1st Franco-Japanese-Vietnamese Symposium on Singularities

Nice, September 16 - 21, 2013

MONDAY, September 16

14h30-15h30 : Takehiko Yasuda (Osaka)
Motivic integration and wild ramification
In this talk, we will discuss how to generalize the motivic integration to wild Deligne-Momford stacks with an eye to an application to the McKay correspondence in the wild case. For this purpose, we have to handle wild ramification and reach a weight function on twisted arc space generalizing the one used by Denef and Loeser. The function is closely related to the Artin conductor studied in the number theory. The talk is partially based on a joint work with Melanie Machett Wood.

16h00-17h00 : Wojciech Kucharz (Kraków)
Stratified-algebraic vector bundles (joint work with Krzysztof Kurdyka)
We investigate stratified-algebraic vector bundles on a real algebraic variety X. A stratification of X is a finite partition of X into Zariski locally closed subvarieties. A topological vector bundle on X is called a stratified-algebraic vector bundle if, roughly speaking, its restriction to each stratum of some stratification of X is an algebraic vector bundle on that stratum. It turns out that stratified-algebraic vector bundles have many surprising properties, which distinguish them from algebraic and topological vector bundles.

17h15-17h45 : Marcin Bilski (Kraków)
Higher order algebraic approximations of complex analytic sets.
We show that germs of complex analytic sets can be approximated by germs of complex algebraic sets with arbitrarily high order of tangency at any fixed point. In the real setting such a result has been recently obtained by M. Ferrarotti, E. Fortuna and L. Wilson who showed that germs of closed semi-analytic sets (of positive codimension) can be approximated by germs of real algebraic sets. The methods used in the complex case are completely different.

TUESDAY, September 17
Special session in honor of Shoji Yokura

9h00-10h00 : Jörg Schürmann (Münster)
Bivariant characteristic classes
In this talk we will survey our joint work Shoji Yokura on the theory of bivariant characteristic classes of singular spaces, starting some years ago with a uniqueness result for bivariant Chern and Todd class transformations. More recently, we constructed a universal oriented bivariant theory and motivic bivariant characteristic class versions of our motivic Chern and Hirzebruch classes for singular spaces. Finally, work in progress deals with a theory of fiberwise cobordism groups and their bivariant analogues in the context of differentiable spaces and manifolds.

10h30-11h30 : Clint McCrory (Athens Georgia)
Stiefel-Whitney homology classes
I will present old and new results, conjectures, and questions about Sullivan's Stiefel-Whitney classes of real algebraic varieties.

11h45-12h15 : Toru Ohmoto (Sapporo)
Shoji Yokura's works and Kagoshima
I will introduce works of Shoji Yokura about Chern class for singular spaces, bivariant theories, Milnor class, and so on, together with related things .

14h30 -15h30 : Shoji Yokura (Kagoshima)
Motivic Donaldson-Thomas type Hirzebruch classes
Donaldson-Thomas invariant is expressed as the weighted Euler characteristic of Behrend (constructible) function. Behrend also introduced a Donaldson-Thomas type invariant for a morphism. Motivated by this invariant, we defined motivic Donaldson-Thomas type Hirzebruch classes and considered Grothendieck-Riemann-Roch type formulas. In this talk I will explain about these classes. If time permits, I will talk about some other related topics. This is joint work with Vittoria Bussi.

16h -17h : Vittoria Bussi (Oxford)
Categorification of DT theory and Lagrangian intersection using perverse sheaves and motives
We study the behaviour of perverse sheaves of vanishing cycles under action of symmetries and stabilization, and we investigate to what extent they depend on the function which defines them. We investigate the relation between perverse sheaves of vanishing cycles associated to isomorphic critical loci with their symmetric obstruction theories, pointing out what Derived Algebraic Geometry has to say about that. Similar results are proved for mixed Hodge modules and motivic Milnor fibres. These results will be used to construct natural perverse sheaves, mixed Hodge modules and motives on moduli schemes of simple coherent sheaves on Calabi-Yau 3-folds equipped with "orientation data", giving a categorification of Donaldson-Thomas invariants. To prove it we need a "Darboux Theorem" for k-shifted symplectic derived schemes for all $k < 0$, which says in particular that a (-1)-shifted symplectic derived scheme (which includes moduli schemes of simple coherent sheaves on a Calabi-Yau 3-fold) is Zariski locally equivalent to the critical locus of a regular function on a smooth scheme. If time permits, we explain also how to construct natural perverse sheaf and motif on intersections of spin Lagrangians in a complex symplectic manifold, describing the relation with Fukaya categories and deformation-quantization.

