d}_{\lambda_r} \\ \mathrm{d}_{\mathbf{E}_c} \\ \mathrm{d}_{\xi} \end{bmatrix}$$

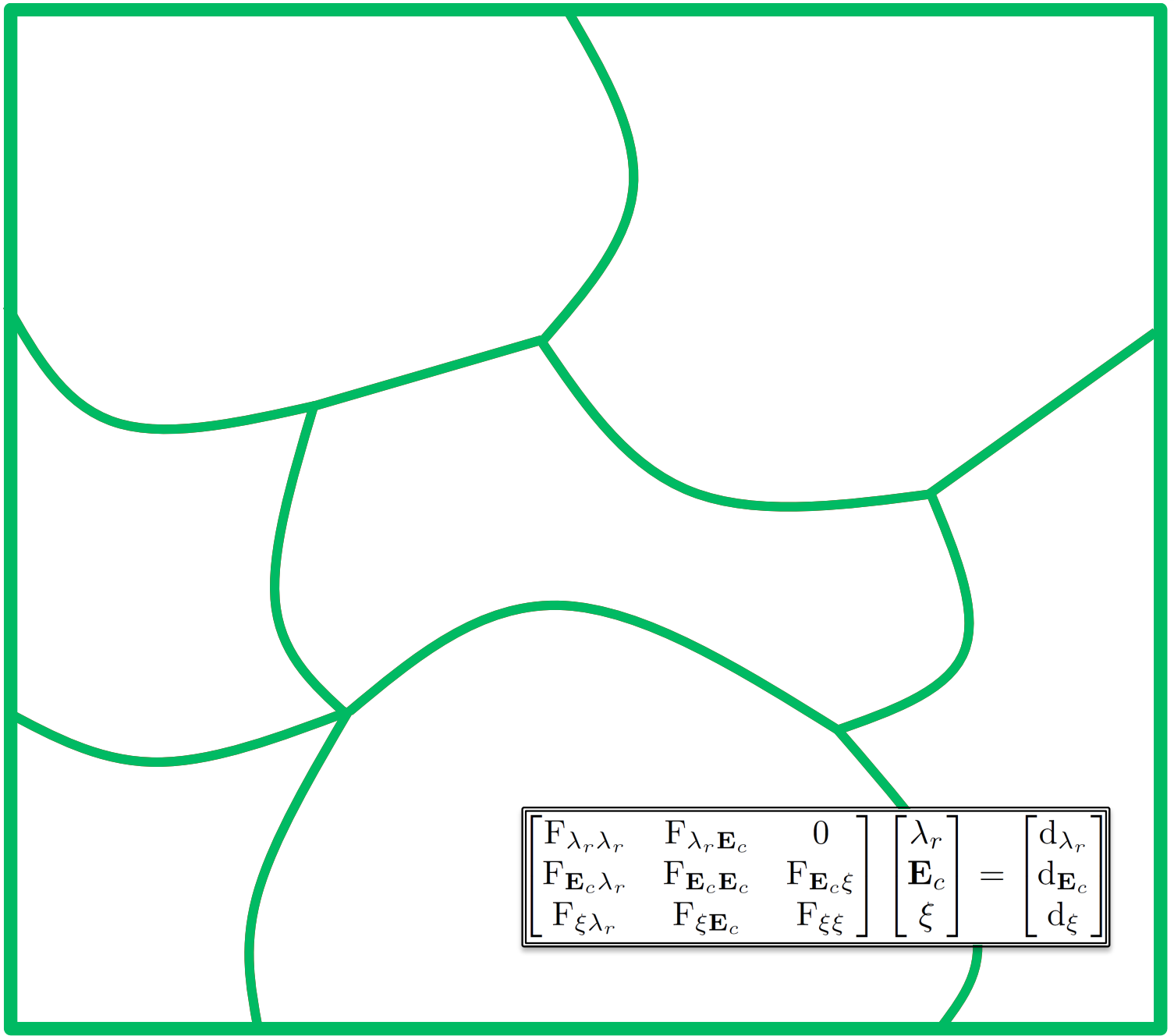
References		
[2]	Lee and Jin	2007
[3]	Voznyuk *et al*	2013
[4]	Voznyuk *et al*	2014
[5]	Xue and Jin	2014

Resolution of the Interface problem



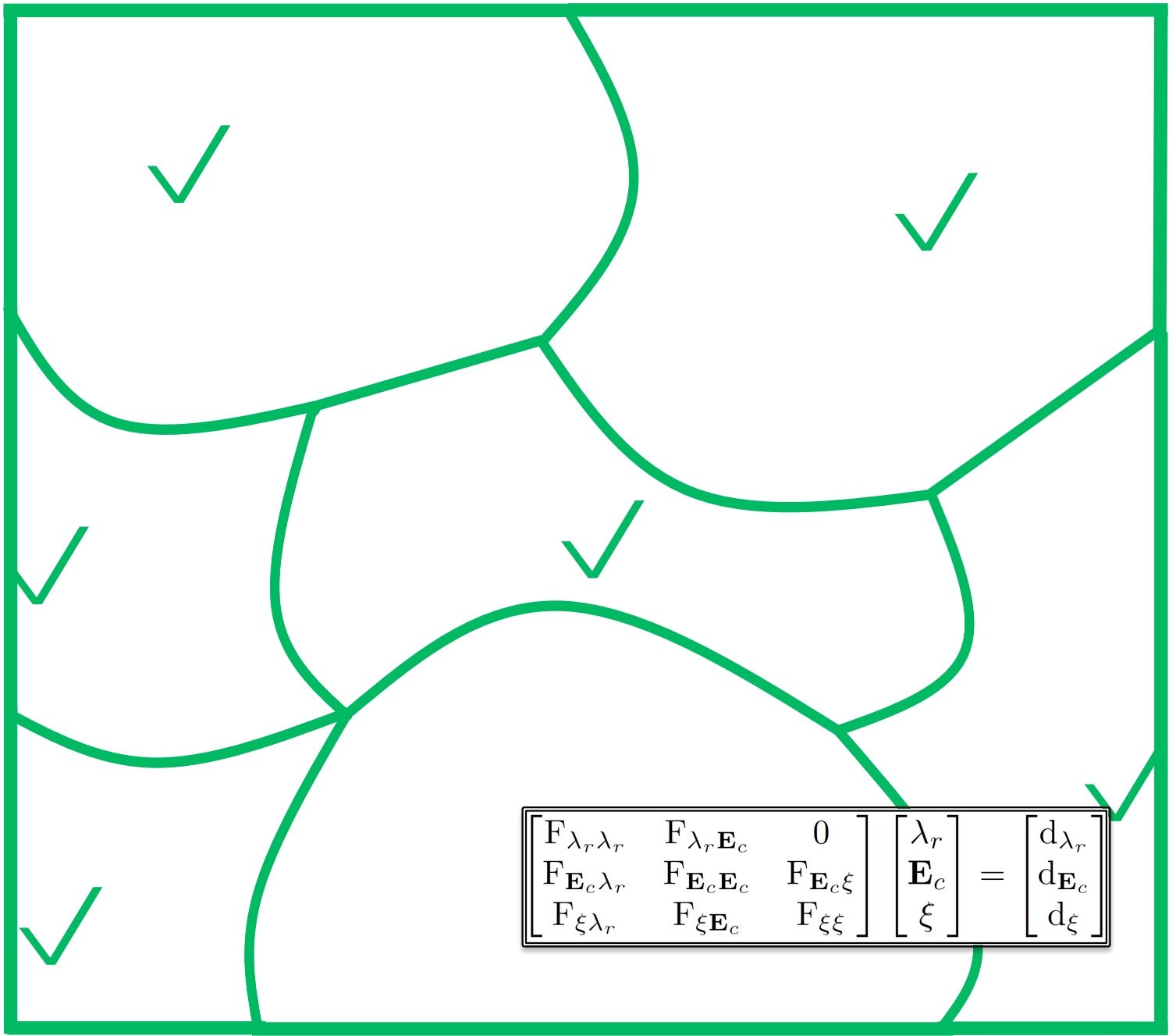
$$\begin{bmatrix} \mathrm{F}_{\lambda_r\lambda_r} & \mathrm{F}_{\lambda_r\mathbf{E}_c} & 0 \\ \mathrm{F}_{\mathbf{E}_c\lambda_r} & \mathrm{F}_{\mathbf{E}_c\mathbf{E}_c} & \mathrm{F}_{\mathbf{E}_c\xi} \\ \mathrm{F}_{\xi\lambda_r} & \mathrm{F}_{\xi\mathbf{E}_c} & \mathrm{F}_{\xi\xi} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \\ \xi \end{bmatrix} = \begin{bmatrix} \mathrm{d}_{\lambda_r} \\ \mathrm{d}_{\mathbf{E}_c} \\ \mathrm{d}_{\xi} \end{bmatrix}$$

Resolution of the Interface problem



$$\begin{bmatrix} \mathrm{F}_{\lambda_r\lambda_r} & \mathrm{F}_{\lambda_r\mathbf{E}_c} & 0 \\ \mathrm{F}_{\mathbf{E}_c\lambda_r} & \mathrm{F}_{\mathbf{E}_c\mathbf{E}_c} & \mathrm{F}_{\mathbf{E}_c\xi} \\ \mathrm{F}_{\xi\lambda_r} & \mathrm{F}_{\xi\mathbf{E}_c} & \mathrm{F}_{\xi\xi} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \\ \xi \end{bmatrix} = \begin{bmatrix} \mathrm{d}_{\lambda_r} \\ \mathrm{d}_{\mathbf{E}_c} \\ \mathrm{d}_{\xi} \end{bmatrix}$$

Last step: solution of independent problems



$$\begin{bmatrix} \mathrm{F}_{\lambda_r\lambda_r} & \mathrm{F}_{\lambda_r\mathbf{E}_c} & 0 \\ \mathrm{F}_{\mathbf{E}_c\lambda_r} & \mathrm{F}_{\mathbf{E}_c\mathbf{E}_c} & \mathrm{F}_{\mathbf{E}_c\xi} \\ \mathrm{F}_{\xi\lambda_r} & \mathrm{F}_{\xi\mathbf{E}_c} & \mathrm{F}_{\xi\xi} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \\ \xi \end{bmatrix} = \begin{bmatrix} \mathrm{d}_{\lambda_r} \\ \mathrm{d}_{\mathbf{E}_c} \\ \mathrm{d}_{\xi} \end{bmatrix}$$

How to test the FETI-methods

How to test the FETI-methods

- Analytical functions

$$\operatorname{div}(\lambda \operatorname{grad} u) + \gamma\, u = -f, \text{ in } \Omega$$

$$\lambda \frac{\partial u}{\partial n} + \alpha\, u = \Lambda, \text{ on } \Sigma$$

How to test the FETI-methods

- Analytical functions
- Previously computed numerical solutions

How to test the FETI-methods

- Analytical functions
- Previously computed numerical solutions
- Comparison with FEM method

$$L^2\text{-error} = \frac{||\mathbb{E}_1 - \mathbb{E}_2||^2}{||\mathbb{E}_1||^2}$$

where $\mathbb{E}_1$ is a solution of FEM

where $\mathbb{E}_2$ is a solution of FETI

How to test the FETI-methods

- Analytical functions
- Previously computed numerical solutions
- Comparison with FEM method
- Measurements

How to test the FETI-methods

- ❑ Analytical functions
- ❑ Previously computed numerical solutions
- ❑ Comparison with FEM method
- ❑ Measurements

Useful packages

GMSH	Mesh construction, fields and geometry representation
METIS	Domain Decomposition
MUMPS MKL	Factorization, Resolution of SLE

