Université Nice Sophia-Antipolis
Statistical leaarning, 2020-2021
`https://math.unice.fr/~rubentha/cours.html`

Answers for final examination, (MPA $\cup$ MATHMODS), A

Documents and calculators forbidden.

1. Quiz

(1) a
(2) a
(3) b
(4) a
(5) c
(6) a
(7) a
(8) a
(9) c
(10) a

2. Exercises

Exercise 1.

(1) We compute

$$\begin{aligned} \mathbb{P}((x_k),\theta|(y_k)) &= \frac{\mathbb{P}((y_k)|(x_k),\theta)\mathbb{P}(x_k,\theta)}{\mathbb{P}((y_k))} \\ \text{(proportional to)} &\propto \mathbb{P}((y_k)|(x_k),\theta)\mathbb{P}(x_k,\theta) \\ &= \mathbb{P}((y_k)|(x_k),\theta)\mathbb{P}(x_k,\theta)\mathbb{P}(\theta)\,. \end{aligned}$$

(2)

- State space : the space of $(x_0,\dots,x_n,\theta)$, that is $\mathbb{R}^{n+2}$.
- Proposal kernel : we choose $Q(x_0,\dots,x_n,\theta;\dots) = \bigotimes_{k=0}^n \mathcal{N}(x_k,1/2)\bigotimes\mathcal{N}(\theta,1/2)$ (we do what we want).
- Target law : $\mathbb{P}((x_k)_{0\le k\le n},\theta|(y_k)_{0\le k\le n})$.

Exercise 2.

(1) The i-th term in the gradient of H is

$$\frac{\partial H}{\partial z_i} = -\frac{y_i}{z_i}\,.$$

So

$$\nabla H = -\begin{pmatrix} \frac{y_1}{z_1} \\ \vdots \\ \frac{y_n}{z_n} \end{pmatrix}\,.$$

(2) The i-th term in the gradient of F is

$$\frac{\partial F}{\partial z_i} = 1\,.$$

So

$$\nabla F = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}\,.$$

(3) The candidates are the points where ∇H and ∇F are co-linear. This means there exists $\lambda \in \mathbb{R}$ such that, for all i,

$$\frac{y_i}{z_i} = \lambda\,.$$

Due to the constrain, there is only one possible λ: $\lambda = 1$.

(4) Our function H is convex (as a sum over k of convex functions of z_k) so our candidate is an absolute minimum.