A dynamic packet-based approach for traffic flow simulation and dynamic traffic assignment

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October 31, 2007

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Motivation

1. Solving simulation-based dynamical traffic assignment problem with travel cost which is non differentiable in multimodal transportation network.
2. Dynamic traffic simulation in large scale multimodal transportation network.
   - Packet-based macroscopic road traffic simulation
   - Public transportation system using Lagrangian coordinates

Traffic assignment problems

1. Travelers behavior hypothesis: maximization of utility/minimization of cost
2. Predictive dynamic user equilibrium condition of Wardrop: "*If, for any travellers between any OD pair leaving their origin at any instant, the actual travel times that these travellers experienced on any used routes are equal and minimal; and the actual travel times that these travellers would experience on any unused routes are greater than or equal to the minimum actual travel time on used routes.*"

User equilibrium conditions of Wardrop

$$
\begin{aligned}
& f_r = 0, \quad \textit{if} \quad C_r(\mathbf{f}) > \pi_k, \quad \forall r \in R_k, \quad \forall k \\
& f_r > 0, \quad \textit{if} \quad C_r(\mathbf{f}) = \pi_k, \quad \forall r \in R_k, \quad \forall k \\
& \sum_{r \in R_k} f_r = d_k, \quad \forall k \\
& f_r \geq 0, \quad \forall r
\end{aligned}
$$

where $\pi_k = \min_{r \in R_k} C_r(\mathbf{f}), \quad \forall k$
r:path
k: pair of origin and destination
f_r: flow over path r
$C_r(\mathbf{f})$: cost of path r

Existing solution methods

- Method of successive averages [Patriksson, 1994]
- Projection-type approaches [Nagurney et Zhang, 1996]

$$f_r = \mathrm{P}_\Omega[f_r - t(C_r(\mathbf{f}) - \pi_k(\mathbf{f})], \quad \forall r \in R_k$$

- Dynamical system approach [Simth, 1984, Jin, 2007]

$$\dot{f}_r = \sum_s f_s(C_s(\mathbf{f}) - C_r(\mathbf{f}))_+ - \sum_s f_r(C_r(\mathbf{f}) - C_s(\mathbf{f}))_+, \quad \forall r, s \in R_k, \forall k$$

where

$$(y)_+ = y, \quad \text{if } y \geq 0; \quad 0, \quad \text{otherwise}$$

- Logit choice model based approaches

$$f_r = d_k * \frac{e^{-C_r(\mathbf{f})}}{\sum_{s \in R_k} e^{-C_s(\mathbf{f})}}, \quad \forall r \in R_k, \quad \forall k$$

Cross Entropy method [Rubinstein, 1999]

1. Originated from Rare Event simulation technique for solving combinatorial and continuous multi-extremal optimization problems.
2. The CE method for Rare Event simulation
   - Generate a set of samples based on probability mechanism,
   - Choosing optimal parameters based on sampling data to produce better estimation in the next iteration
3. Numerous applications based on CE method to solve NP-hard problems.

The CE method for Rare Event simulation

- Let ℓ be the expected performance of a stochastic system:

$$\ell = \mathbb{E}_f(H(\mathbf{x})) \tag{1}$$

where $H(x) = \mathbf{I}_{\{S(\mathbf{x}) \geq \gamma\}}$ or $e^{-S(\mathbf{x})/\gamma}$

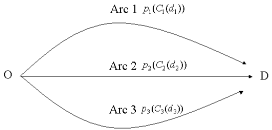
- By changing measure from f to g, we get:

$$\mathbb{E}_f(H(\mathbf{x})) = \mathbb{E}_g(H(\mathbf{x})\frac{f(\mathbf{x})}{g(\mathbf{x})}) \tag{2}$$

- The optimal Importance Sampling probability is derived based on minimization of Kullback-Liebler cross entropy between two probability distributions:

$$g^*(\mathbf{x}) = \arg\min_g D(f,g) = \arg\min_g \mathbb{E}_f ln\frac{f(\mathbf{x})}{g(\mathbf{x})} \tag{3}$$

Solving traffic assignment problems based on CE



- The performance function of path i is defined by Boltzmann function :

$$H_i(\gamma) = e^{-C_i(d_i)/\gamma} \quad (4)$$

where γ is a temperature parameter.