Inside air

$f = 4$ GHz

Domain of $\approx 4.2\lambda \times 4.6\lambda \times 3.0\lambda$

The wavelength $\lambda \approx 0.075$ m

Excitation

Source moving $\theta_s = 0° \div 180°$

Receivers are $\theta_r = 0° \div 360°$

Polarization E_ϕ

Antenna step $2°$

Scatterers

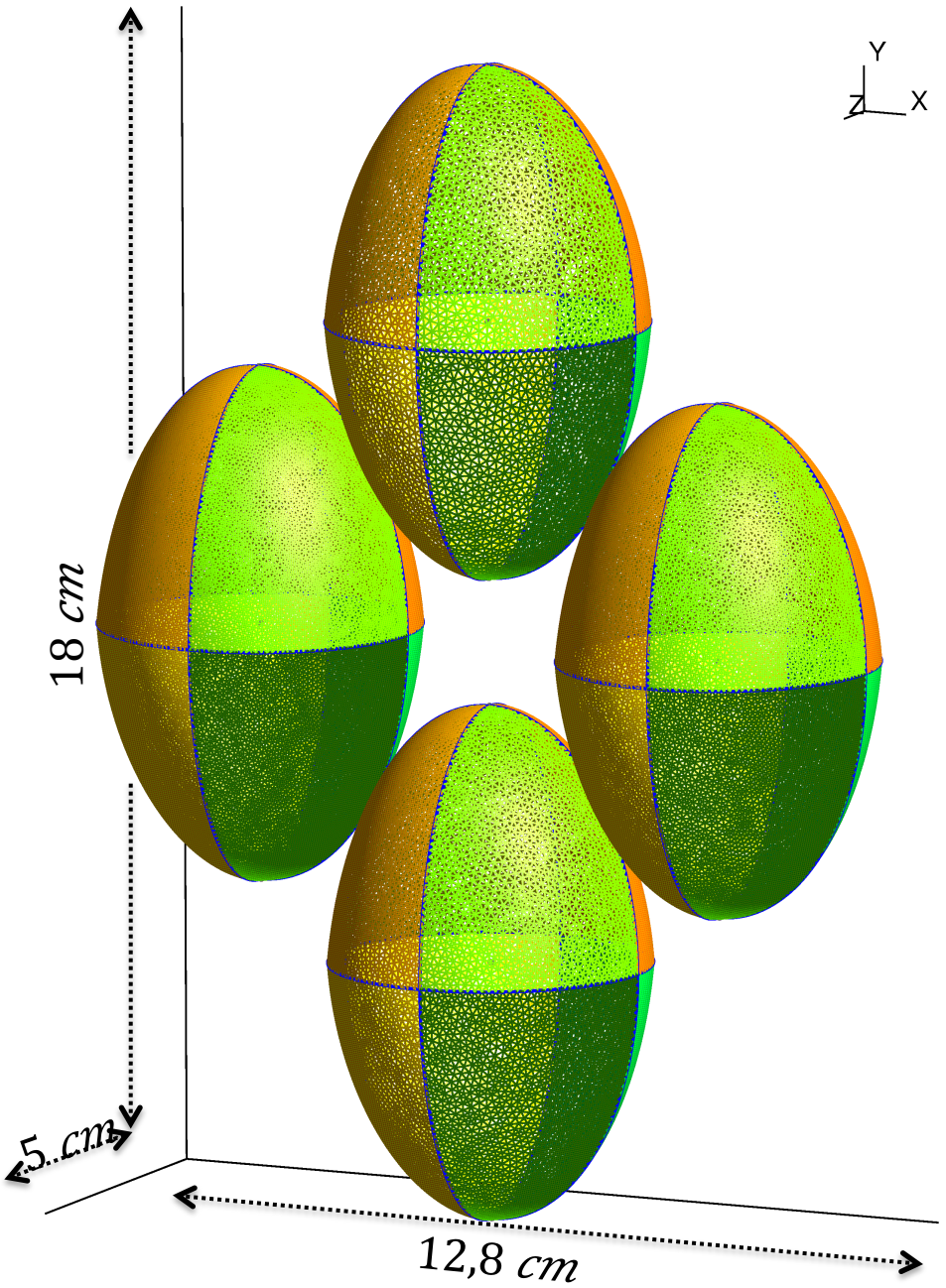
4 ellipsoids

$\varepsilon_r^1 = 2.5$

$\varepsilon_r^2 = 3.0$

$\varepsilon_r^3 = 3.5$

$\varepsilon_r^4 = 4.0$



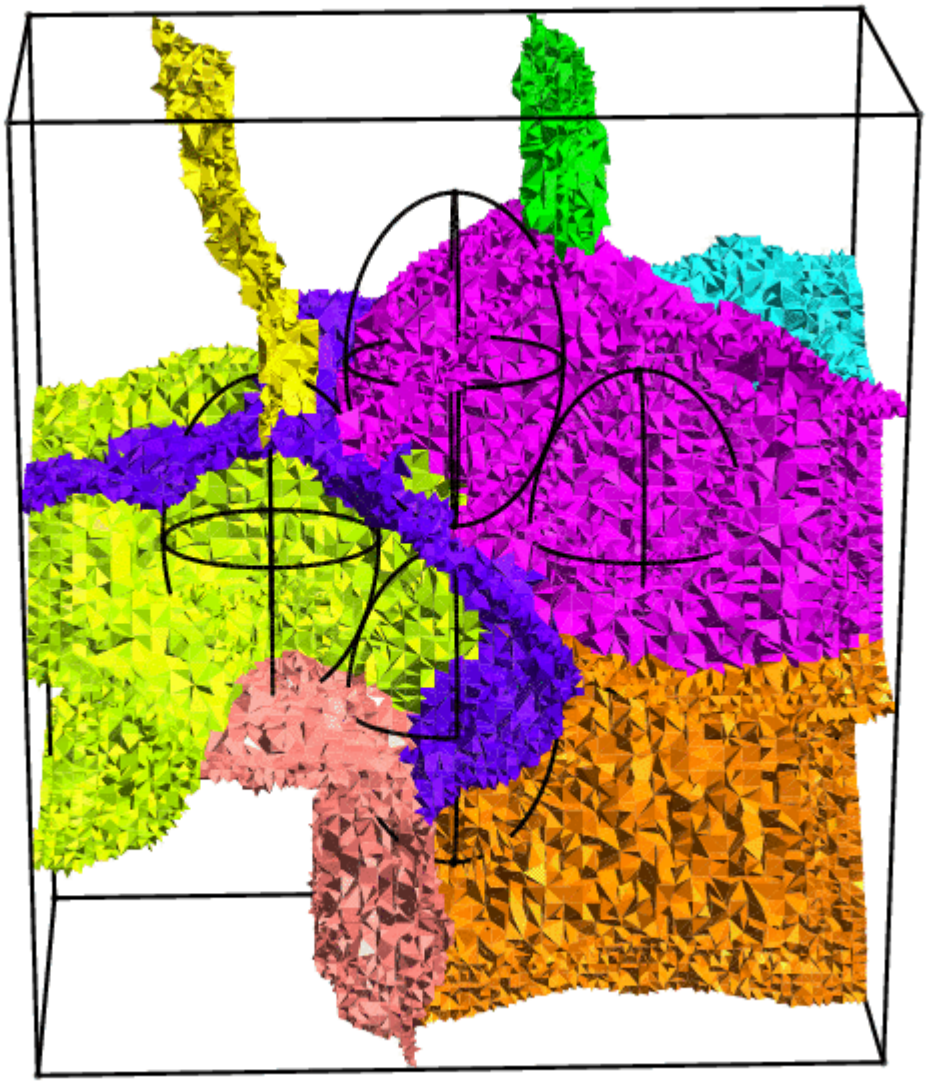
10 subdomains

Approximate size of one subdomain: 40 000

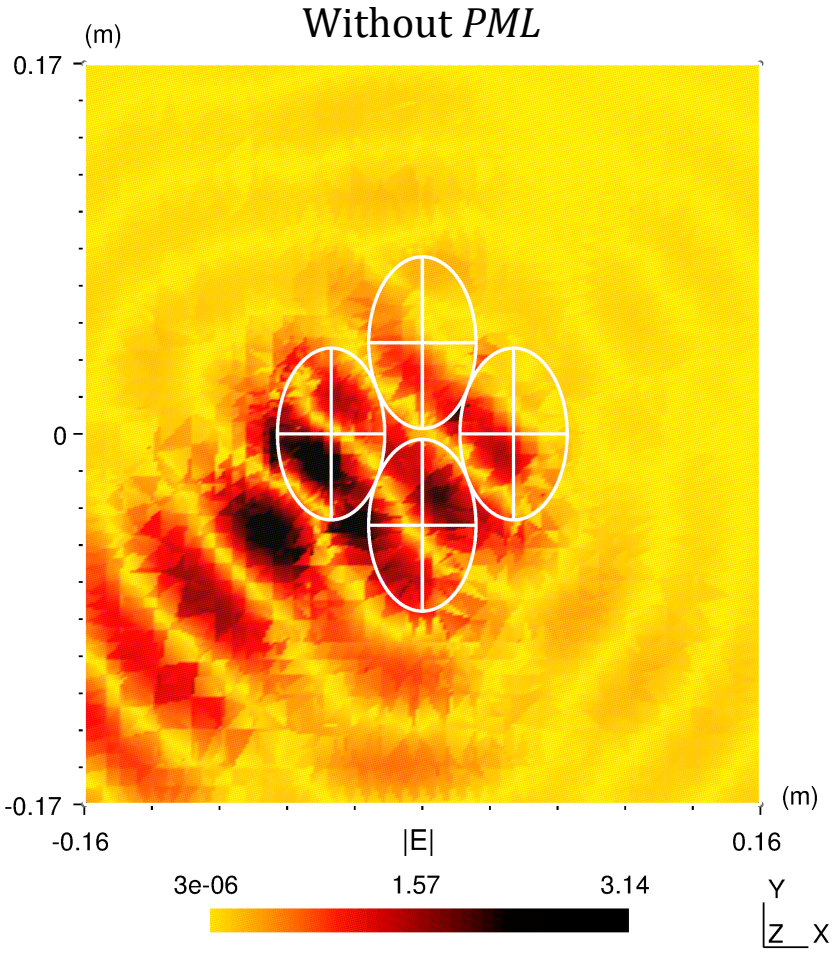
Size of interface problem (number of λ_r): 46 500

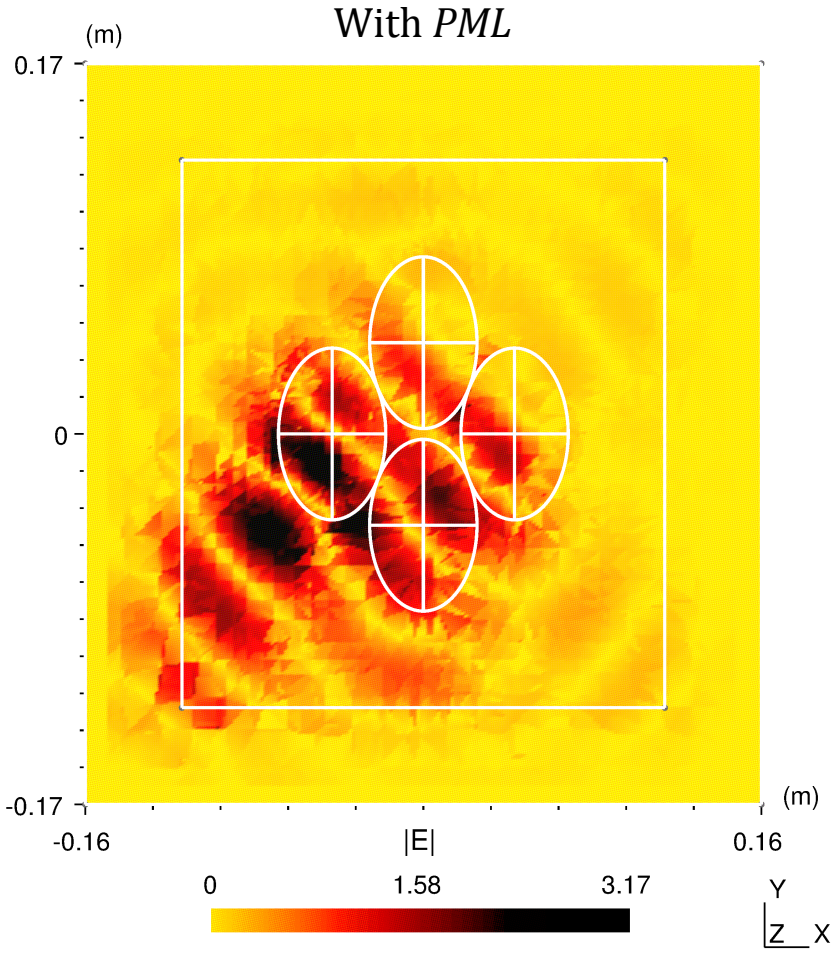
Total number of corner-edges: 580

Total number of ξ: 1 605



3D Direct problem: field distribution





The interface problem is solved with MUMPS based on ***LU-decomposition***

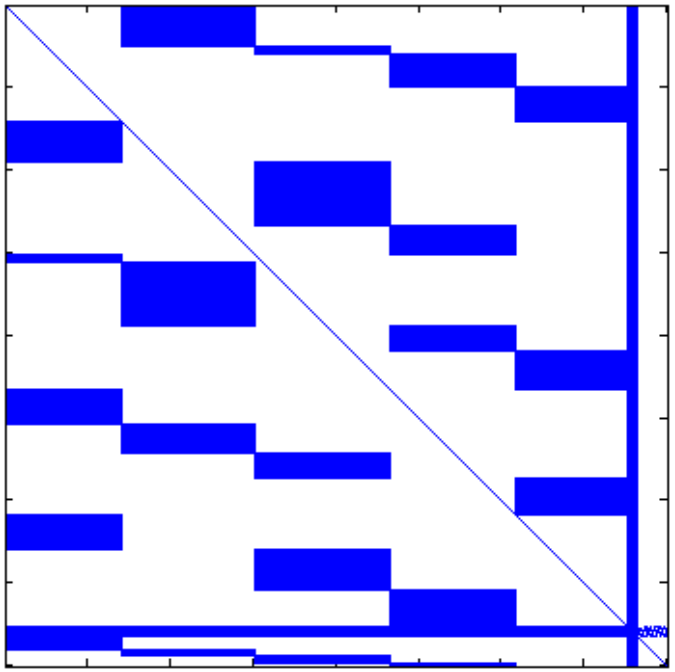
N_s	FETI-DPEM2 (a)	FETI-DPEM2 (b)	FETI-DPEM2 *full*
5	5.11E-002	9.12E-013	2.92E-012
7	8.98E-002	1.59E-012	7.66E-013
10	1.73E-001	9.78E-013	3.46E-012

The Interface Problem (IP) Matrix

$$\begin{bmatrix} \mathrm{F}_{\lambda_r\lambda_r} & \mathrm{F}_{\lambda_r\mathbf{E}_c} & 0 \\ \mathrm{F}_{\mathbf{E}_c\lambda_r} & \mathrm{F}_{\mathbf{E}_c\mathbf{E}_c} & \mathrm{F}_{\mathbf{E}_c\xi} \\ \mathrm{F}_{\xi\lambda_r} & \mathrm{F}_{\xi\mathbf{E}_c} & \mathrm{F}_{\xi\xi} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \\ \xi \end{bmatrix} = \begin{bmatrix} \mathrm{d}_{\lambda_r} \\ \mathrm{d}_{\mathbf{E}_c} \\ \mathrm{d}_{\xi} \end{bmatrix}$$

Bottlenecks
Inverting and storing matrices
Storing the Interface Problem (IP) matrix
Computing and storing LU of the IP matrix

An iterative method?



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GMRES

iterative method

The Interface Problem (IP) Matrix

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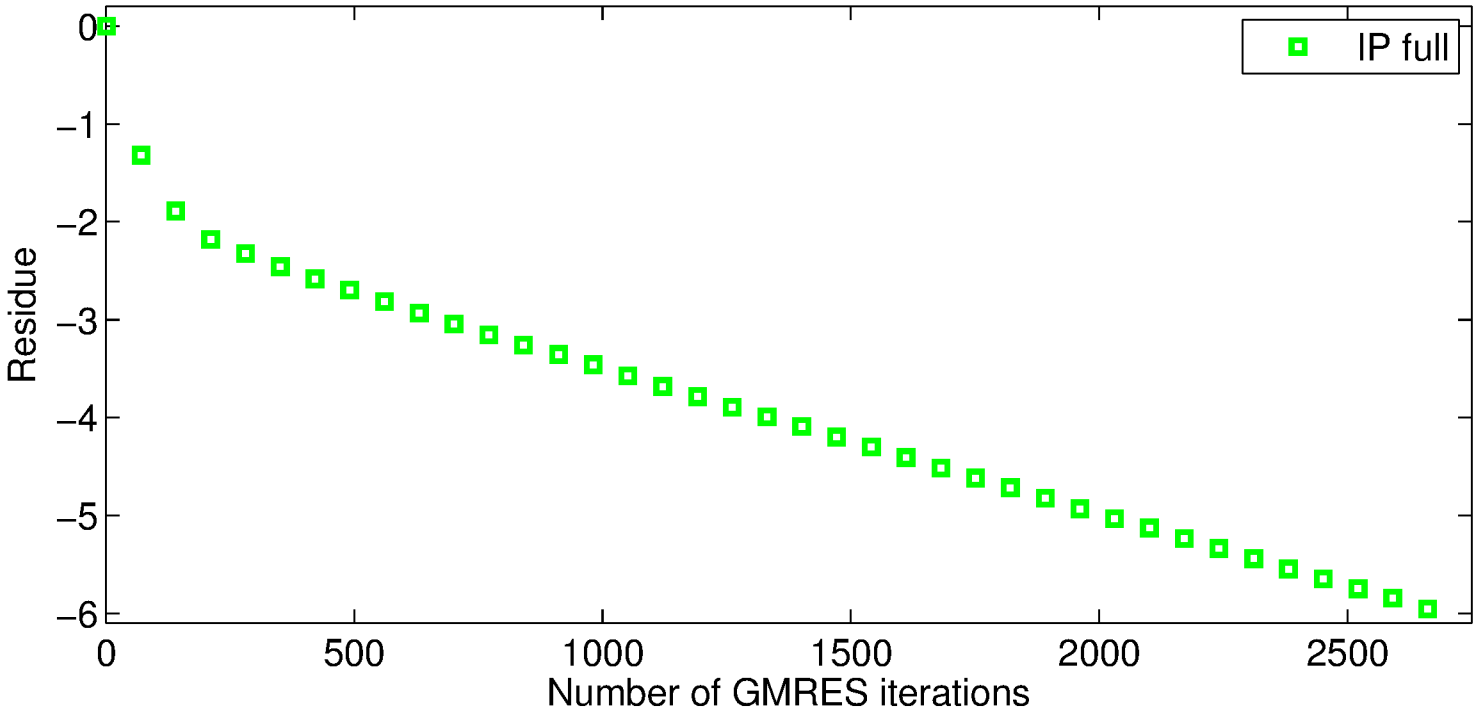


