Traffic Flow on Single Links with Bottlenecks:
Variational Theory, Analysis,
Application and Empirical Evidence

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References

A Single Stream: Description

Moskowitz's Function

\[ n = N(t,x) \]
A Single Stream: Prediction

Solution Domain, $S$

Boundary, $D$

$N_B(t, x)$

$N_P = N(t, x)$

$P = (t, x)$
Variational Theory (VT): Basic Ideas

\[ N(t,x) \text{ is } L\text{-Continuous} \quad \frac{\Delta N}{\Delta x} \in [0, \kappa] \quad \frac{\Delta N}{\Delta t} \in [0, q_{\text{max}}] \]

Relative Capacity

\[ r(u) \]

\[ q_{\text{max}} \]

\[ N_P \text{ is largest possible subject to capacity constraints} \]

(Daganzo, 2005; 2006)
Variational Theory (VT): Mathematical Expression

- Observer $\equiv$ Valid Path: $\mathcal{P}$, $x(t)$
- Observer Speed: $x'(t)$
- Observer Bound:
  $$\int_{\mathcal{P}} r(x') dt \equiv \Delta(\mathcal{P})$$
- Observer Constraint:
  $$N_P \leq N_{B(\mathcal{P})} + D(\mathcal{P})$$

VT Expression: $N_{P}^{\text{VT}} = \inf \{ N_{B(\mathcal{P})} + D(\mathcal{P}) : \forall \mathcal{P} \}$

(Daganzo, 2005;2006)
Simple Case: Linear

Relative Capacity

\[ q_{\text{max}} \]

\[ r(u) = q_{\text{max}} - k_0 u \]

Bound is Path-Independent:

\[ D(\mathcal{P}) = q_{\text{max}} \Delta t - k_0 \Delta x \]

(Daganzo, 2005a)
Network Solution with Dynamic Programming (DP)

Sufficient network:

\[ x \]

\[ t \]

(Daganzo, 2005a)
Network Solution with Dynamic Programming (DP)

Sufficient network:

\[ N_P = \min \{ N_{P'}, N_{P''} + r\varepsilon'' \} \]

Stencil:
Bottlenecks in VT

Sufficient network with shortcut:

(Daganzo, 2005a); (Daganzo & Menendez, 2005b)
Bottlenecks in VT

Sufficient network with shortcut:

$B, x_B(t), r_B(t)$

Still a DP Problem

(Daganzo, 2005a); (Daganzo & Menendez, 2005b)
Questions about $N^{VT}(t,x)$

1. Well-posed?

2. Related to known models: KW ; CF ?

3. Realistic?
(Q1) Well-posed?

Initial Value Problem

Finite Highway Problem

Composite Highway Problem

(Daganzo, 2006)
(Q1) Well-posed?

**Assume:** Data are L-Continuous  
Bottlenecks have non-negative speeds & rel. capacities

Then:

Well-posed  
Well-posed if:  
\[ N_P \geq N_P^{VT} \]  
Well-posed if:  
\[ N_P = \min \{ N_P^U, N_P^D \} \]
(Q2) Related to KW?

\[ \frac{\partial N}{\partial t} = Q \left( -\frac{\partial N}{\partial x} \right) \]

Flow \hspace{1cm} Density

(Daganzo, 2005; 2006)
(Q2) Related to KW?

\[ \frac{\partial N}{\partial t} = Q \left( -\frac{\partial N}{\partial x} \right) \]

Flow \quad Density

KW has a relative capacity function

(Daganzo, 2005; 2006)
(Q2) Related to KW?

Solution must satisfy the relative capacity constraint:

\[ N_P^{KW} \leq \inf \left\{ N_{B(P)} + \Delta(P) : \forall P \right\} = N_P^{VT} \]

Solution defined by waves that satisfy:

\[ N_P^{KW} = N_{B(w)} + \Delta(W) \geq \inf \left\{ N_{B(P)} + \Delta(P) : \forall P \right\} = N_P^{VT} \]

Hence: \( N_P^{KW} = N_P^{VT} \)
(Q2) Related to CF?

Expressions of the same $M$ to be found

Vehicles at given positions  Positions of given vehicles

$N(t,x)$  $X(t,n)$
(Q2) Related to CF?

\[ N(t,x) \]

\[ X(t,n) \]

Flow vs. Speed

\[ r(u) \]

Speed vs. Flow

\[ r(u) \]

(Daganzo, 2006)

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(Q2) Related to CF?

\[ N_P = \min\{N_P, N_P^{\prime} + r\varepsilon^{\prime}\prime\} \]

\[ X_P = \min\{X_P, + v\varepsilon^{\prime} ; X_P^{\prime} - w\varepsilon^{\prime}\prime\} \]
(Q3) Realistic?

Right lane

Left lane

(Laval & Daganzo, 2006)
(Q3) Realistic?

Method is parsimonious:
- Optional moves $\rightarrow$ 1 behavior parameter (when)
- Mandatory moves $\rightarrow$ 1 behavior parameter (where)
A Lane Drop Bottleneck

(Laval & Daganzo, 2006)
Capacity of Moving Bottlenecks

(Munoz & Daganzo, 2002); (Laval & Daganzo, 2006)
Causes of Moving Bottlenecks

(Laval & Daganzo, 2002)
HOV and On-ramps: Mandatory and Optional Moves

Data

Cumulative flows

Lane changes

Model

Cumulative flows

Lane changes

(Laval & Cassidy and Daganzo, 2006)
(Menendez & Daganzo, 2007)
Discharge through bottleneck is greater with separation of traffic than without.

(Menendez & Daganzo, 2007)
The Smoothing Effect

(Cassidy & Daganzo & Jang and Chung, 2006)
Summary

- Variational Theory unifies different views of traffic
- Parallel streams can be composed with VT
- Results are parsimonious and realistic
- Larger systems, scaling?