

# Degrees of parallelism and Boolean algebras

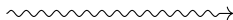
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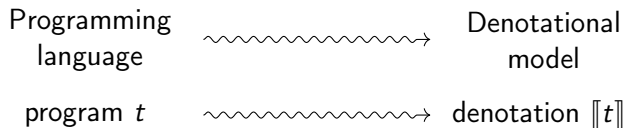
# Denotational models

Programming  
language

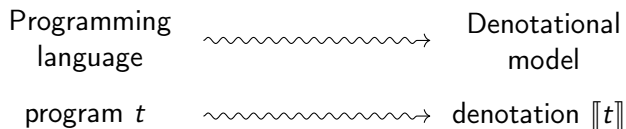


Denotational  
model

# Denotational models

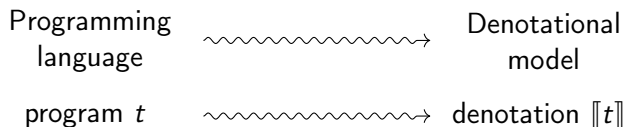


## Denotational models



$$\llbracket (\text{fun } x \rightarrow t) u \rrbracket = \llbracket t[x := u] \rrbracket$$

## Denotational models



$$\llbracket (\text{fun } x \rightarrow t) u \rrbracket = \llbracket t[x := u] \rrbracket$$

etc.

## Left-first or

```
fun x y → if (x) then true  
         else y
```

## Left-first or

fun  $x\ y$   $\rightarrow$  if ( $x$ ) then true  
else  $y$

$y \backslash x$	true	false
true	true	true
false	true	false

## Left-first or

fun  $x\ y \rightarrow$  if ( $x$ ) then true  
else  $y$

$y \backslash x$	true	false
true	true	true
false	true	false
?	true	



## Left-first or

fun  $x\ y$   $\rightarrow$  if ( $x$ ) then true  
else  $y$

$y \backslash x$	true	false
true	true	true
false	true	false
?	true	?

## Left-first or

fun  $x\ y \rightarrow$  if  $(x)$  then true  
else  $y$

$y \backslash x$	true	false	?
true	true	true	?
false	true	false	?
?	true	?	?

## Right-first or

fun  $x \ y \rightarrow$  if  $(y)$  true  
else  $x$

$y \backslash x$	true	false	?
true	true	true	true
false	true	false	?
?	?	?	?

## Parallel (symmetric) or

$y \backslash x$	true	false	?
true	true	true	true
false	true	false	?
?	true	?	?

# Parallel (symmetric) or

fun x y → ???

$y \backslash x$	true	false	?
true	true	true	true
false	true	false	?
?	true	?	?

# Voting function

true	true	?	→	true
?	true	true	→	true
true	?	true	→	true
false	false	?	→	false
?	false	false	→	false
false	?	false	→	false

Voting and parallel or can simulate each other

Voting and parallel or can simulate each other

$$\overleftarrow{\text{or}} = \text{fun } x \ y \rightarrow \text{vote } (\overrightarrow{\text{or}} \ x \ y) \ (\overleftarrow{\text{or}} \ x \ y)$$



## Voting and parallel or can simulate each other

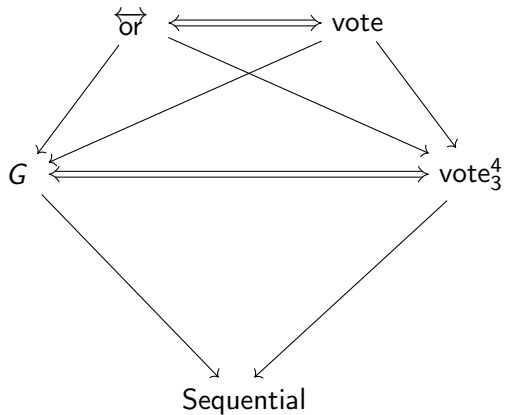
$\overleftarrow{\text{or}} = \text{fun } x \ y \rightarrow \text{vote } (\overrightarrow{\text{or}} \ x \ y) \ (\overleftarrow{\text{or}} \ x \ y) \ \text{true}$

## Voting and parallel or can simulate each other

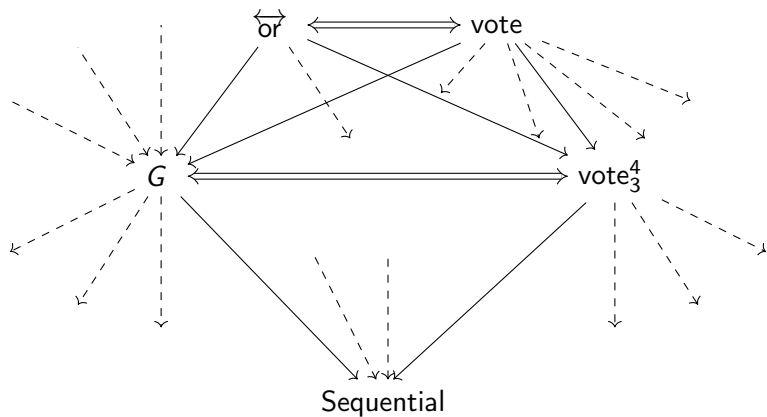
$\overleftarrow{\text{or}} = \text{fun } x \ y \rightarrow \text{vote } (\overrightarrow{\text{or}} \ x \ y) \ (\overleftarrow{\text{or}} \ x \ y) \ \text{true}$

$\text{vote} = \text{fun } x \ y \ z \rightarrow \overleftarrow{\text{or}} \left( \overleftarrow{\text{or}} \left( (\overleftrightarrow{\text{and}} \ x \ y) \ (\overleftrightarrow{\text{and}} \ y \ z) \right) \right) \ (\overleftrightarrow{\text{and}} \ z \ x)$

## Degrees of parallelism



## Degrees of parallelism: the wild west



## Degrees of parallelism and Boolean algebras

Very particular  
denotational models



Formulas in the language  
of Boolean algebras

# Degrees of parallelism and Boolean algebras

Very particular  
denotational models



Formulas in the language  
of Boolean algebras

denotation  $d$



formula  $F_d$

# Degrees of parallelism and Boolean algebras

Very particular  
denotational models



Formulas in the language  
of Boolean algebras

denotation  $d$



formula  $F_d$

$d$  simulates  $e$

*iff*

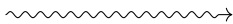
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Very particular  
denotational models



Formulas in the language  
of Boolean algebras

denotation  $d$



formula  $F_d$

$d$  simulates  $e$

*iff*

$F_d$  entails  $F_e$