GMRES

iterative method

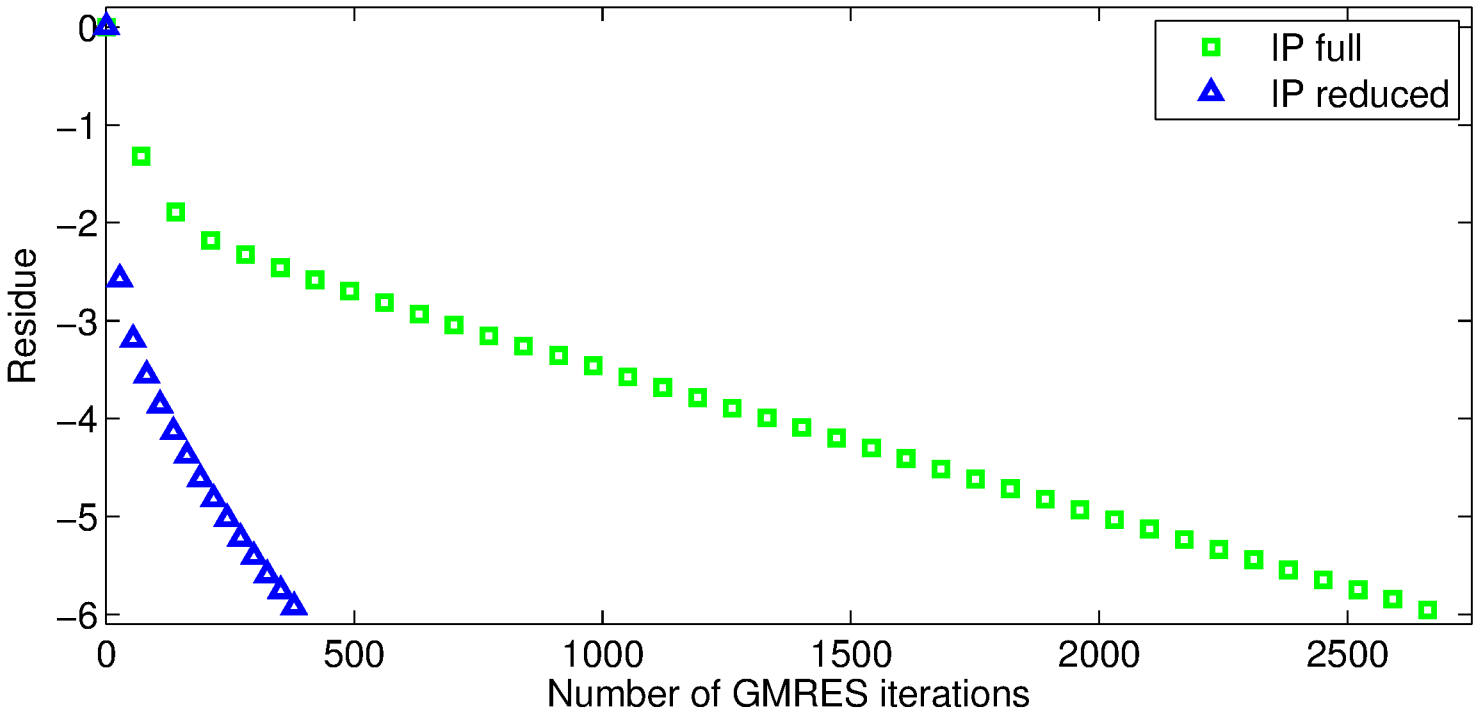
Interface problem: iterative solution

The Interface Problem (IP) Matrix

$$\begin{bmatrix} \mathrm{F}_{\lambda_r\lambda_r} & \mathrm{F}_{\lambda_r\mathbf{E}_c} & 0 \\ \mathrm{F}_{\mathbf{E}_c\lambda_r} & \mathrm{F}_{\mathbf{E}_c\mathbf{E}_c} & \mathrm{F}_{\mathbf{E}_c\xi} \\ \mathrm{F}_{\xi\lambda_r} & \mathrm{F}_{\xi\mathbf{E}_c} & \mathrm{F}_{\xi\xi} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \\ \xi \end{bmatrix} = \begin{bmatrix} \mathrm{d}_{\lambda_r} \\ \mathrm{d}_{\mathbf{E}_c} \\ \mathrm{d}_{\xi} \end{bmatrix}$$

The Reduced IP Matrix [1,2]

$$\left(\mathrm{F}_{\lambda_r\lambda_r} + \mathrm{F}_{\lambda_r\mathbf{E}_c}\left[\hat{\mathrm{F}}^{-1}_{\mathbf{E}_c\mathbf{E}_c}\right]\hat{\mathrm{F}}_{\mathbf{E}_c\lambda_r}\right)\lambda_r = -\hat{\mathrm{d}}_{\lambda_r}$$



References		
[1]	Li and Jin	2007
[2]	Xue and Jin	2012

GMRES

iterative method

Interface problem: iterative solution

The Interface Problem (IP) Matrix

$$\begin{bmatrix} \mathrm{F}_{\lambda_r\lambda_r} & \mathrm{F}_{\lambda_r\mathbf{E}_c} & 0 \\ \mathrm{F}_{\mathbf{E}_c\lambda_r} & \mathrm{F}_{\mathbf{E}_c\mathbf{E}_c} & \mathrm{F}_{\mathbf{E}_c\xi} \\ \mathrm{F}_{\xi\lambda_r} & \mathrm{F}_{\xi\mathbf{E}_c} & \mathrm{F}_{\xi\xi} \end{bmatrix} \begin{bmatrix} \lambda_r \\ \mathbf{E}_c \\ \xi \end{bmatrix} = \begin{bmatrix} \mathrm{d}_{\lambda_r} \\ \mathrm{d}_{\mathbf{E}_c} \\ \mathrm{d}_{\xi} \end{bmatrix}$$

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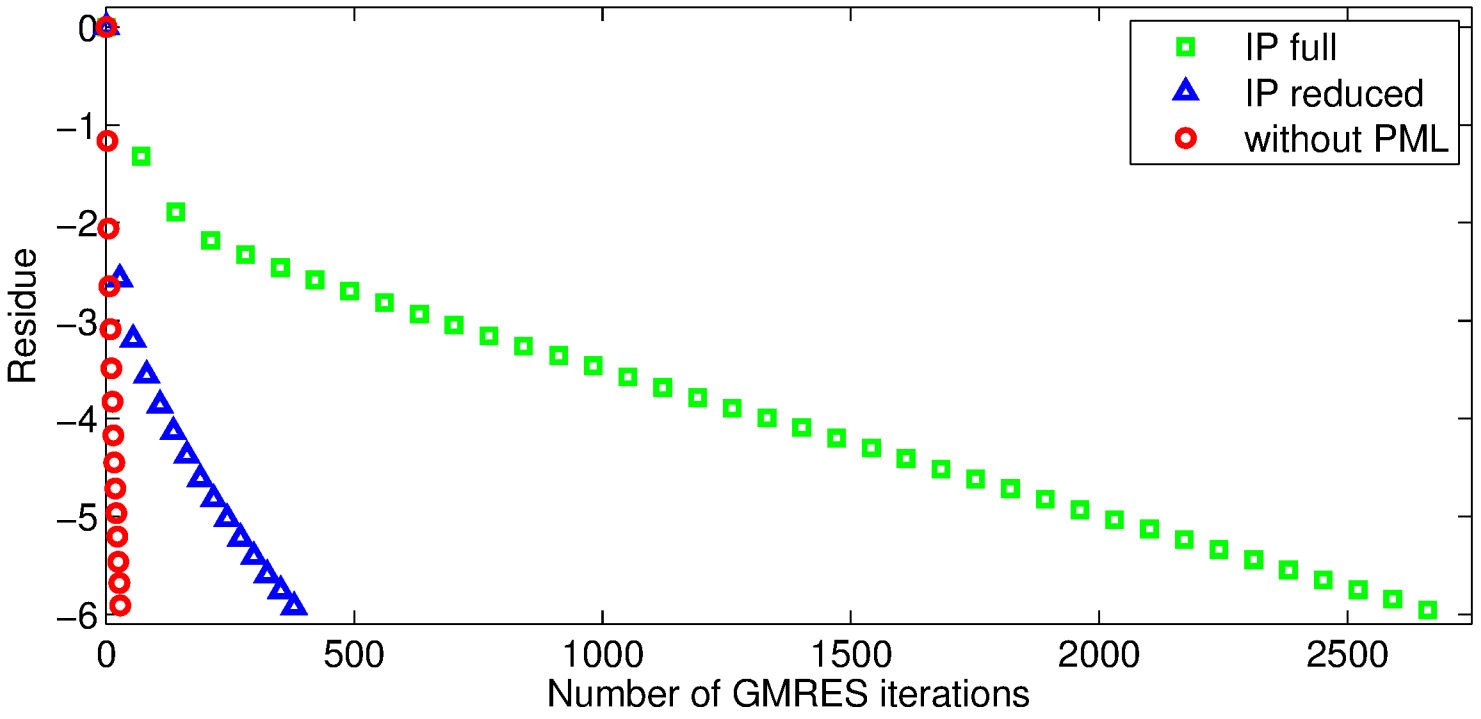

References		
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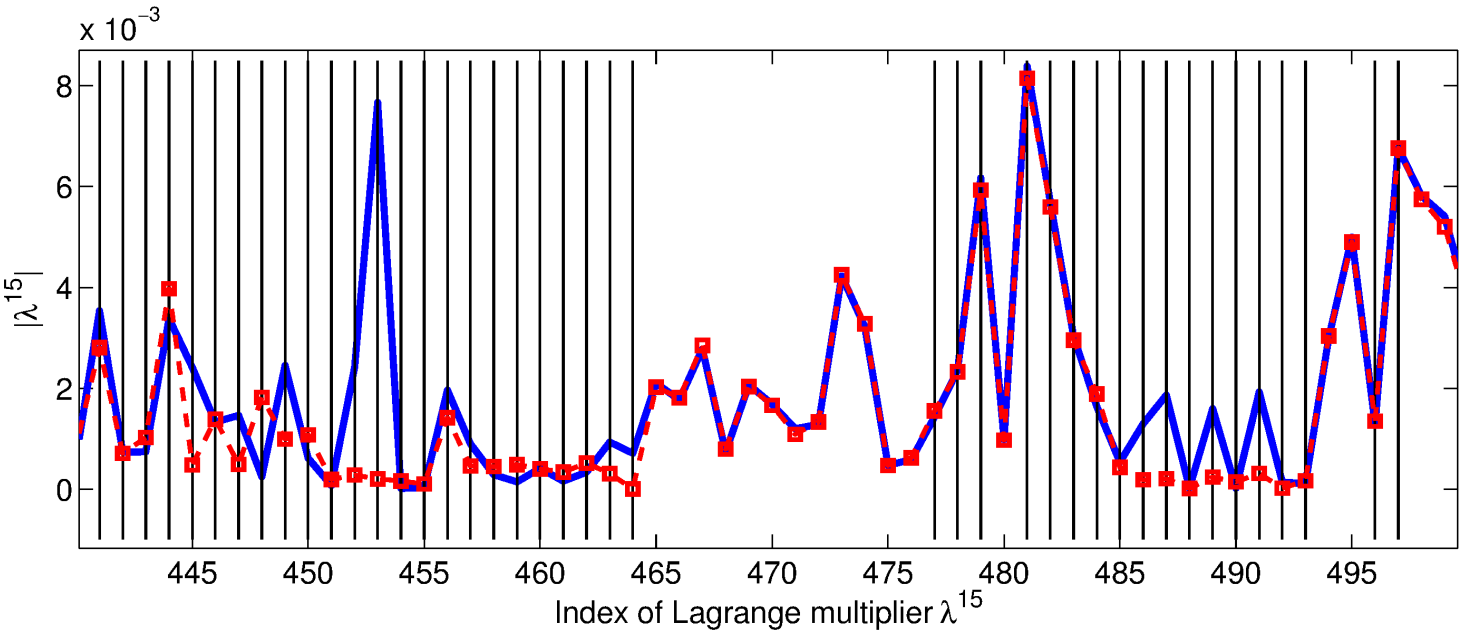
GMRES

iterative method

Convergence: influence of PML

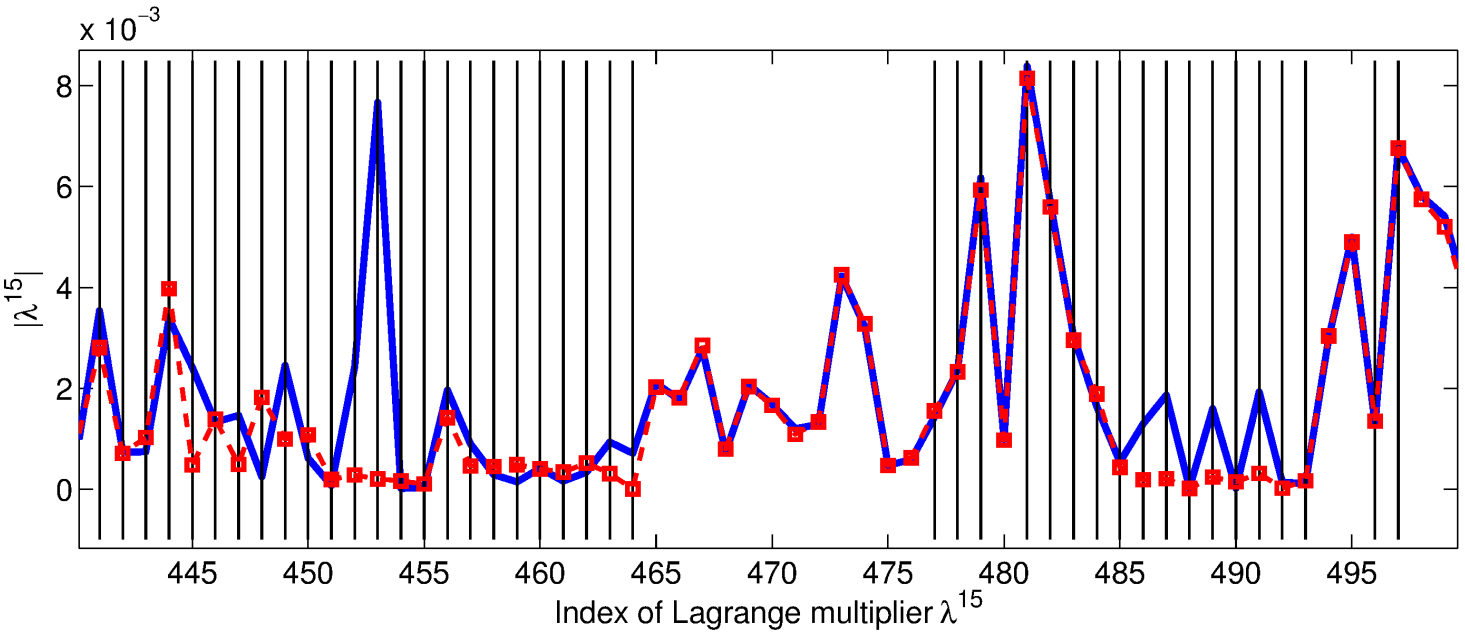


Exact values of the Lagrange multipliers (*LU-decomposition*)

Lagrange multipliers obtained after 10 iterations

Presence of PML area

Convergence: influence of PML



Conclusion so far

- ❑ Bad influence of PML
- ❑ The error does not spread

Exact values of the Lagrange multipliers (*LU-decomposition*)

