Bending metal sheets, Riemann surfaces and integrable systems

When you bend a metal sheet, without stretching, it deforms through isometric immersions of a Riemannian metric. Problem: for which surfaces is the differential equation of isometric immersion an integrable system? We find the first examples. We use ideas of Darboux relating complex geometry and integrable systems. Joint work with Jeanne Clelland, Tom Ivey and Peter Vassiliou.