Influence of a mean shear on the spectrum of passive scalar fluctuations

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The form of the spectrum of passive scalar fluctuations in the inertial–convective range is investigated. In the presence of a mean shear, there exists a critical wave number k_c. At wave numbers greater than k_c, the mean shear does not influence the spectrum of scalar fluctuations and the Obukhov–Corrsin prediction holds. At wave numbers smaller than k_c, the mean shear dominates and the spectrum is less steep. These predictions are based on the study of Lagrangian dynamics of fluid particles and account for the experimental observations.

1 Introduction

The temperature of a fluid, the concentration of an atmospheric pollutant, the density of a dye are described mathematically by a scalar field depending on space and time. The scalar field satisfies the advection–diffusion equation with a possible source term. The scalar is referred as passive if it does not influence the carrier flow. The temperature field, for instance, can be regarded as passive if buoyancy effects are negligible.

A systematic study of the spectrum of scalar fluctuations in a turbulent flow was developed independently by Obukhov [1] and Corrsin [2]. They considered wave numbers k much greater than the ones associated with the integral scale of the flow and than the injection scale of the scalar, and much smaller than the wave numbers beyond which fluid viscosity and scalar diffusivity become important. This "intermediate" range is usually referred as the inertial–convective range, and corresponds to very large local Reynolds and Péclet numbers. Obukhov and Corrsin assumed small-scale homogeneity and isotropy in accordance with Kolmogorov's theory. Under this assumptions, they showed that the one-dimensional spectrum of scalar fluctuations, $E(k)$, is a power law with exponent $-5/3$. (It should be noted that the scaling of $E(k)$ in the inertial–convective range had been previously obtained by Onsager [3] and Weizsäcker [4] by dimensional arguments.)

The spectrum of scalar fluctuations has been examined in several experiments (see the reviews by Sreenivasan [5] and by Warhaft [6]). The scalar fields most often considered were the temperature field and the concentration field of a dye. Experiments do not support the Obukhov–Corrsin (OC) prediction, the deviation being stronger in shear flows than in grid generated turbulence. Furthermore, measurements performed with different mean flows give different scaling exponents for $E(k)$. For example, both the experiment by Mestayer [7] in an air-sea interaction wind tunnel and the one by Sreenivasan [5] in the wake of a circular cylinder show clear power-law scalar spectra in the inertial–convective range. The power law, however, is less steep than $k^{-5/3}$ in both cases, namely $k^{-1.49}$ in the former experiment and $k^{-1.33}$ in the latter one. Villermaux and collaborators even reported a spectrum $E(k) \propto k^{-1}$ for a dye released in a turbulent jet [8]. In general, the experimental observations can be summarised as follows [5, 6]: a) in the inertial–convective range the spectrum of scalar fluctuations is a power law $E(k) \propto k^{-\alpha}$ with $\alpha < 5/3$; b) the scaling exponent α depends on the characteristics of the mean flow; c) the deviation from the OC prediction decreases with increasing Reynolds number; d) the deviation from the OC prediction is stronger for shear flows than for grid turbulence.

2 Spectrum of scalar fluctuations

In the presence of a mean shear, the experimental observations can be explained by the existence of a critical wave number $k_c \equiv (\sigma/D)^{3/2}$, where σ denotes the amplitude of the mean shear and D measures the intensity of turbulent fluctuations ($\sqrt{\langle[\boldsymbol{u}(\boldsymbol{x}+\boldsymbol{r}) - \boldsymbol{u}(\boldsymbol{x})]^2\rangle} \sim Dr^{1/3}$). At wave numbers smaller than k_c, the statistics of scalar fluctuations is dominated by the shear, the spectrum is less steep than $k^{-5/3}$, and the scaling exponent depends on the details of the mean flow. At scales greater than k_c, turbulent fluctuations prevail, small-scale isotropy is restored, and the OC prediction is recovered (see Fig. 1).