Lagrange multipliers obtained after 10 iterations

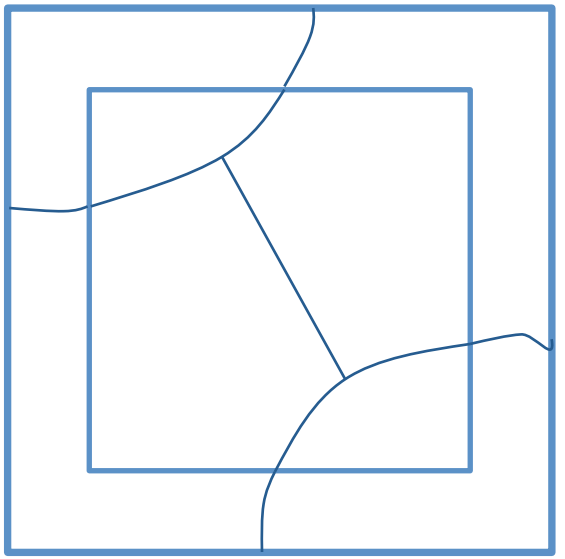
Presence of PML area

What we can play with

- Robin-type boundary conditions

$$\vec{n} \times \left(\frac{1}{\mu_r} \nabla \times \mathcal{E}^i \right) + \alpha^i \vec{n} \times \vec{n} \times \mathcal{E}^i = \Lambda^i \text{ on } \Gamma^i$$

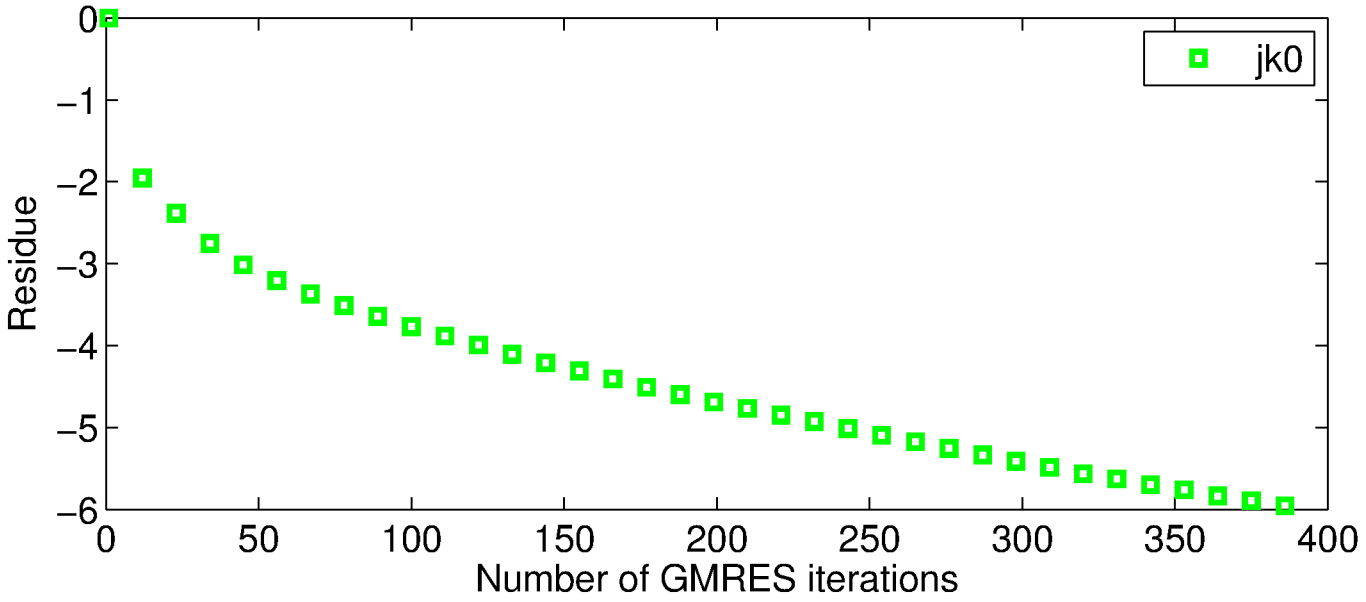
Després approach [1]	EMDA* approach [2]
$\alpha^i = jk_0$	$\alpha^i = jk_0(1 + j\chi)$



* Evanescent Modes Damping Algorithm

References		
[1]	Després	1991
[2]	Boubendir *et al*	2000

Convergence: influence of the -parameter



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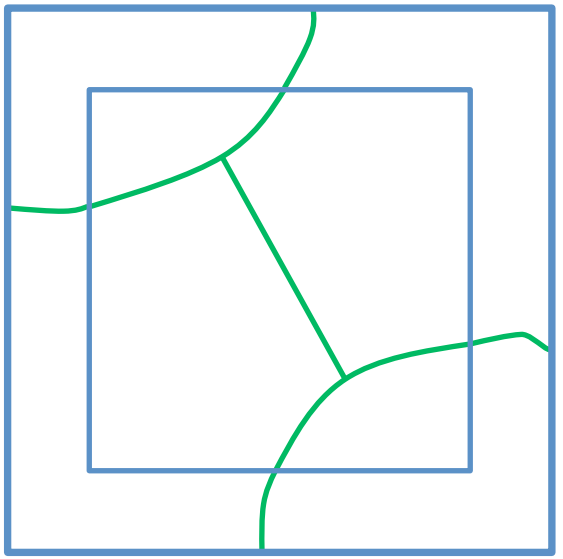
$\alpha^i = jk_0$	$\alpha^i = jk_0(1 + j\chi)$



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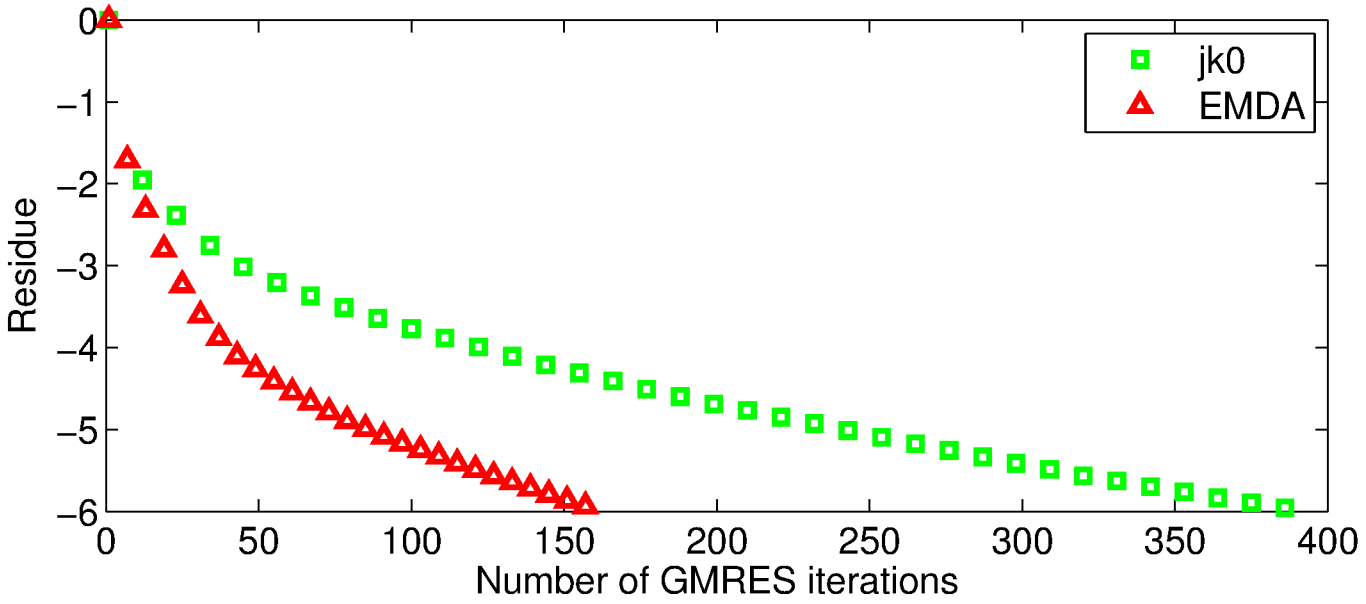
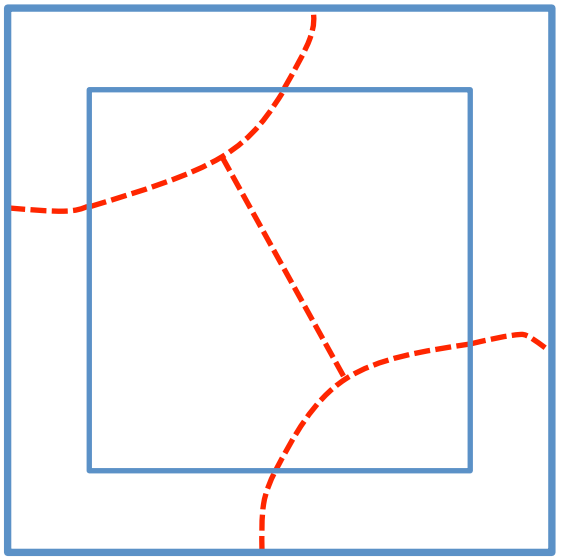
Convergence: influence of the -parameter



Després approach [1]	EMDA* approach [2]
$\alpha^i = jk_0$	$\alpha^i = jk_0(1 + j\chi)$



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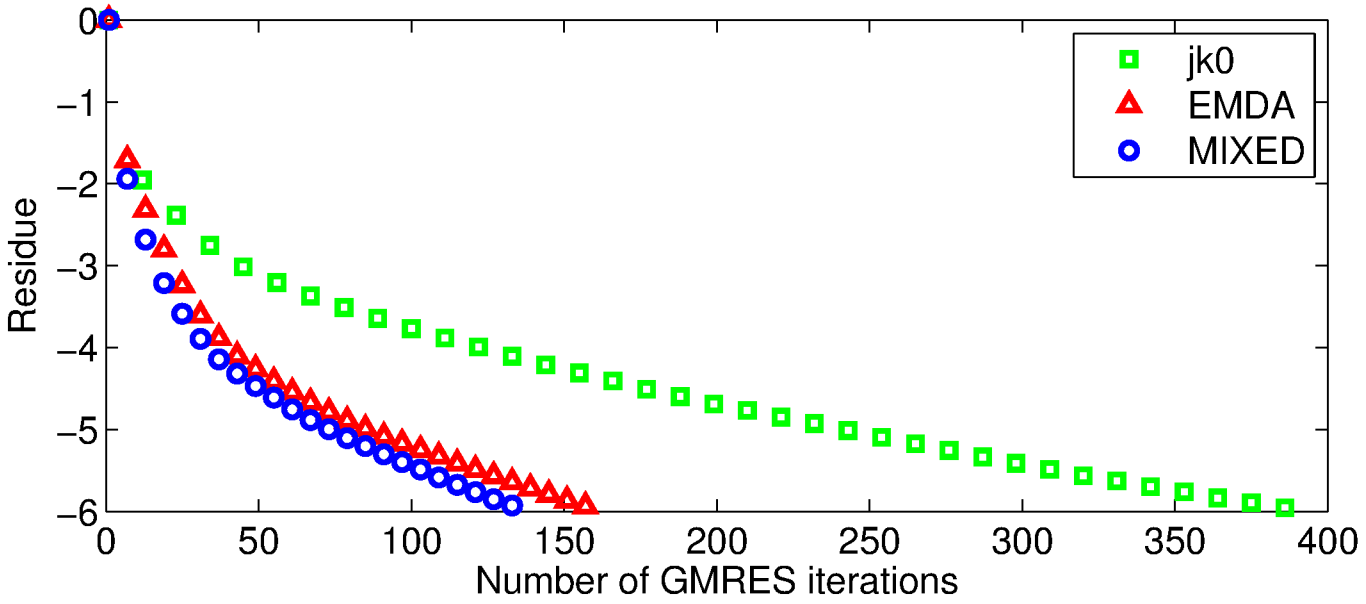
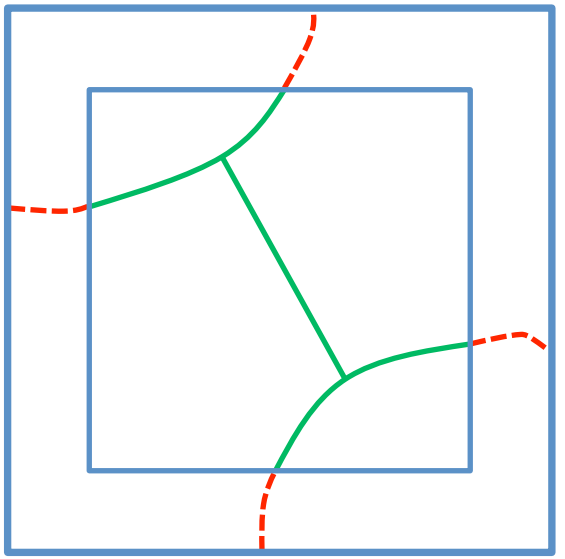
Convergence: influence of the -parameter



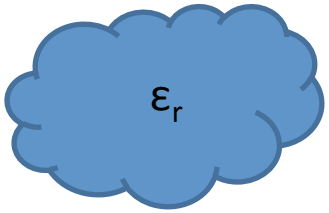
Després approach [1]	EMDA* approach [2]
$\alpha^i = jk_0$	$\alpha^i = jk_0(1 + j\chi)$
MIXED approach [3]	



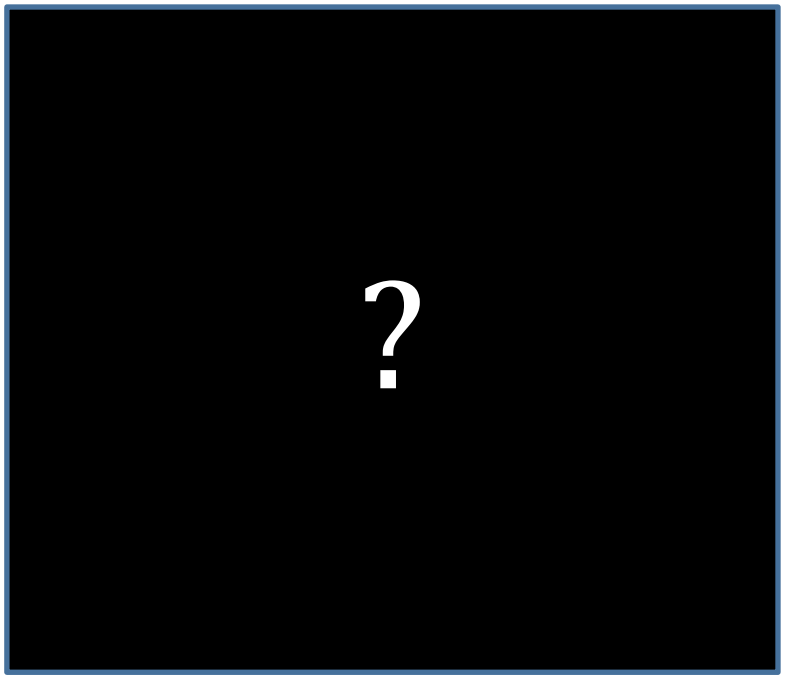
* Evanescent Modes Damping Algorithm

References		
[1]	Després	1991
[2]	Boubendir *et al*	2000
[3]	Voznyuk *et al* (submitted)	2014

