Modeling highway bottlenecks in balanced vehicular traffic

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• Balanced vehicular traffic
• Coupling conditions at intersections
• Modeling highway bottlenecks
Balanced vehicular traffic
Aw, Rascle, Greenberg model

- Macroscopic model
  - continuity equation: \( \frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0 \)
  - pseudomomentum equation: \( \frac{\partial (\rho (v - u(\rho)))}{\partial t} + \frac{\partial (\rho v (v - u(\rho)))}{\partial x} = \frac{\rho (u(\rho) - v)}{T} \)

- hyperbolic system of balance laws
- references:
  - A. Aw and M. Rascle (SIAP 2000)
  - J. Greenberg (SIAP 2001)
  - M. Zhang (Transportation Research B 2002)

- Motivation for an extension
  - multi-valued fundamental diagram
    - Greenberg, Klar, Rascle (SIAP 2002)
  - instabilities
    - Greenberg (SIAP 2004)
  - instantaneous reaction to the current traffic situation

- \( \rho \): vehicle density
- \( v \): dynamical velocity
- \( u(\rho) \): equilibrium velocity
- \( T > 0 \): relaxation time

-- M. Koshi et al (1983)
Balanced vehicular traffic

- Extended Aw-Rascle-Greenberg model
  \[
  \frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0
  \]
  \[
  \frac{\partial (\rho (v - u(\rho)))}{\partial t} + \frac{\partial (\rho v (v - u(\rho)))}{\partial x} = b(\rho, v) \rho (u(\rho) - v)
  \]
  - continuity equation
  - pseudomomentum equation

- Characteristic speeds
  - \( \lambda_1 = v + \rho u'(\rho) \leq v \)
  - \( \lambda_2 = v \)

- Effective relaxation coefficient \( b(\rho, v) \)
  - ARG: constant, inverse relaxation time
  - here: function of density \( \rho \) and velocity \( v \)

- Papers
  - F. Siebel, W. Mauser, PRE 73, 066108 (2006)
Steady-state solutions

- **Trivial steady-state solutions:**
  - **equilibrium velocity** \( v = u(\rho) \)
  - **zeros** of the effective relaxation coefficient \( b(\rho, v) \)
    - jam line
    - high flow branch

- **Non-trivial steady-state solutions:**
  - lie on straight lines in the fundamental diagram
  - cover in particular regions II and III

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Steady-state solutions

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  - equilibrium velocity $v = u(\rho)$
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    - jam line
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- Non-trivial steady-state solutions:
  - lie on straight lines in the fundamental diagram
  - cover in particular regions II and III
- Characteristic curves:
  - $\lambda_1 = v + \rho u'(\rho) \leq v$
  - $\lambda_2 = v$
- Stability:
  - sub-characteristic condition (Whitham 1974)
Synchronized flow and wide moving jams

• Simulation setup:
  - periodic boundaries
  - speed limit between 5 and 6 km
  - initially free flow data

• Traffic dynamics:
  - synchronized flow:
    - bottleneck
    - narrow moving jams
      - pinch region
      - merging
      - catch effect
  - wide moving jam:
    - speed -15 km/h
    - robust
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Synchronized flow and wide moving jams

- Measurements of a virtual detector located at $x=0$ km:
Coupling conditions at intersections
Riemann problem at intersections

• Coupling conditions at the interface: Riemann problem for principal part
  - literature for the Aw, Rascle model:
  - boundary values for the fluxes at a junction
  - source term of BVT model treated separately
  - macroscopic description: lane changes neglected
  - generalization of the coupling conditions for the LWR model

\[
\begin{align*}
\left(\rho_i^-, v_i^-, u_i\right) & \quad \left(\rho_k^+, v_k^+, u_k\right) \\
\text{incoming } i & \quad \text{outgoing } k \\
\text{i = 1, \ldots, J} & \quad \text{k = J+1, \ldots, J+L}
\end{align*}
\]

• Solution of the Riemann problem:
  - shock / rarefaction waves and contact discontinuities
  - definitions:
    - \( q_{i/k} \): outflow from / inflow to road section \( i/k \)
    - distance from equilibrium on road section \( i/k \)
      \[ w_{i/k}(\rho, v) = v - u_{i/k}(\rho) \]
Principles of the coupling conditions

1) Flow conservation
\[ \sum_{i=1}^{J} q_i = \sum_{k=J+1}^{J+L} q_k \]

2) Conservation of pseudomomentum flow (no source term!)
\[ \sum_{i=1}^{J} q_i w_i (\rho_i^-, v_i^-) = \sum_{k=J+1}^{J+L} q_k w_k \]

- define \( \beta_{ik} \) as the portion of the flow on the outgoing road \( k \) coming from road \( i \)

\[ q_i = \sum_{k=J+1}^{J+L} \beta_{ik} q_k \ \forall i \quad \text{with} \quad \sum_{i=1}^{J} \beta_{ik} = 1 \ \forall k \]