- The average performance of this distribution is defined by :

$$h(p,\gamma) = \sum_{i \in I} p_i H_i(\gamma) \quad (5)$$

$$\text{subject to } \sum_{i \in I} p_i = 1, p_i \geq 0, \forall i \in I \quad (6)$$

Solving traffic assignment problems based on CE

Derivation of optimal *pdf* by minimizing CE

- The optimal $\mathbf{P}^*$ for next iteration is derived based on minimizing cross entropy :

$$\mathbf{P}^* = \arg\max_{\mathbf{P}} \mathbb{E}_{\mathbf{P}^{w-1}}[H(\gamma) \ln \mathbf{P}] \tag{7}$$

  subject to eq. (6)
  where w denotes an iteration.

- The solution of the previous minimization problem is :

$$p_i^w = p_i^{w-1} \times \frac{e^{-C_i^{w-1}/\gamma^w}}{\sum_{j \in I} p_j^{w-1} e^{-C_j^{w-1}/\gamma^w}}, \forall i \in I \tag{8}$$

Solving traffic assignment problems based on CE

Determine the parameter γ

- To determine the temperature parameter γ^w at each iteration, we consider following minimization problem :

$$\min \gamma^w \text{ subject to } \sum_{i \in I} |p_i^w - p_i^{w-1}| \leq \alpha^w \tag{9}$$

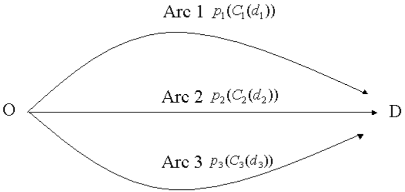
where α^w is a divergent series: $\sum_w \alpha^w = \infty, \quad \alpha^w \to 0, \quad$ as $w \to \infty$

- By applying first order approximation of e^x, we can get the following result:

$$\dot{p} = -p(C - \overline{C}) \tag{10}$$

Solving nonlinear STAP based on CE

Numerical studies : example network 1 [Jin, 2007]



$$t_1 = 10(1.0 + 0.15(\frac{x_1}{2})^4)$$
$$t_2 = 20(1.0 + 0.15(\frac{x_2}{4})^4)$$
$$t_3 = 25(1.0 + 0.15(\frac{x_3}{3})^4)$$

$$x_1 + x_2 + x_3 = 10$$

where x is link flows, t is travel time.

Solving nonlinear STAP based on CE

Convergence results

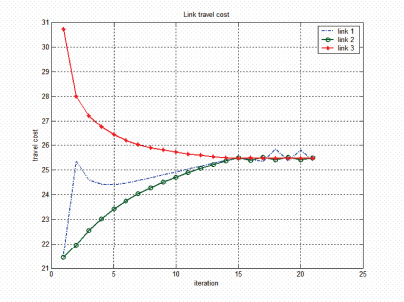


Figure: Link travel time evolution

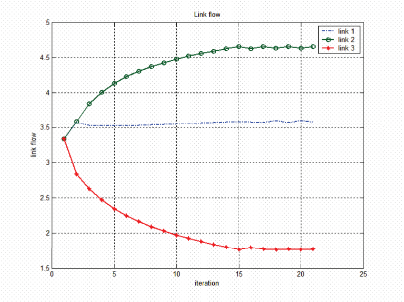
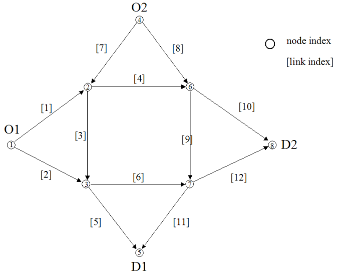


Figure: Link flow evolution

Numerical example

non linear static network

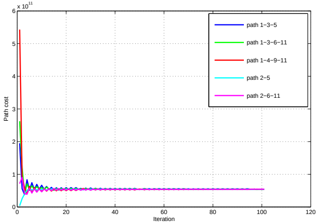


Link cost function c_a is defined by :