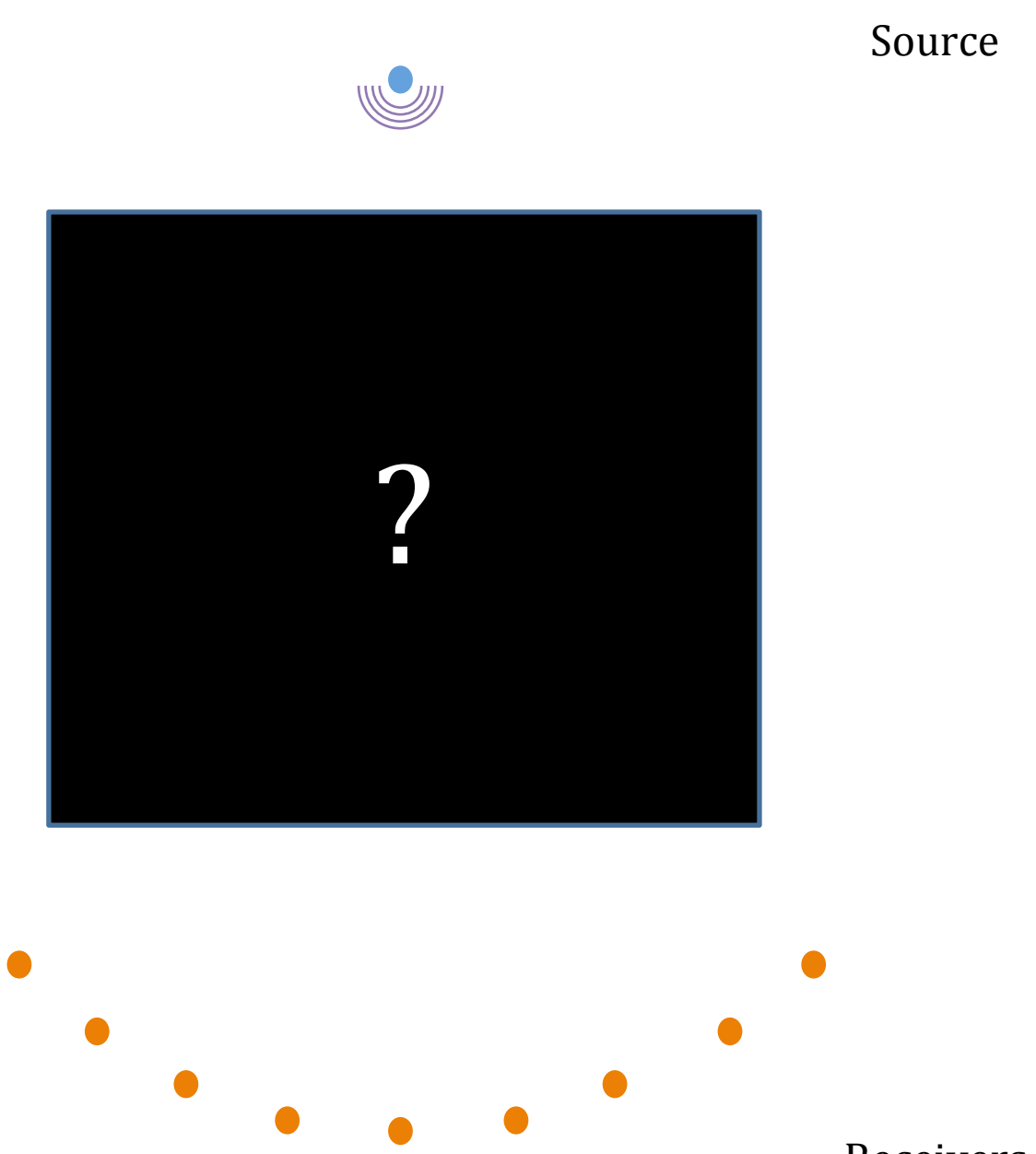
Just an object



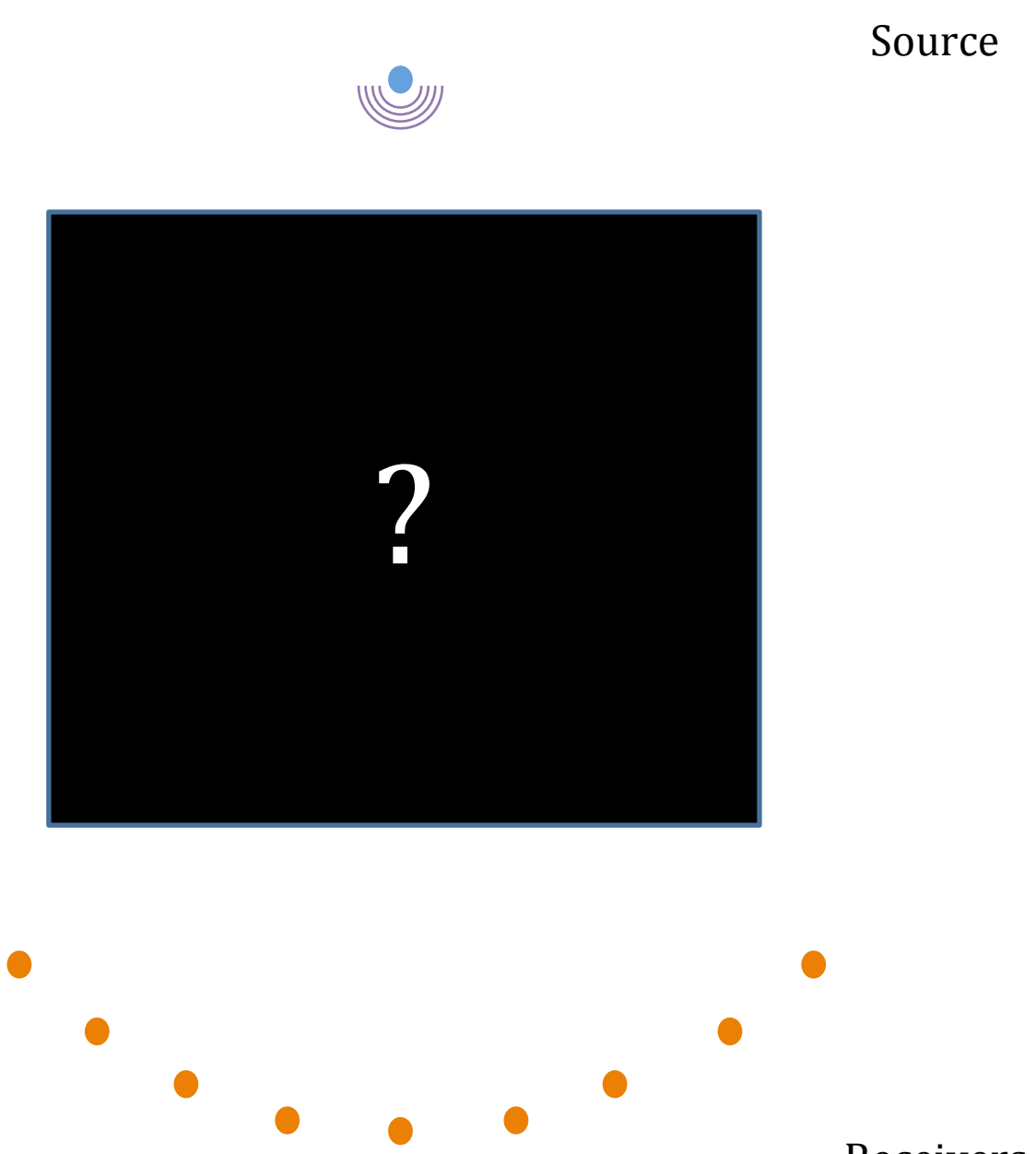
3D Inversion: schematic configuration



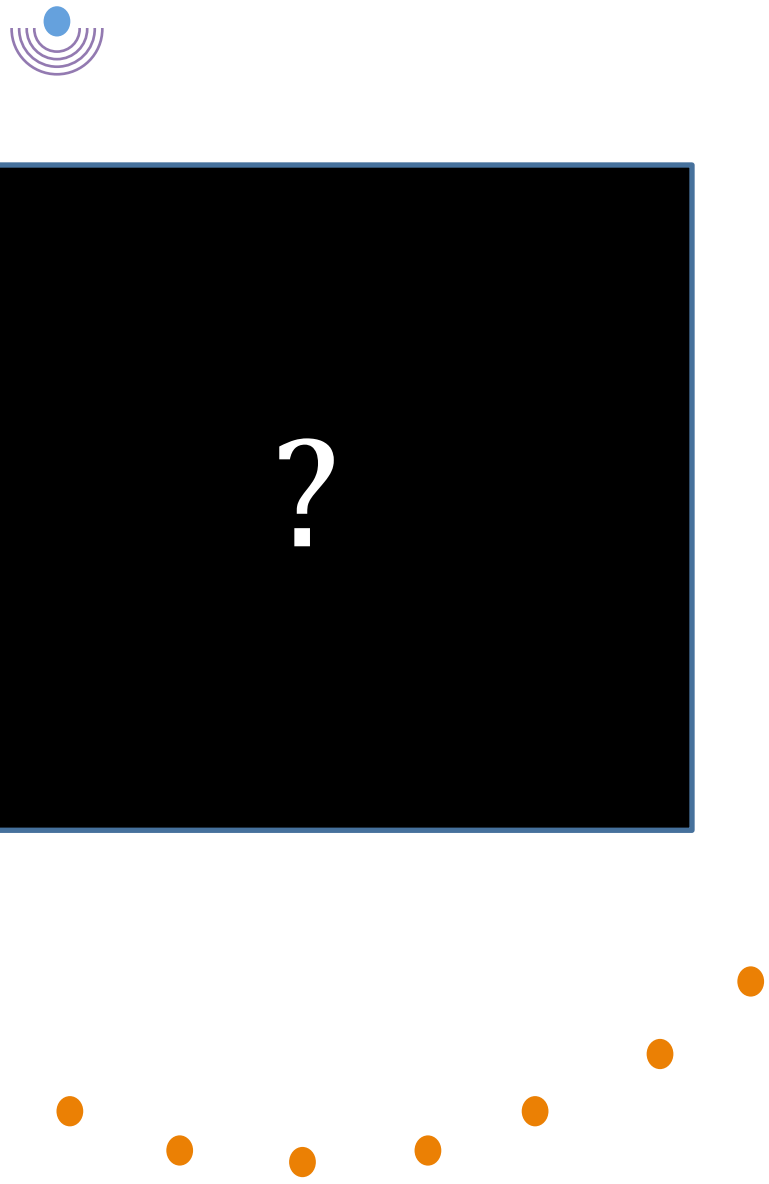
3D Inversion: schematic configuration



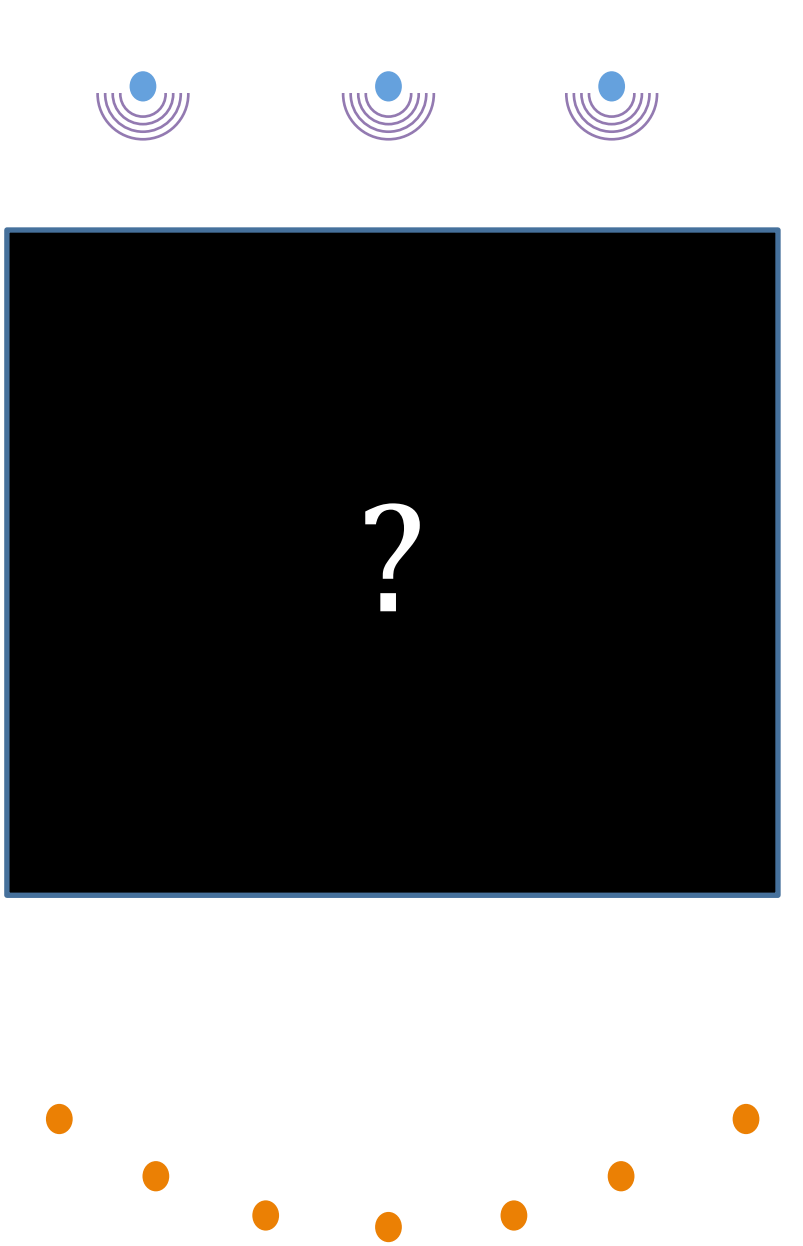
3D Inversion: schematic configuration



3D Inversion: schematic configuration



3D Inversion: schematic configuration

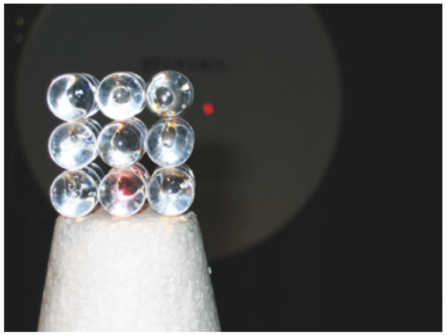


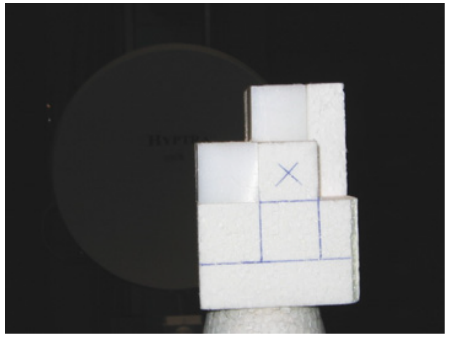
3D Inversion: measurement setup

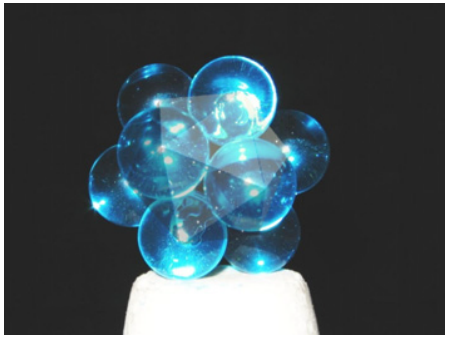
$$\mathcal{E}^{mes} : N^{src} \times N^{rec}$$

3D Fresnel database [1]

- Set of homogeneous targets
- 162 transmitting dipoles
- 32 receiver positions
- Distance:







Anechoic chamber

References

[1] J.-M. Geffrin and P. Sabouroux 2009

$\mathcal{E}^{mes} : N^{src} \times N^{rec}$

Problem statement:

Find such as the cost functional

$\mathcal{J}(\varepsilon_r, \mathcal{E}) = \| \mathcal{E}^{mes} - \mathcal{E}^{calc} \|_W^2$ is minimized

- W – covariance matrix, related to + noise
- – scattered far-field, computed thanks to Near-to-Far-Field transformation

$\mathcal{E}^{mes} : N^{src} \times N^{rec}$

Problem statement:

Find such as the cost functional

$\mathcal{J}(\varepsilon_r, \mathcal{E}) = \| \mathcal{E}^{mes} - \mathcal{E}^{calc} \|^2_W$ is minimized

- W – covariance matrix, related to + noise
- – scattered far-field, computed thanks to Near-to-Far-Field transformation

Characteristics of problem

- nonlinear
- ill posed
- underdetermined

There is not a unique solution

$\mathcal{E}^{mes} : N^{src} \times N^{rec}$

Problem statement:

Find such as the cost functional

$\mathcal{J}(\varepsilon_r, \mathcal{E}) = \| \mathcal{E}^{mes} - \mathcal{E}^{calc} \|_W^2$ is minimized

- An iterative ***quasi-Newton*** method based on L-BFGS [1] approach with constraints

Constraints

$$\mathcal{L}(\varepsilon_r, \mathcal{E}, \mathcal{P}) = \mathcal{J}(\varepsilon_r, \mathcal{E}) + \mathcal{R}e\left(\mathcal{P} \mid \nabla \times \nabla \times \mathcal{E} + k^2 \varepsilon_r \mathcal{E} - S\right)$$

References

[1] R. Byrd *et al* 1995

$\mathcal{E}^{mes} : N^{src} \times N^{rec}$

Problem statement:

Find such as the cost functional

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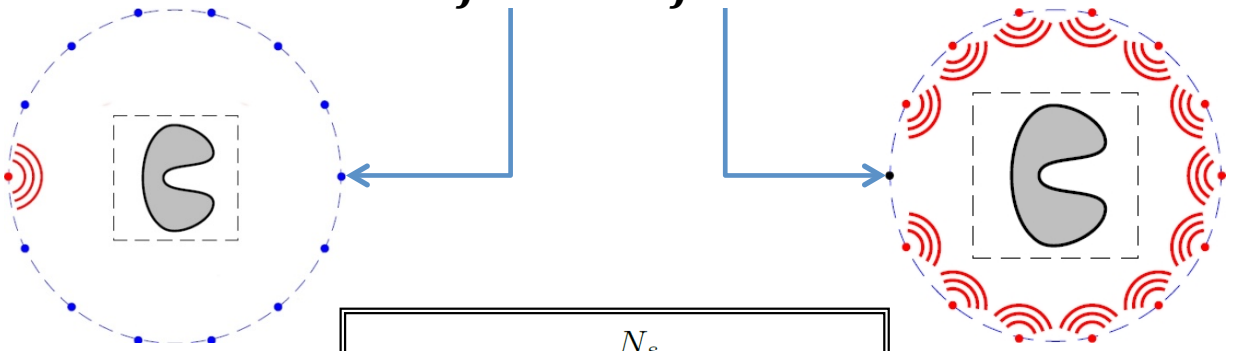
Total field ***Adjoint field***

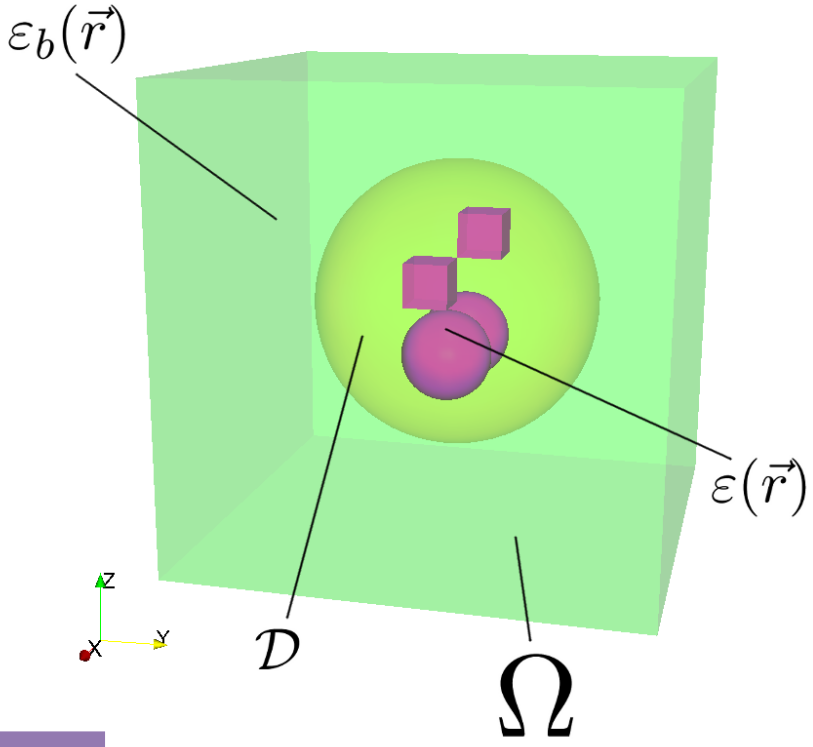


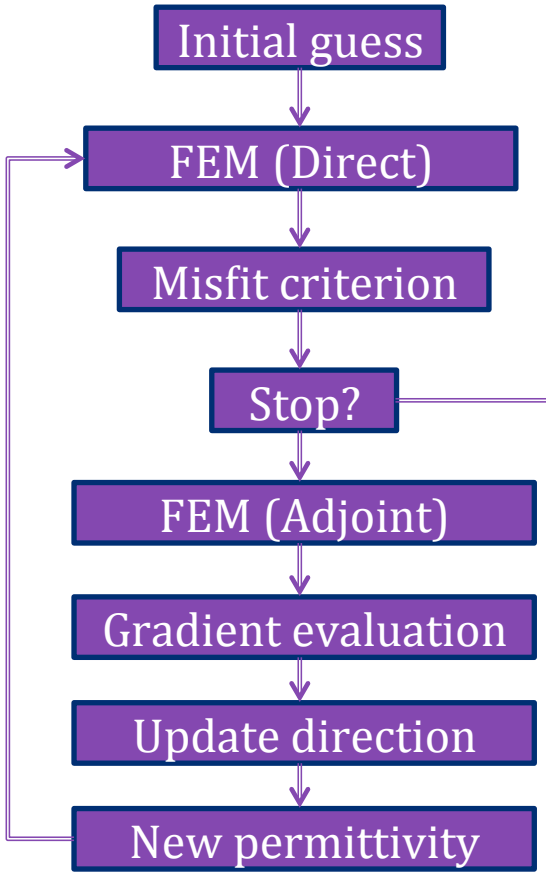
$$\nabla_{\varepsilon_r} \mathcal{J}(\varepsilon_r) = \sum_{s=1}^{N_s} \overline{\mathcal{E}_s^{\text{tot}}} \cdot \mathcal{P}_s^{\text{tot}}$$

References

[1] R. Byrd *et al* 1995

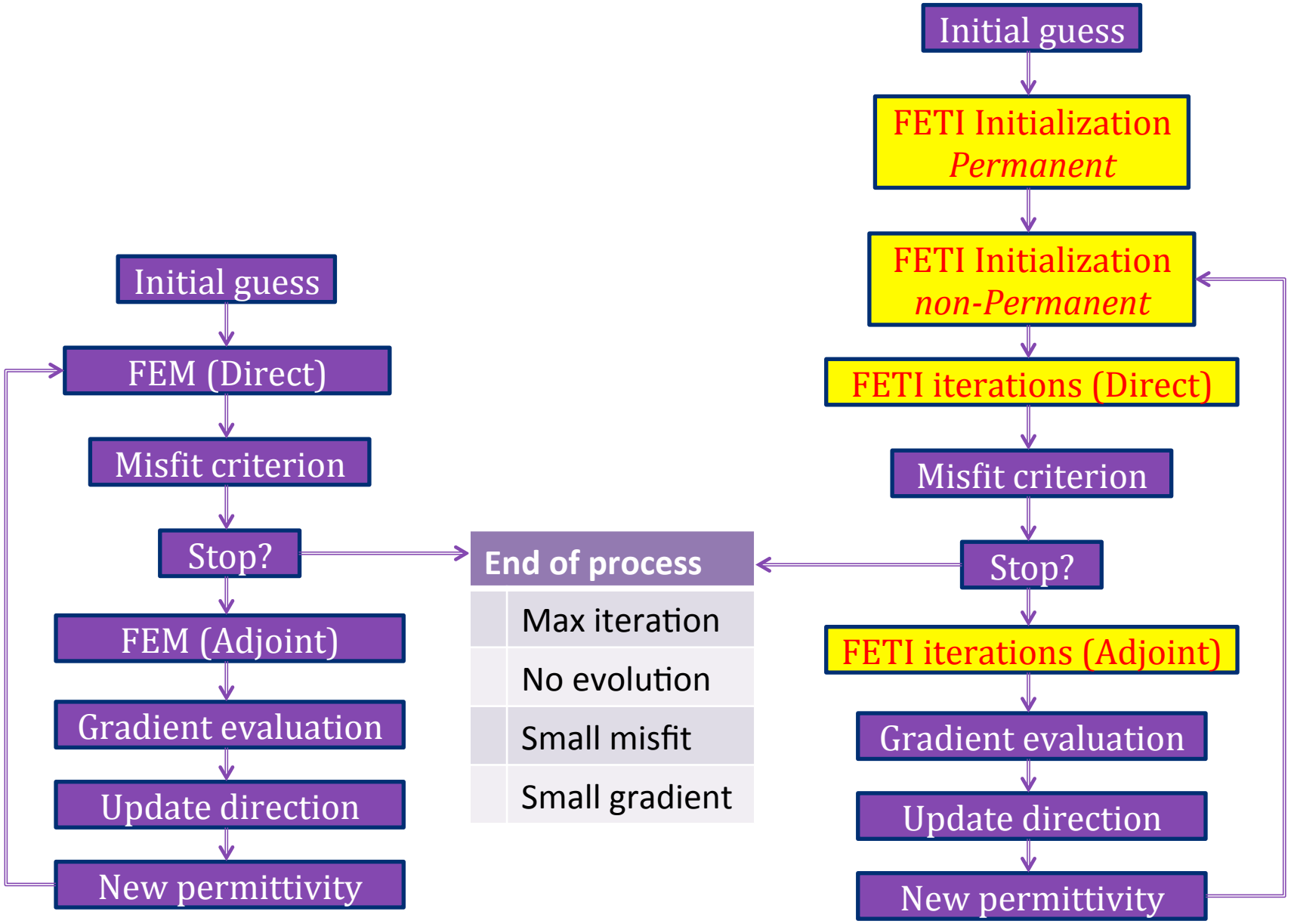
3D Inversion: Iterative FEM algorithm

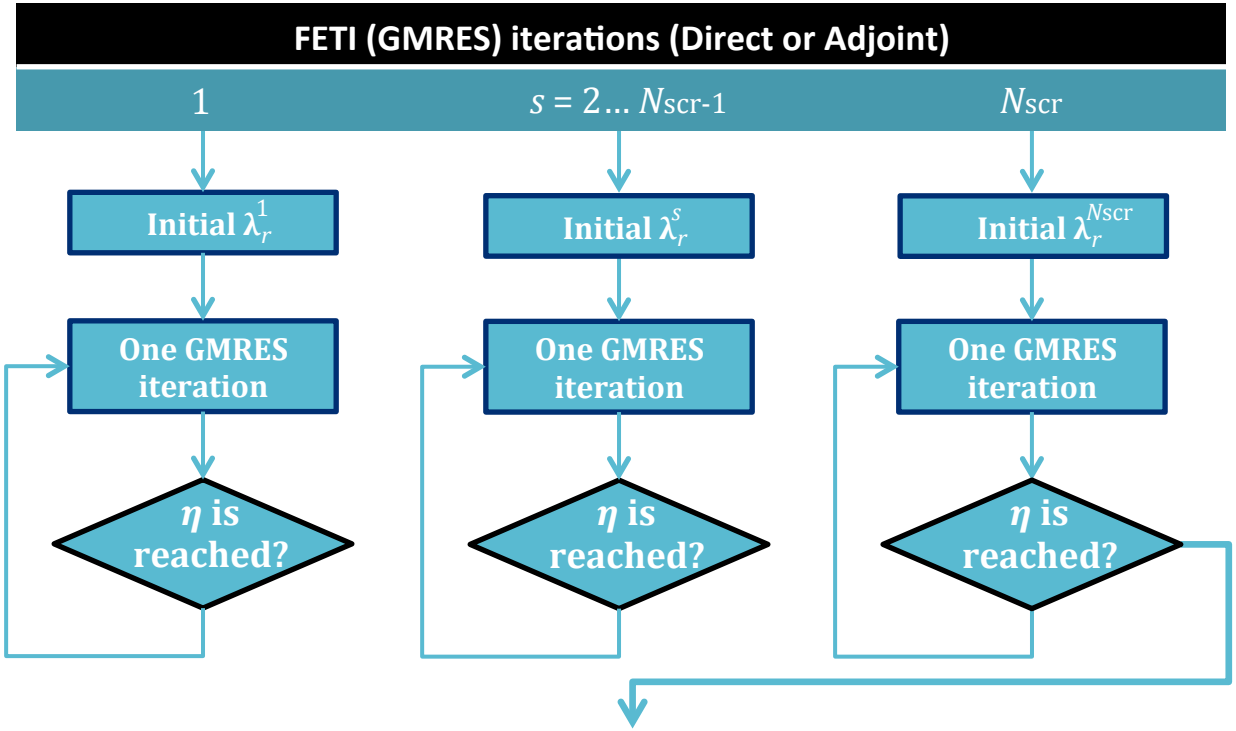




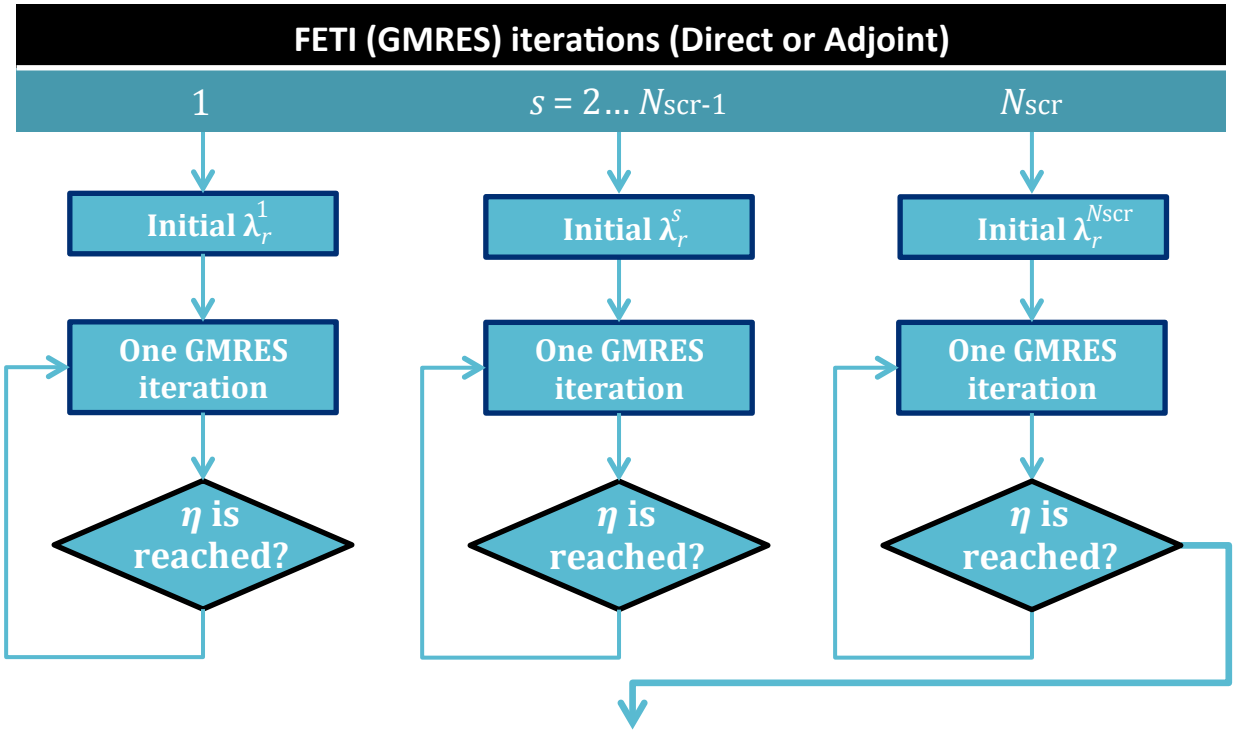
End of process
Max iteration
No evolution
Small misfit
Small gradient