WEDNESDAY, September 18

9h00-10h00 : Morihiko Saito (Kyoto)
Graded Koszul cohomology and pole order spectrum
Using the graded duality of the cohomology of the Koszul complexes defined by the partial derivatives of homogeneous polynomials with one-dimensional singular loci, we get some formulas for their Poincare series generalizing a well-known result in the isolated singularity case. We also show some relations with the spectrum in the sense of Steenbrink by introducing the pole order spectrum. This is by joint work with A. Dimca.

10h30-11h30 : Dinh Si Tiep (Hanoi)
Łojasiewicz inequality on some non-compact domains
Let $f(x)$ be a real analytic function such that $f(0) = 0$. Let $K \subset \mathbb{R}^n$ be a compact set. Then the Classical Łojasiewicz inequality says that there exist some constants $\alpha, c > 0$ such that for every $x \in K$, we have

$$|f(x)| \geq cd(x, V)^\alpha,$$

where $V = f^{-1}(0)$ and $d(., V)$ is the usual distance in R^n. In this talk, we present some recent results on Łojasiewicz inequality for polynomial functions and polynomial maps on some non-compact domain K. Then we give an application in the studies of singularities at infinity.

11h45-12h15 : Fabien Priziac (Brest)
Weight filtration for real algebraic varieties and group action
In 1974, P. Deligne established the existence of a weight filtration on the rational cohomology of complex algebraic varieties. An analog of this filtration for real algebraic varieties was introduced by Totaro in 2002. In a paper published in 2011, C. McCrory and A. Parusinski enriched the understanding of this real weight filtration, realizing it through a filtered chain complex, verifying properties that can be read on the induced spectral sequence. Consider now real algebraic varieties equipped with an algebraic group action. The functoriality of the weight complex allows to equip it with the induced action. This filtered weight complex with action is the first step for the construction of an equivariant weight filtration for real algebraic varieties with action. In this talk, we will establish several properties of these equivariant objects, notably in the case of the two-elements group. We will see that a result on the splitting of Nash manifolds implies a filtered version of the Smith short exact sequence. We use it to extract from some spectral sequence additive invariants on real algebraic varieties equipped with an algebraic involution.

14h30-15h30 : Ta Le Loi (Dalat)
On the Łojasiewicz inequalities in o-minimal structures
Łojasiewicz inequalities were introduced by Hörmander and Łojasiewicz in 1958-1962. From that time it has been recognized that they have many interesting relations to various branches of mathematics : Differential Analysis, Dynamical Systems, Algebraic Geometry, Optimization, ... This talk is devoted to the generalization of the inequalities for functions definable in o-minimal structures, which is, roughly speaking, a generalization for semialgebraic or global subanalytic functions. To compare two functions f and g on some set X we find a inequality of the form : $|g(x)| \geq \phi(|f(x)|)$, for $x \in X$, and ϕ being a reasonable function (e.g. a positive monotone function). The main idea is to consider the infimum of $|g|$ or the supremum of $|f|$ on the fibers of the other function, then investigate the behaviour of that optimal one-variable

function. In general, one can not get any desired inequality, e.g. consider the C^∞ functions given by $f(x) = e^{-\frac{1}{|x|}} sin\frac{1}{x}$, when $x \neq 0$, $f(0) = 0$, and $g(x) = f'(x)$ in a neighborhood of 0. However, if the objects involved are definable in a o-minimal structure, then one can use tame properties of them to estimate the optimal function (from below or from above) to get some useful inequalities. The talk is prepared in tree parts. In Part 1 we give some versions of Łojasiewicz inequalities to compare two definable functions in some neighborhoods of the zero-sets of the functions. Part 2 is devoted to o-minimal versions of gradient inequalities (Kurdyka- Łojasiewicz inequality and Bochnak-Łojasiewicz inequality), we also mention some incorrect results of D. D'Acunto and V. Granjean in "A Gradient Inequality at infinity for Tame Functions" , Rev. Mat. Complut. 18, No. 2 (2005). In Part 3 we give some applications of the inequalities.

16h00-16h30 : Nguyen Thi Bich Thuy (Marseille)
A singular set associated to a polynomial map (partially with Anna and Guillaume Valette)
In their paper (preprint) *"Geometry of polynomial mappings at infinity via intersection homology"*, Anna and Guillaume Valette constructed a pseudomanifold N_F associated to a polynomial map $F : \mathbb{C}^2 \to \mathbb{C}^2$ with nowhere vanishing Jacobian. They showed that if the intersection homology of the set N_F is not trivial, then F is not an isomorphism. This talk provides a generalization of this result and a method to stratify the set N_F for computing its intersection homology, in the case of a polynomial mapping $F : \mathbb{C}^n \to \mathbb{C}^n$.