- conservation of pseudomomentum flow
\[ \sum_{k=J+1}^{J+L} q_k \sum_{i=1}^{J} \beta_{ik} w_i (\rho_i^-, v_i^-) = \sum_{k=J+1}^{J+L} q_k w_k \ \forall q_k \]
\[ \Rightarrow \sum_{i=1}^{J} \beta_{ik} w_i (\rho_i^-, v_i^-) = w_k \ \forall k \]

(homogenized distance from equilibrium)
Traffic demand (sending flow) on incoming roads

3) Demand functions on incoming roads : i:

let \( w_i = w_i(\rho_i, v_i) \),
\[ \eta_{di}(\rho) = \rho u_i(\rho) + \rho w_i \] and \( \tilde{\rho}_i = \arg \max_{\rho} (\eta_{di}(\rho)) \)

demand functions :
\[ d_i(\rho) = \begin{cases} \eta_{di}(\rho) & \text{for } \rho \leq \tilde{\rho}_i \\ \eta_{di}(\tilde{\rho}_i) & \text{for } \rho > \tilde{\rho}_i \end{cases} \]

4) Prescription \( \beta_{ik} \) of the portion of the flow on road \( k \) from road \( i \)

- let \( \alpha_{ik} \) the portion of the cars on road \( i \) intending to enter road \( k \)
\[ \sum_{k=J+1}^{l+1} \alpha_{ik} = 1 \quad \forall i \] (all cars on road \( i \) intend to go to one of the roads \( k \))

- distribution according to demands (Haut, Bastin 2005)
\[ \beta_{ik} = \frac{\alpha_{ik} d_i(\rho_i)}{\sum_{j=1}^{l} \alpha_{jk} d_j(\rho_j)} \] (can be adjusted by traffic management)
Traffic supply (receiving flow) on outgoing roads

5) Supply functions on outgoing roads $k$:
let $\eta_{sk} (\rho) = \rho u_k (\rho) + \rho w_k$
and $\tilde{\rho}_k = \arg \max_{\rho} \rho (\eta_{sk} (\rho))$
supply functions:
$$s_k (\rho) = \begin{cases} \eta_{sk} (\rho) & \text{for } \rho > \tilde{\rho}_k \\
\eta_{sk} (\tilde{\rho}_k) & \text{for } \rho \leq \tilde{\rho}_k \end{cases}$$

6) Density on outgoing roads $k$, where supply function is evaluated:
solution $\rho_k^\uparrow$ of $v - u_k (\rho_k^\uparrow) = w_k$, $v = v_k^+$

7) Optimization problem
maximize flow on the outgoing roads $\max \sum_{k=J+1}^{J+L} q_k$
under the boundary conditions
$$0 \leq q_i \leq d_i (\rho_i^-) \quad \forall i,$$  \hspace{1cm} (flow from incoming road $i$ bounded by demand)
$$0 \leq q_k \leq \min \left( s_k (\rho_k^\uparrow), \sum_{i=1}^{L} \alpha_{ik} d_i (\rho_i^-) \right) \quad \forall k,$$  \hspace{1cm} (flow to outgoing road $k$ bounded by supply ... and by total demand for road $k$)
Modeling highway bottlenecks
Capacity drop at lane drop bottlenecks

- **Measurements:**
  (Bertini, Leal, Journal of Transportation Engineering 2005)

- **Capacity drop:**
  - The outflow in the downstream section is below the maximum free flow of that section after synchronized flow has formed upstream of the bottleneck

- **Simulation setup:**
  - highway sections: three-lane section 1, two-lane section 2
  - periodic boundary conditions: simulation determined by initial data
Lane drop bottleneck: Aw, Rasce, Greenberg model

- Simulation results:

\[ \rho_0 = 100 \text{ [1/km]} \]

\[ \rho_0 = 150 \text{ [1/km]} \]
Lane drop bottleneck: Aw, Rascle, Greenberg model

- **Static solution:**
  - piecewise constant solution with constant total flow
  - shock discontinuity at about 4.2 km and 1.3 km
  - Rankine-Hugoniot jump conditions:
    - flow $\rho v$ and distance from equilibrium $w$ constant across the shock
  - synchronized flow region in front of the bottleneck
  - maximum outflow from the bottleneck region in section 2
    - no capacity drop
Lane drop bottleneck: BVT model

- Simulation results:

\[
\rho_0 = 50 \text{ [1/km]}
\]

\[
\rho_0 = 100 \text{ [1/km]}
\]
Lane drop bottleneck: BVT model

- **Static solution:**
  - von Neumann state downstream of the shock, followed by a section of a nontrivial steady-state solution

- **capacity drop:**
  - flow value below maximum in downstream section 2
  - determined by the crossing of the static solutions with the jam line
    - similar to wide cluster solutions:

- on/off-ramps:
Conclusion

- Balanced vehicular traffic model
  - hyperbolic system of balance laws
    - macroscopic
    - deterministic
    - effective one lane
    - no distinction between different vehicle types
    - nonlinear dynamics
  - model results
    - multi-valued fundamental diagrams
    - metastability of free flow at the onset of instabilities
    - wide moving jams
    - synchronized flow
    - capacity drop

Michel Rascle, Salissou Moutari, Wolfram Mauser