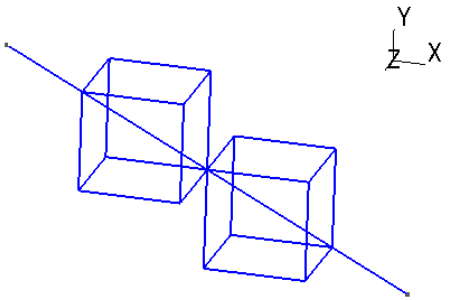
3D Inversion: Iterative FETI algorithm

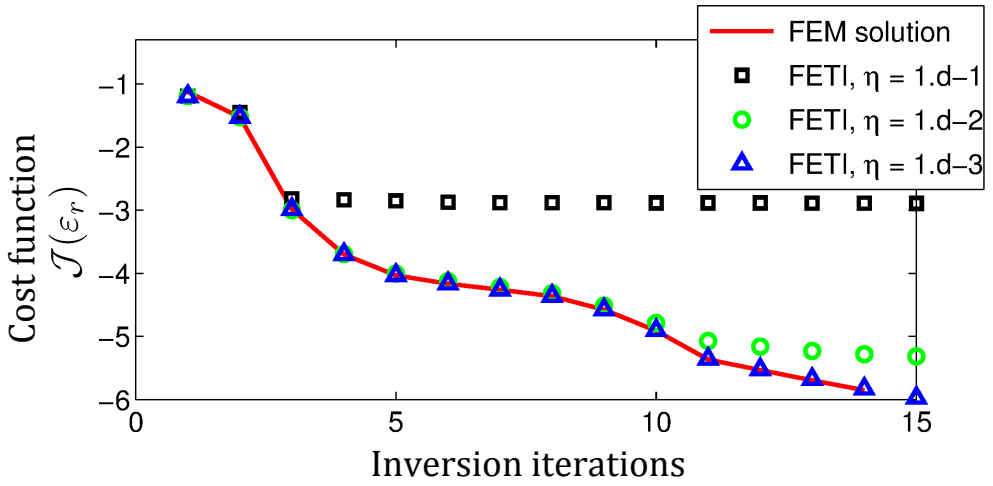



FETI (GMRES) iterations (Direct or Adjoint)
1
s = 2... Nscr-1
Nscr
Initial λ_r^1
Initial λ_r^s
Initial λ_r^{Nscr}
One GMRES iteration
One GMRES iteration
One GMRES iteration
η is reached?
η is reached?
η is reached?

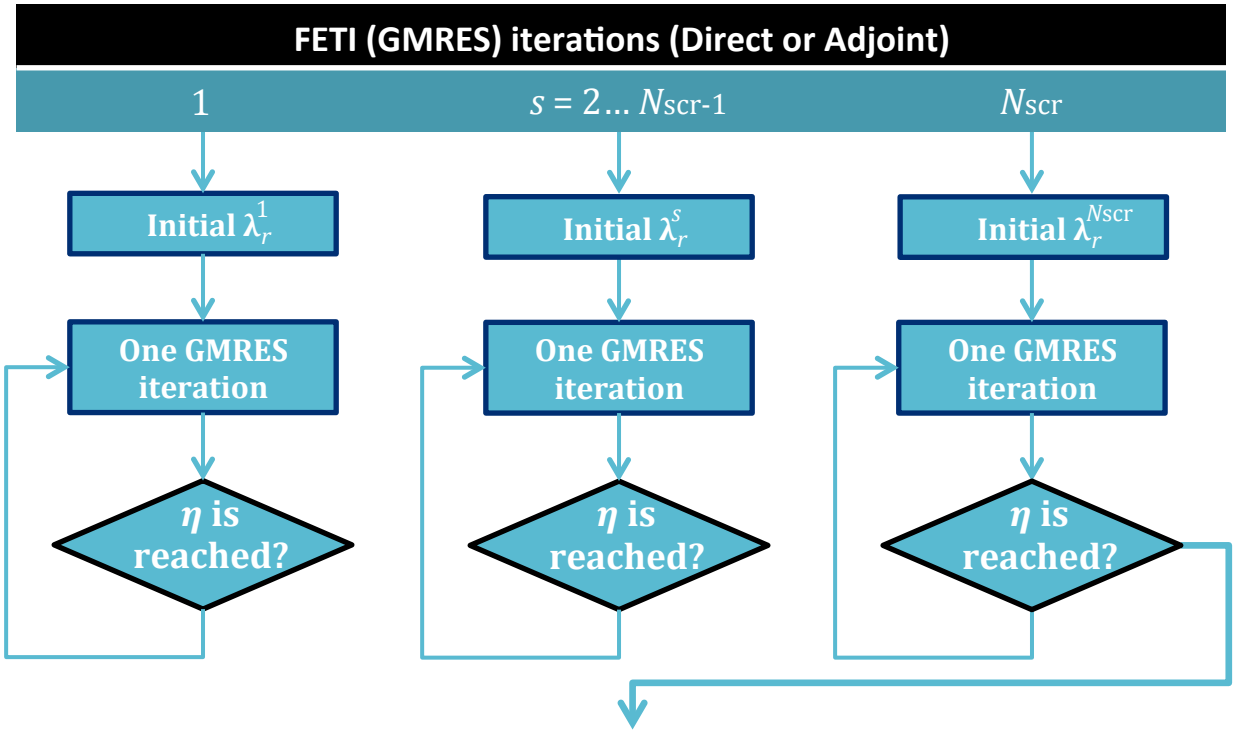
FETI optimization: GMRES stopping criterion

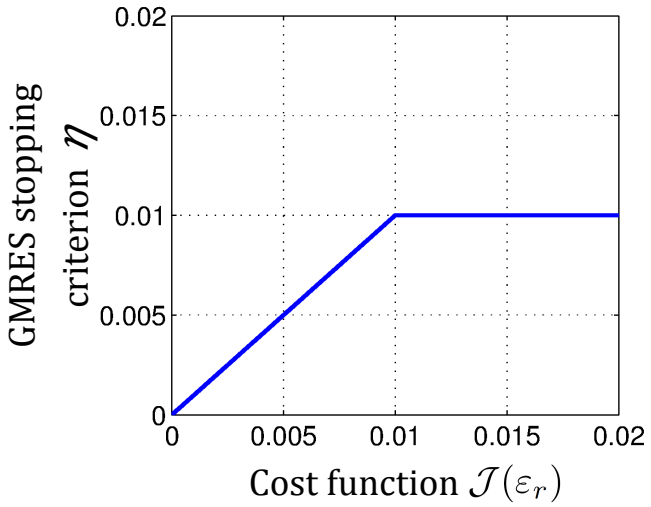


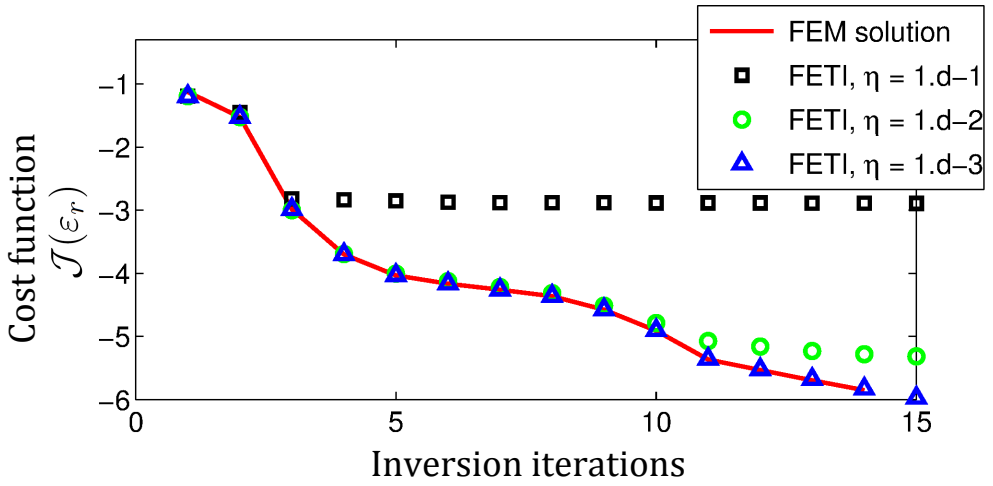




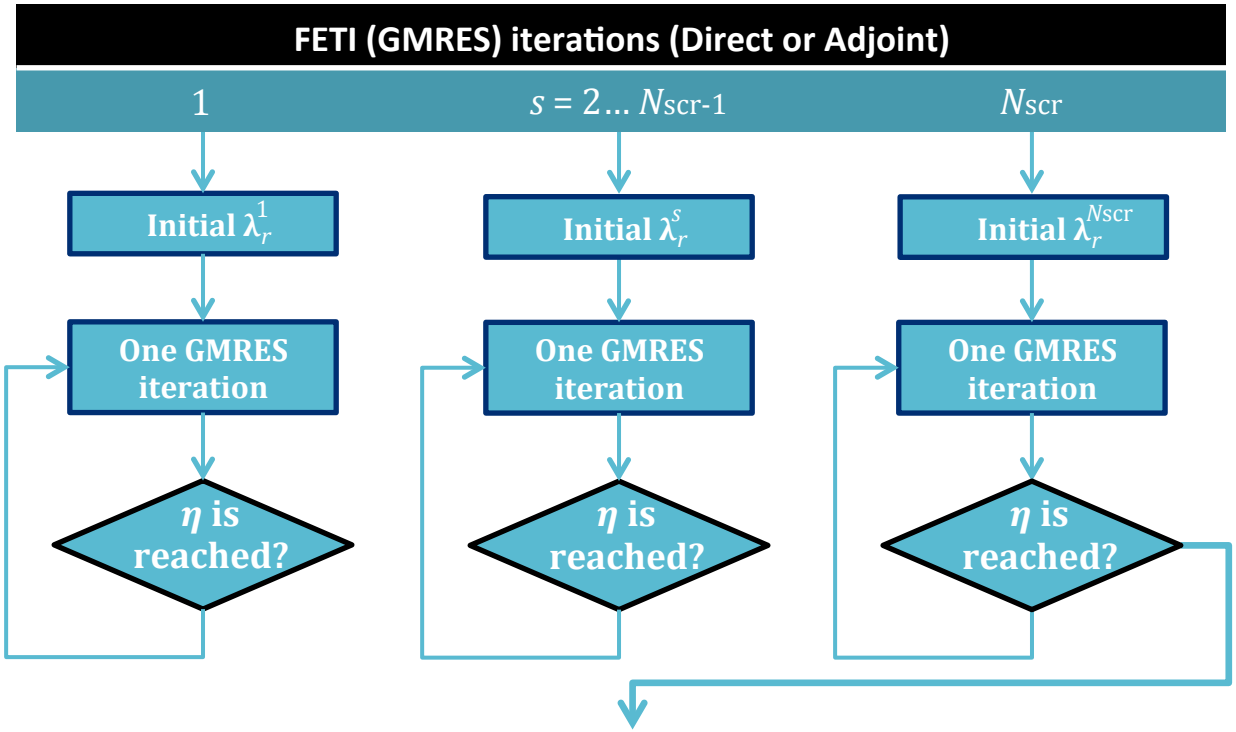
FETI optimization: GMRES stopping criterion