16h45-17h15 : Nguyen Hong Duc (Kaiserslautern)
Right uni- and bi- modal singularities in positive characteristic
The problem of classification of real and complex singularities was initiated by Arnold in the sixties who classified simple, unimodal and bimodal singularities w.r.t. right equivalence. The classification of right simple singularities in positive characteristic was achieved by Greuel and Nguyen in 2012. We classify right unimodal and bimodal singularities in positive characteristic.

THURSDAY, September 19
Special session in honor of Michel Merle

9h00-10h00 : François Loeser (IMJ, Paris 6)
Motivic height zeta functions and motivic Poisson formula
The motivic height zeta function is a generating series of moduli spaces of curves on a polarized variety. It has been introduced by Peyre in analogy with Manin's problem on counting points of bounded height for varieties over number fields. In some cases, for instance equivariant compactifications of algebraic groups, adelic harmonic analysis allows to solve Manin's problem. For equivariant compactifications of vector spaces, using the motivic Poisson formula of Hrushovski and Kazhdan, we prove that (a version of) the motivic height zeta function is rational and determine its largest pole. We deduce in particular estimates for the growth of the dimension of the moduli spaces.This is joint work with Antoine Chambert-Loir.

10h30-11h30 : Kiyoshi Takeuchi (Tsukuba University)
Toric compactifications for polynomial maps and their applications
In this talk we will review our recent results on
1 : Bifurcation loci and singularities at infinity of polynomial maps
2 : Global monodromies of tame and non-tame polynomial maps
3 : Integral representations of confluent A-hypergeometric functions etc.
Nice toric compactifications for polynomial maps as well as methods from D-module theory will play key roles in their proofs. This is a joint work with Chen, Dias, Esterov, Matsui, Sabbah and Tibar.

11h45 - 12h15 : Le Quy Thuong (IMJ, Paris 6)
Motivic Milnor fiber from point of view of Hrushovski-Kazhdan's integration
In this talk, mentioned in a short survey some points in the work of Hrushovski-Loeser which indicate how to describe motivic Milnor fiber using Hrushovski-Kazhdan's integration in valued fields. We shall emphasis recent applications of this method in algebraic geometry, such as proofs of the integral identity conjecture, the motivic Thom-Sebastiani theorem, the motivic version of Steenbrink's conjecture and so on.

14h30-15h30 : Masahiko Yoshinaga (Hokkaido University)
Milnor fibers of real line arrangements.
The Milnor fiber of a line arrangement is a certain cyclic covering space of the complexified complement equipped with monodromy action. We will present a new algorithm computing eigen spaces of the first cohomology group with respect to the monodromy action. The algorithm uses real and combinatorial structures (chambers). I will also give some applications and several conjectures.

16h00-17h00 : Goulwen Fichou (Rennes)
Real Milnor fibres and Puiseux series
Given a polynomial with real coefficients, we define a weakly o-minimal object composed of Puiseux series that takes into account the homology of the real Milnor fibres. One goal is to rely it to motivic zeta function in the real context.

17h15-17h45 : Michel Raibaut (Chambéry)
Motivic Milnor fibers of a rational fonction

Let P and Q be two complex polynomials and f the induced rational function which is well defined out of the common zeros of P and Q. During the nineties Gusein-Zade, Luengo and Melle introduced Milnor fibers of the germs of f at an indeterminacy point, and gave an A'Campo's formula for its monodromy zeta function. They also studied the global Milnor fiber and in each case proved existence of a bifurcation set. In this talk using Denef-Loeser (and more recently Guibert-Loeser-Merle) motivic integration approach we will construct corresponding motivic Milnor fibers which are motives containing ad- ditive and multiplicative invariants of these Milnor fibers. As an example we will obtain the A'Campo's formula for the monodromy zeta function.

FRIDAY, September 20

9h00-10h00 : Hiroo Tokunaga (TMU)
Geometry of sections of elliptic surfaces and its application
We consider the geometry of sections of elliptic surfaces and elementary arithmetic on the Mordell-Weil group in order to investigate the existence problem of dihedral covers with given reduced curves as the branch loci. We apply our result to study problems on the topology of plane algebraic curves such as Zariski pairs.