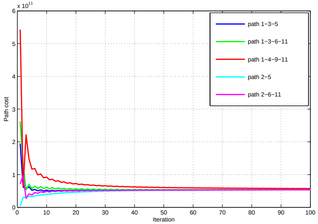
$$c_a = \alpha_a(\beta_a + \gamma_a(\frac{x_a}{\delta_a})^4)$$

where a denotes an arc

Comparative study of CE method and dynamical system approach

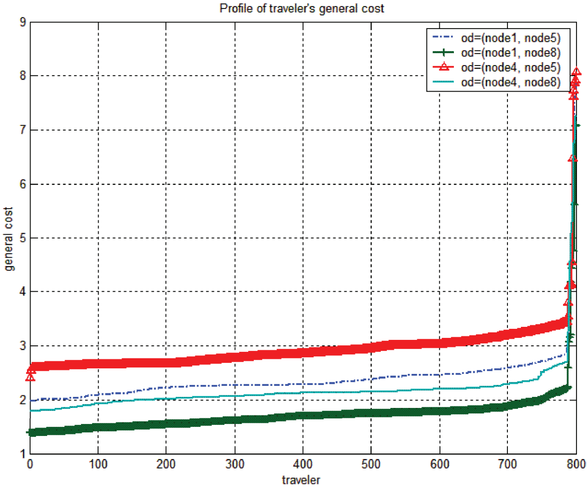


(CE method)

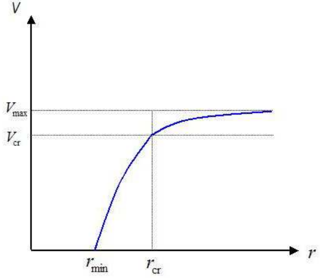


(Dynamical system approach [Smith, 1979])

Travelers' OD cost based on queue model



Change of coordinates



LWR model in Lagrangian coordinates (Aw et al.,2000):

$$\begin{cases} \partial_t r + \partial_N v = 0 \\ v = V_e(r, x) \end{cases}$$

where $N = \int_x^{\infty} \rho(x,t)dt$
r: spacing of two consecutive vehicles
v: vehicle speed

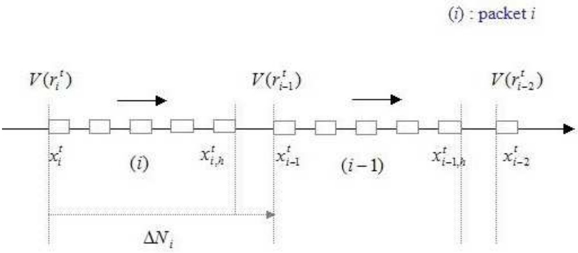
Lagrangian discretization based on Godunov scheme

$r_i^{t+1} = r_i^t + \frac{\Delta t}{\Delta N_i}(V(r_{i-1}^t) - V(r_i^t))$

where

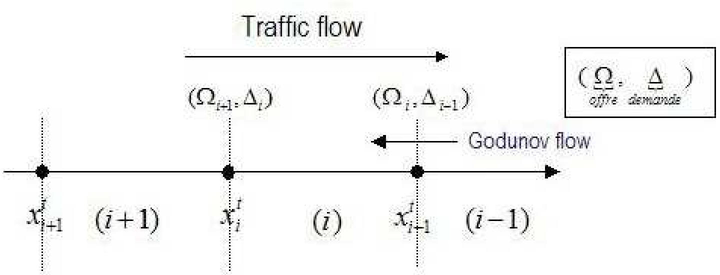
$r_i^t = \frac{x_{i-1}^t - x_i^t}{\Delta N_i}$