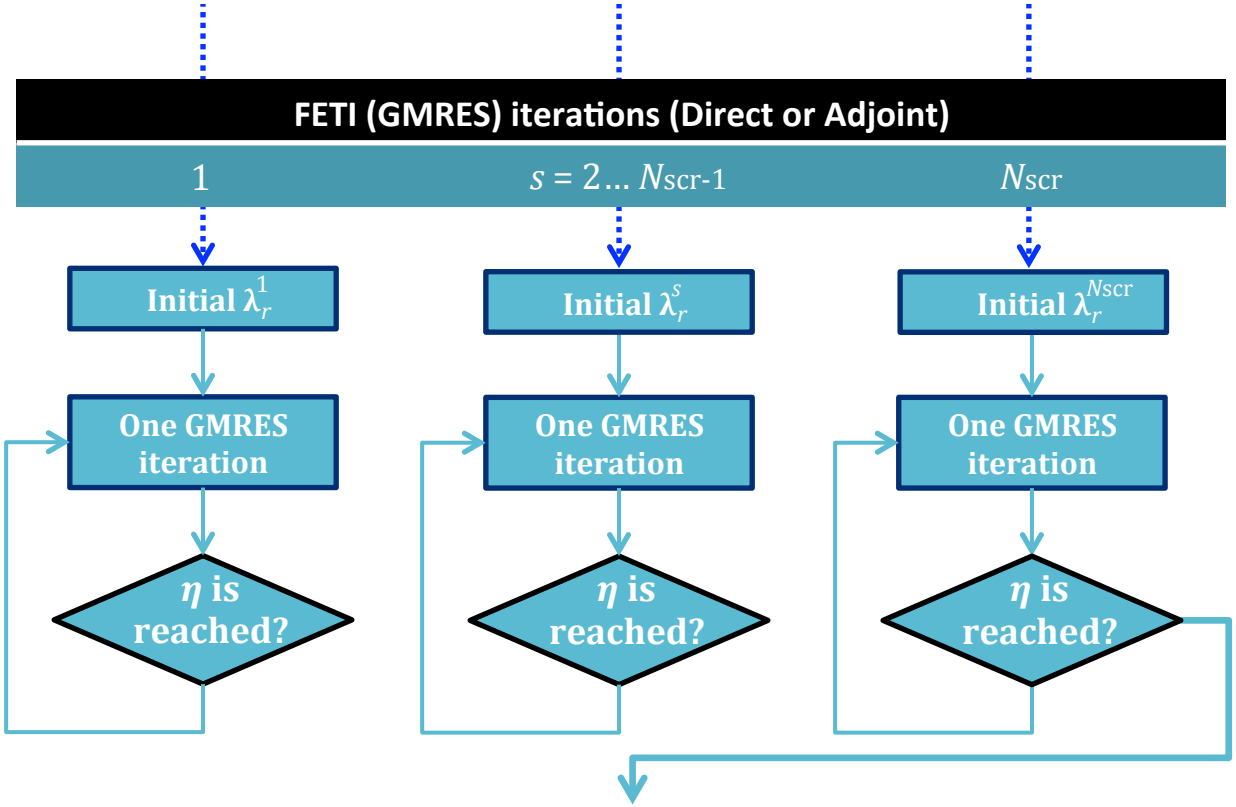


FETI optimization: starting point



Without initialization

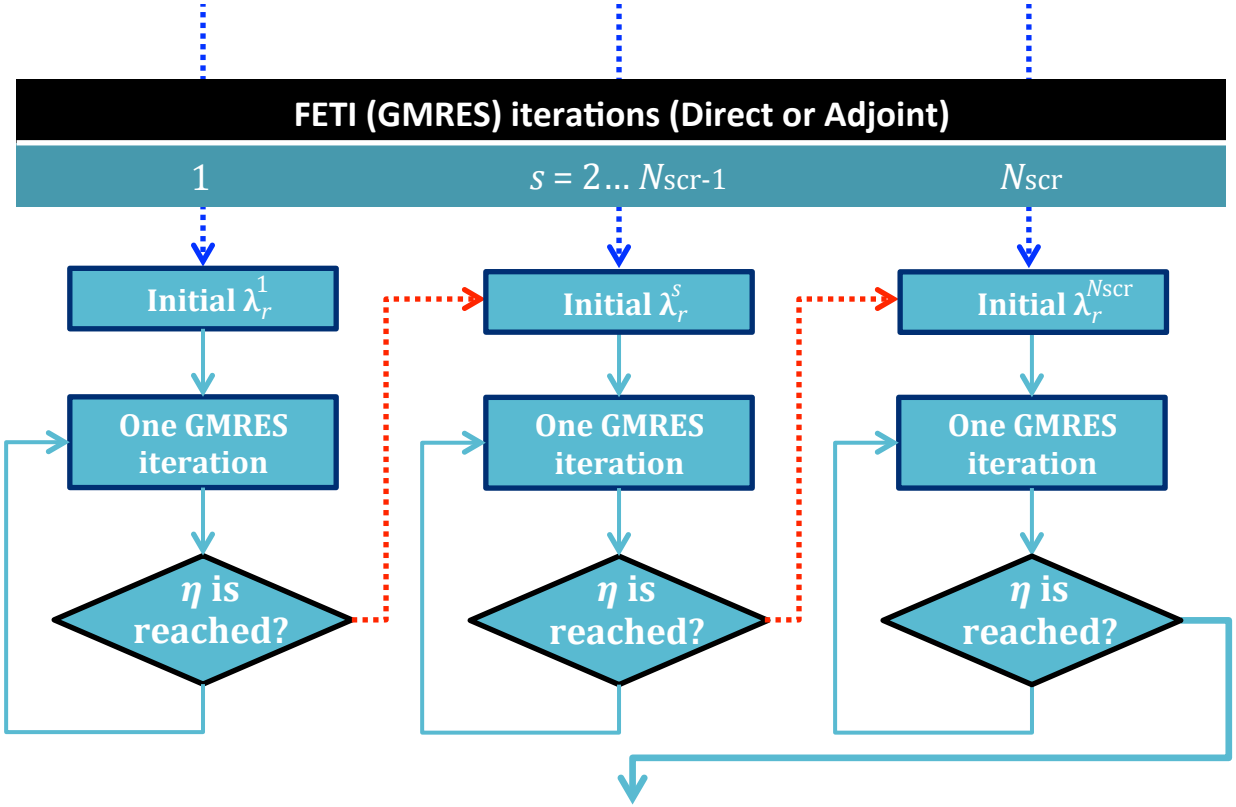
FETI optimization: starting point



Without initialization

lambdaR initialization

FETI optimization: starting point

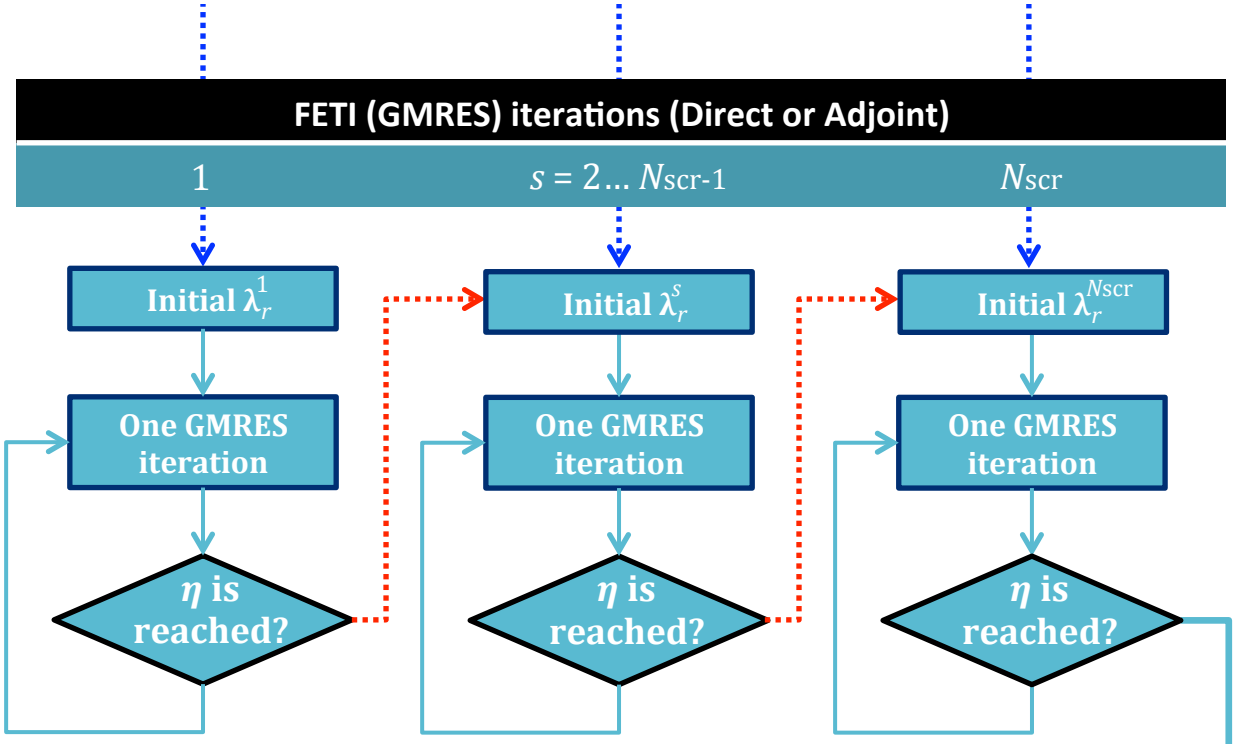


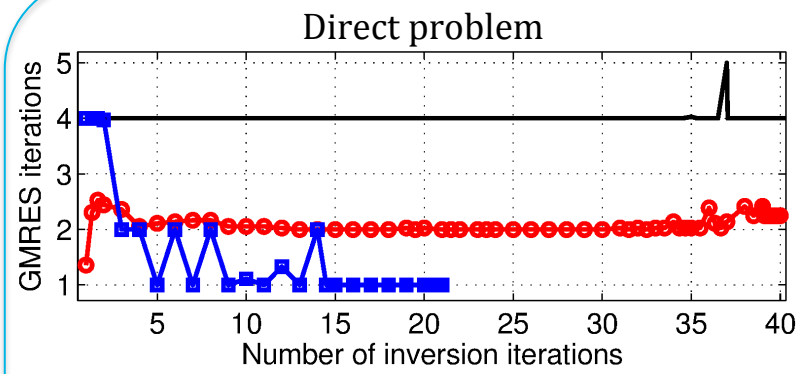
Without initialization

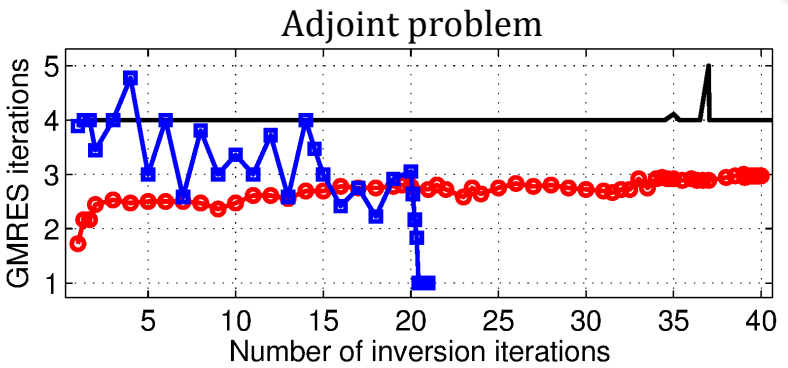
lambdaR initialization

Sources initialization

FETI optimization: starting point





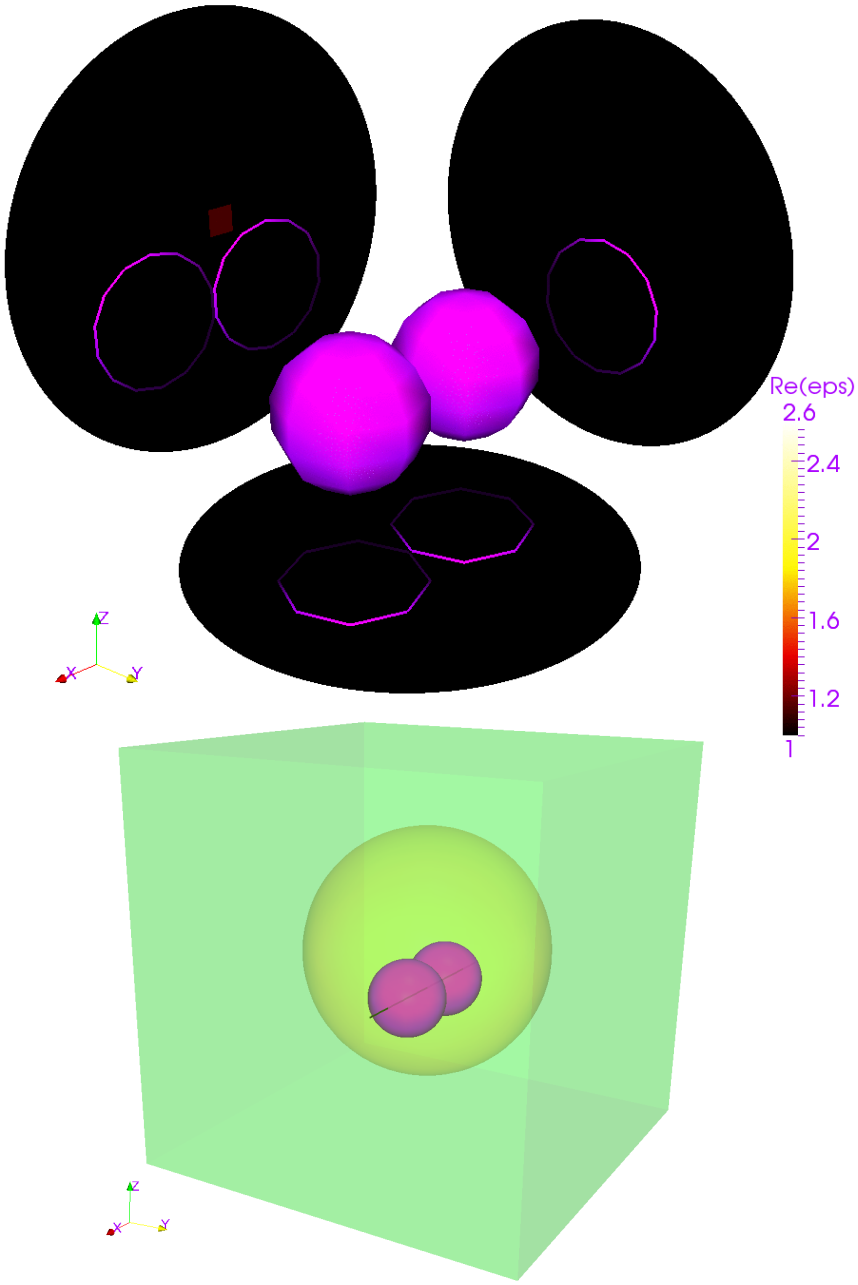


Without initialization

lambdaR initialization

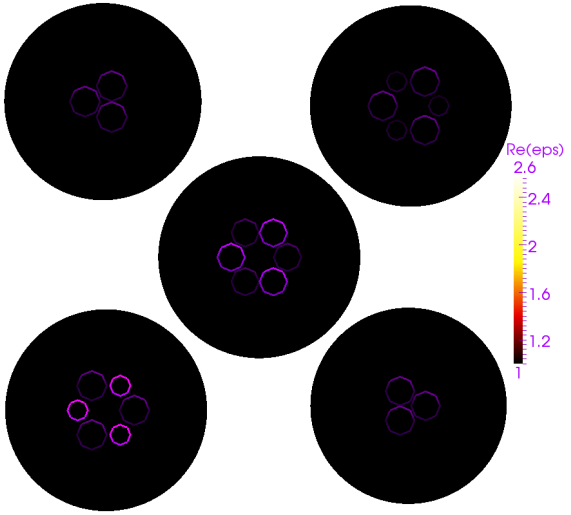
Sources initialization

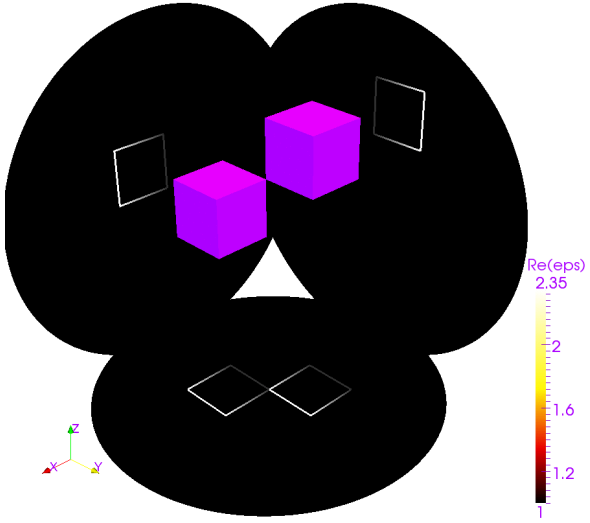
3D Inversion from measurements

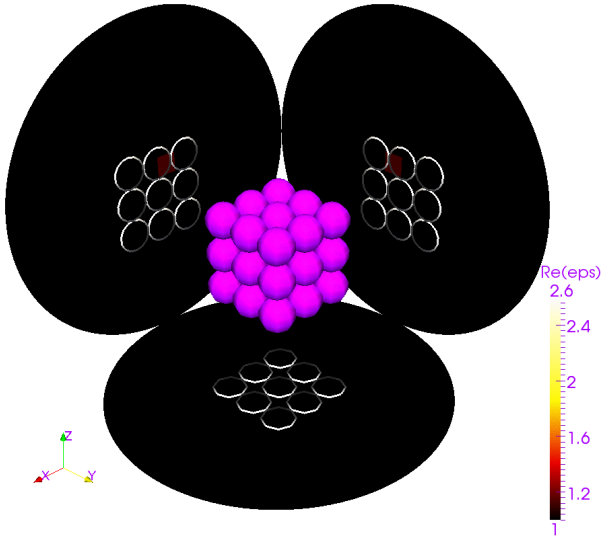


Conclusion

- Successful combination of the Inversion algorithm and FETI method







General conclusions and perspectives

Conclusions

- ❑ Development of the 3D FETI method
- ❑ Implementation to the Large-Scale
  - ❖ Direct problems
  - ❖ Inverse quantitative problems

Perspective

- ❑ Play with the transmission conditions
- ❑ Introduce *a-priori* information in inversion

Merci beaucoup