The form of $E(k)$ can be obtained by studying the Lagrangian dynamics of fluid particles. We consider the equal-time structure function of scalar fluctuations, $S_2(r) \equiv \langle[\theta(\boldsymbol{x}+\boldsymbol{r},t) - \theta(\boldsymbol{x},t)]^2\rangle$, where θ denotes the fluctuating component of the scalar field. The scaling of $E(k)$ is related to that of $S_2(r)$: $S_2(r) \propto r^{\alpha-1}$ corresponds to $E(k) \propto k^{-\alpha}$ (see e.g. Ref. [9]). The

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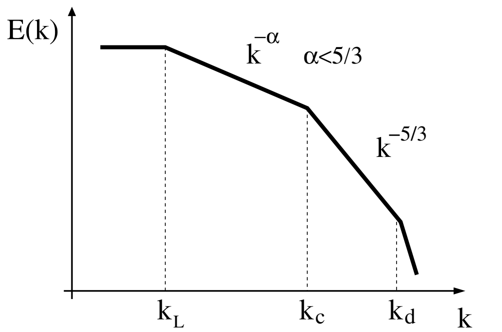

Fig. 1 Schematic representation of the spectrum of scalar fluctuations in the presence of a mean shear; k_L denotes the wave number associated with the integral scale of the flow and k_d is the diffusive wave number.

structure function $S_2(r)$ has a simple Lagrangian interpretation: it is proportional to the time $T_{\mathcal{L}}(r)$ needed for two coinciding particles to separate to a distance r [10]. For a given mean flow, it is easy to compute how $T_{\mathcal{L}}(r)$ scales with the separation r; one just have to consider the evolution equation for Lagrangian trajectories. Knowing the scaling exponent of $T_{\mathcal{L}}(r) \propto S_2(r)$ thus allows the scaling exponent of $E(k)$ to be computed. If, for example, the mean shear is constant in time and space, then $E(k) \propto k^{-\alpha}$ with $\alpha = 4/3$ for $k \ll k_c$ and $E(k) \propto k^{-5/3}$ for $k \gg k_c$. Furthermore, $\alpha = 13/9$ for a mean shear with constant direction and rapidly changing intensity, and $\alpha = 1$ for a rapidly rotating shear. In all these cases, the scaling exponent α is smaller than 5/3 for wave numbers less than k_c, while the OC law is obtained at greater k.

Detecting two power laws and showing the existence of the critical wave number may be difficult in numerical simulations of the advection–diffusion equation coupled with the Navier–Stokes equations. To confirm our predictions, we therefore considered a random velocity field which can reproduce some of the properties of turbulent flows and, at the same time, allows a semi-analytical computation of $E(k)$. We studied a scalar field transported by a random flow with Kraichnan's statistics [11] (see also Ref. [10]). The velocity field is Gaussian, zero-mean, white-in-time, and has a power law spectrum. Kraichnan's model has the advantage that if also the forcing driving the scalar is white-in-time, then the scalar structure function (and hence the spectrum) satisfies a partial differential equation. Solving such partial differential equation numerically is less demanding than performing direct simulations of the advection–diffusion equation and the Navier–Stokes equations. Moreover, the Reynolds number is formally infinite for Kraichnan's model, and therefore it is in principle easier to resolve two power laws in the scalar spectrum. The numerical computation of the spectrum of scalar fluctuations in Kraichnan's model confirm our predictions and their interpretation in terms of Lagrangian dynamics [12]. We note that the crossover region between the two scaling behaviours separated by k_c is quite wide; detecting the transition between them thus requires a broad range of scales.

The implications of our study may be summarised as follows: a-b) a mean shear influences the statistics of scalar fluctuations in the inertial–convective range up to a critical wave number k_c depending on the relative intensity of the mean flow and turbulent fluctuations. Owing to the limited range of accessible scales, experiments have measured a power law resulting from a combination of $k^{-5/3}$ for $k \gg k_c$ and $k^{-\alpha}$ with $\alpha < 5/3$ for $k \ll k_c$, and hence a scalar spectrum less steep than $k^{-5/3}$ and with slope depending on the mean flow; c) k_c decreases with increasing Reynolds number. The experimental slope, therefore, becomes closer and closer to $-5/3$ with increasing Reynolds number, and reaches the OC prediction when k_c becomes smaller than the inverse of the integral scale of the flow; d) in grid turbulence, the flow is nearly isotropic, and therefore there is no strong mean flow that could influence the scalar statistics in the inertial–convective range, hence the better agreement with the OC theory. These results account for the form of the scalar spectra observed in experiments.

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