10h30-11h30 : Tomohiro Okuma (Yamagata)
Good ideals for Gorenstein surface singularities
S. Goto, S. Iai, and K. Watanabe introduced the notion of good ideals in Gorenstein local rings as good ones next to the parameter ideals, and characterized good ideal for rational surface singularities in terms of desingularization. I will discuss the existence of good ideals for non-rational surface singularities. This is a joint work with Kei-ichi Watanabe and Ken-ichi Yoshida.

11h45-12h15 : Ann Lemahieu (Lille)
On the monodromy conjecture
The monodromy conjecture predicts a relation between the poles of certain zeta functions at a singular point of a complex hypersurface $X = V(f)$ and the eigenvalues of monodromy of f in a neighbourhood of that point. Thanks to the famous theorem of A'Campo, one can try to prove this conjecture by studying an embedded resolution of singularities of that singularity. In this talk I want to explain a strategy how to show the monodromy conjecture for the class of singularities that are nondegenerate with respect to their Newton polyhedron. In particular we show the monodromy conjecture for nondegenerate surface singularities.

14h00-15h00 : Anne Pichon (Marseille)
Bilipschitz geometry of complex surfaces and equisingularity
The question of defining a good notion of equisingularity of a reduced hypersurface $\mathfrak{X} \subset \mathbb{C}^n$ along a non singular complex subspace $Y \subset \mathfrak{X}$ in a neighbourhood of a point $0 \in \mathfrak{X}$ has a long history which started in 1965 with works of Zariski. One of the central concepts introduced by Zariski is the algebro-geometric equisingularity, called nowadays Zariski equisingularity, which defines the equisingularity inductively on the codimension of Y in $\mathfrak{X}$ by requiring that the reduced discriminant locus of a suitably general projection $p\colon \mathfrak{X} \to \mathbb{C}^{n-1}$ be itself equisingular along $p(Y)$.

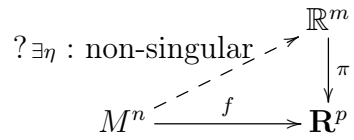
When Y has codimension one in $\mathfrak{X}$, *i.e.*, when dealing wih a family of plane curves transversal to the parameter space Y, it is well known that Zariski equisingularity is equivalent to the main notions of equisingularity such as Whitney conditions for the pair $(\mathfrak{X} \setminus Y, Y)$ and topological triviality. However these properties fail to be equivalent in higher codimension.

I will present a recent joint work with Walter Neumann in which we prove that in codimension 2, for a family of hypersurfaces in $\mathbb{C}^3$ with isolated singularities, Zariski equisingularity is equivalent to the constancy of the family up to bilipschitz semi-algebraic homeomorphism with respect to the outer metric.

15h30-16h30 : Osamu Saeki (Kyushu)
Desingularizing special generic maps

This is a joint work with Masamichi Takase (Seikei University, Japan).

Given a singular map $f : M^n \to \mathbb{R}^p$ of a closed manifold of dimension n with $n \geq p \geq 1$, we consider the following problem : for a standard projection $\pi : \mathbb{R}^m \to \mathbb{R}^p$ with $m > n \geq p \geq 1$, determine if there exists an immersion or embedding $\eta : M^n \to \mathbb{R}^m$ such that $f = \pi \circ \eta$.



Such a map η can be considered as a "resolution of singularities" of f.

For generic maps of surfaces into the plane, Haefliger (1960) obtained a necessary and sufficient condition for such a map to be lifted to an immersion into $\mathbb{R}^3$. Burlet-Haab (1985) showed that any Morse function on a surface can be lifted to an immersion into $\mathbb{R}^3$. Other than these, some results are known, but most of them concern maps between equidimensional manifolds.

In this talk, we consider special generic maps $f : M^n \to \mathbf{R}^p$ that have only definite fold as their singularities. For various dimension pairs (n, p), we give answers to the existence problem of immersion or embedding lifts into $\mathbf{R}^{n+1}$. In particular, for the cases where $p = 1$ and 2 we obtain complete results. Our techniques are related to Smale–Hirsch theory of immersions, topology of the space of immersions, relation between the space of topological immersions and that of smooth immersions, sphere eversions, differentiable structures of homotopy spheres, diffeomorphism group of spheres, free group actions on the sphere, etc.

16h45 - 17h15 : Shinzo Bannai (TMU)

Zariski N-plets for quartic-conic configurations from a view point of sub-configurations

We introduce a method to study the topology of the complement of reducible plane curves by considering sub-configurations of the curve. This method enables us to distinguish plane curves and lets us prove that two curves form a Zariski pair. We also give examples of quartic-conic configurations where the above method can be applied, and in effect give new examples of Zariski N-plets.