ΔN_i: packet of vehicles

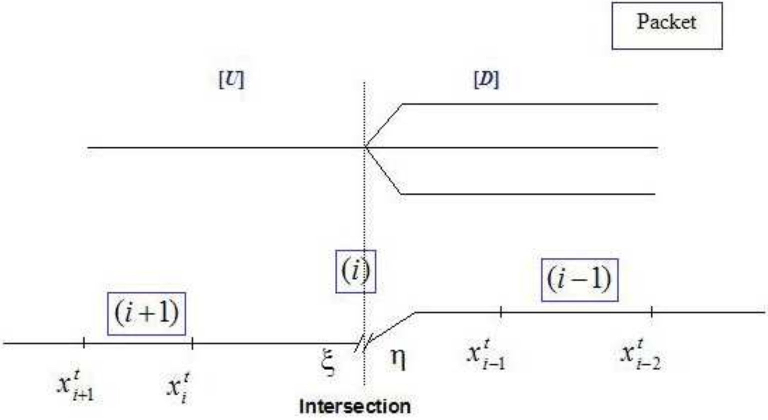


Supply-demand approach

$$\begin{cases} x_i^{t+1} = x_i^t + \Delta t v_i^t \\ v_i^t = \min\{V(\frac{N_i}{x_{i-1}^t - x_i^t}, \frac{x_{i-1}^t + x_i^t}{2}), V_{max}(\frac{x_i^t + x_{i+1}^t}{2})\} \end{cases}$$



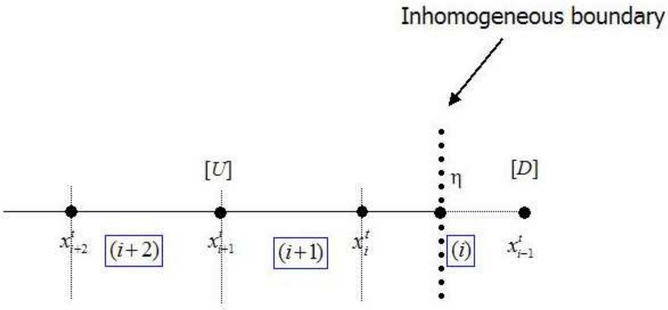
Intersection modeling: divergent model



Inhomogeneous link (1)

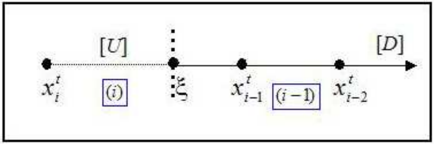
V_{max} (supply in Godunov flow) change at the boundary:

$$V_{max} = \begin{cases} V_{max}(\frac{x_i^t + x_{i+1}^t}{2}), & if \quad \frac{x_i^t + x_{i+1}^t}{2} < \eta \\ V_{max}(\eta), & otherwise \end{cases}$$



Inhomogeneous link (2)

Demand (in Godunov flow) change at the boundary:



(i) Fluid regime ($r_{i-1} > r_{cr}$)

Packet enter in [D] if

$$\frac{x_{i-1}^{t+1} - \xi}{N_i} \geq r_{cr} = r_i^t = \frac{x_{i-1}^t - x_i^t}{2}$$

(ii) Congestion regime

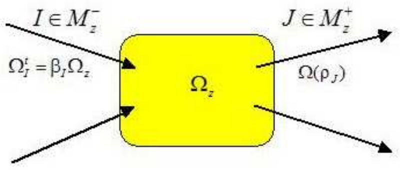
Packet enter in [D] if

$$\frac{x_{i-1}^{t+1} - \xi}{N_i} \geq r_{i-1}^t = r_i^t$$

otherwise

$$x_i^{t+1} = x_i^t$$

Intersection modeling: convergent model (1)



$\Omega_z^t = \Omega_z^t(N_z^t, N_z, \Omega_j(\rho_j))$

Supply for entering link I at t:

$$\begin{cases} \Omega_I^t = \beta_I \Omega_z^t \\ \sum_{I \in M_z^-} \beta_I = 1 \end{cases}$$

where

β_i: split coefficient

Intersection modeling: convergent model (2)

(i) Compute $x_{p,z}^{t+1}$

if $n_{p,z}^{t} \geq \Omega_I^t \Delta t$

then compute:

$n_{p,z}^{t+1} = n_{p,z}^{t} - \Omega_I^t \Delta t$, and

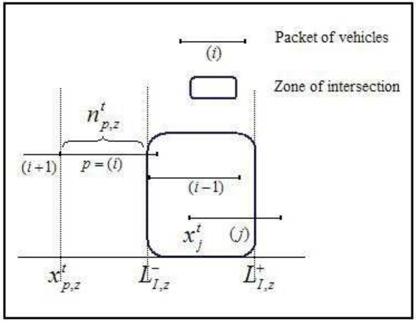
$x_{p,z}^{t+1} \approx \frac{n_{p,z}^{t+1}}{n_{p,z}^{t}}$

otherwise

$n_{p+1,z}^{t+1} = \Delta N_{i+1} - (\Omega_I^t \Delta t - n_{p,z}^{t})$

$p := p + 1$ (ii) Compute x_j^{t+1}

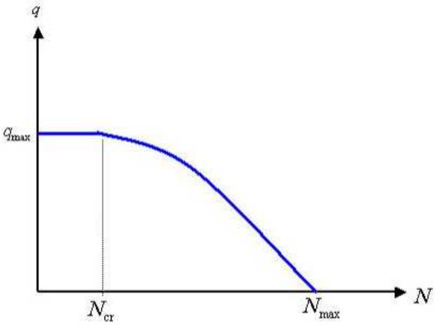
$\Rightarrow$ `Divergent model`



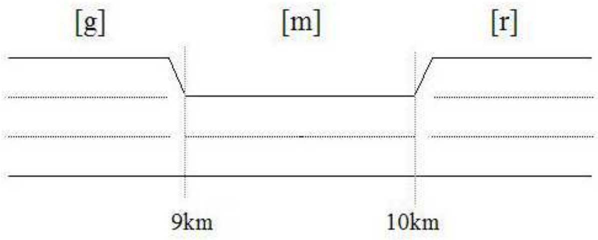
Intersection modeling: convergent model (3)

(iii) Update N_z^{t+1}

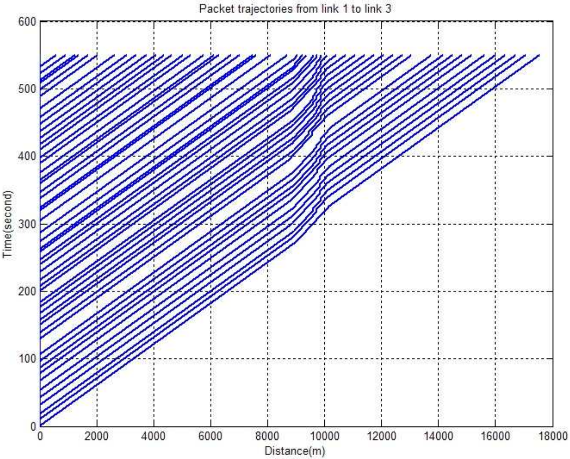
$$N_z^{t+1} = N_z^t + \sum_{I \in M_z^-} \Omega_I^t \Delta t - \sum_{J \in M_z^+} \delta_J^t \Delta t$$



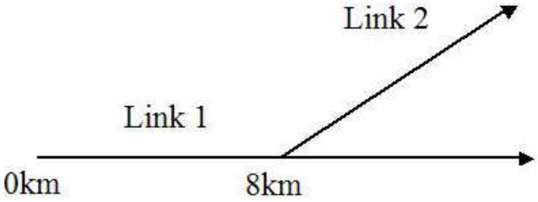
Numerical example



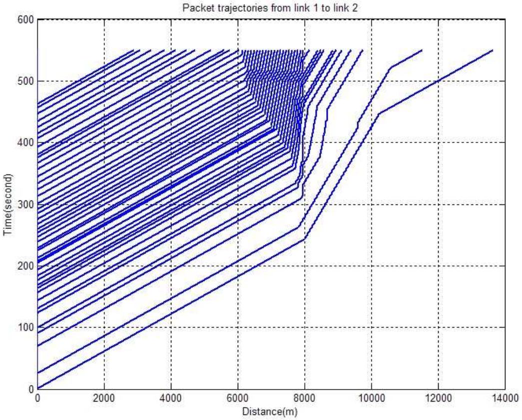
Simulation results



Divergent example



Simulation results



Conclusion

1. Packet-based approach provide fast traffic flow simulation in large scale network
2. Lagrangian discretisation is appropriate for dynamic public transportation simulation
3. CE method provide fast convergence result for solving dynamic traffic assignment problems based on more realistic traffic flow model