Take a generic curve $C$ in a linear system $|L|$ on a toric surface $X$. Can any simple closed curve in $C$ be contracted to a nodal point along a degeneration of $C$ inside $|L|$? Equivalently, is any simple closed curve a vanishing cycles of $C$ (relatively to $|L|$)? The latter question is equivalent to the surjectivity of the monodromy map from the complement of the discriminant $D \subset |L|$ into the mapping class group of $C$. The answer to this question depends on the pair $(X, L)$ and amounts to an obstruction/construction business.

In this talk, we will determine all the $(X, L)$ for which the monodromy is surjective. To this aim, we will construct explicit elements in the image of the monodromy by tropical means. If time permits, we will also discuss the image of the monodromy for obstructed pairs $(X, L)$. 