Université de Nice-Sophia Antipolis
L3 MIAGE - Processus stochastiques et analyse de données
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Examen, corrigé du sujet C

Durée : 1h. Documents, calculatrices et téléphones interdits.

1. $\mathbb{P}(\{X = 3\} \cup \{Y = 4\}) = \mathbb{P}(Y = 3) + \mathbb{P}(Y = 4) = C_4^3(1/4)^3(3/4) + C_4^4(1/4)^4 = 4\frac{1}{4^3}\frac{3}{4} + \frac{1}{4^4} = \frac{13}{4^4} = \frac{13}{256}$
2. $\mathbb{P}(X \in [0; 2]) = \int_0^2 2e^{-2x}dx = [-e^{-2x}]_0^2 = 1 - e^{-4}$
3. $\mathbb{P}(Z > 2,86) = 1 - \mathbb{P}(Z \leq 2,86) = 1 - 0,9979 = 0,0021$
4.
   (a) La variable Z peut prendre les valeurs : 0, -1, 2, 1.
   (b) $\mathbb{P}(Z = 0) = \mathbb{P}(X = 0, Y = 0) = (1/3)(1/4) = 1/12$, $\mathbb{P}(Z = -1) = \mathbb{P}(X = 0, Y = -1) = (1/3)(3/4) = 1/4$, $\mathbb{P}(Z = 2) = \mathbb{P}(X = 2, Y = 0) = (2/3)(1/4) = 1/6$, $\mathbb{P}(Z = 1) = \mathbb{P}(X = 2, Y = -1) = (2/3)(3/4) = 1/2$
5.
   (a) $\mathbb{E}(X) = 0 \times (1/6) + 1 \times (4/6) + (-1) \times (1/6) = 3/6 = 1/2$
   (b) $\mathbb{E}(X^2) = 0^2 \times (1/6) + 1^2 \times (4/6) + (-1)^2 \times (1/6) = 5/6$
   (c) $\mathrm{Var}(X) = \frac{5}{6} - \frac{1}{4} = \frac{10-3}{12} = \frac{7}{12}$
6. $C_4^2(1/2)^4 = \frac{4\times 3}{2}\frac{1}{2^4} = \frac{3}{2^3} = \frac{3}{8}$
7. Notons F la fonction de répartition de X.
   (a) $\mathbb{P}(X \leq 1/2) = F(1/2) = 1/2$
   (b) $\mathbb{P}(X < -1/2) = F(-1/2) = 0$
   (c) $\mathbb{P}(X \leq 2) = F(2) = 1$