Modeling highway bottlenecks in balanced vehicular traffic

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- Balanced vehicular traffic
- Coupling conditions at intersections
- Modeling highway bottlenecks

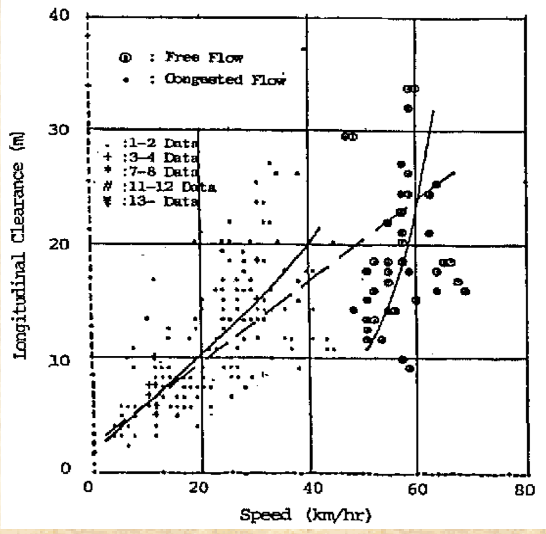
Balanced vehicular traffic

Aw, Rascle, Greenberg model

- Macroscopic model
  - continuity equation: $$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0$$
    pseudomomentum equation: $$\frac{\partial(\rho(v-u(\rho)))}{\partial t} + \frac{\partial(\rho v(v-u(\rho)))}{\partial x} = \frac{\rho(u(\rho)-v)}{T}$$

ρ: vehicle density
v: dynamical velocity
u(ρ): equilibrium velocity
T>0: relaxation time

  - hyperbolic system of balance laws
  - references:
    - A. Aw and M. Rascle (SIAP 2000)
    - J. Greenberg (SIAP 2001)
    - M. Zhang (Transportation Research B 2002)
- Motivation for an extension
  - multi-valued fundamental diagram
    - Greenberg, Klar, Rascle (SIAP 2002)
  - instabilities
    - Greenberg (SIAP 2004)
  - instantaneous reaction to the current traffic situation



M. Koshi et al (1983)

Balanced vehicular traffic

- Extended Aw-Rascle-Greenberg model

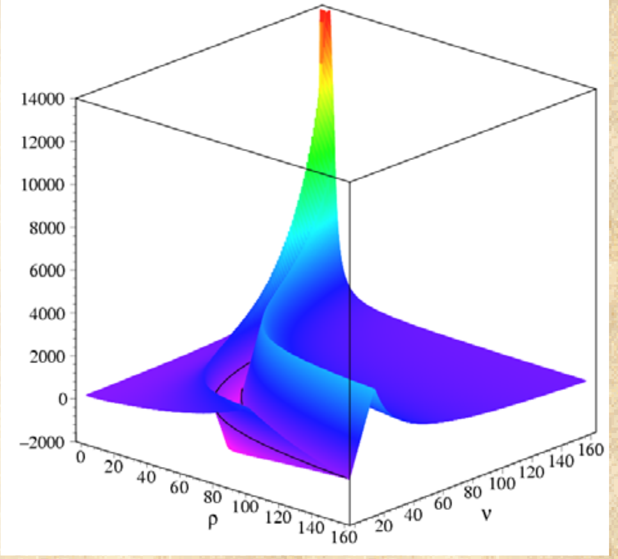
$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0$$

continuity equation

$$\frac{\partial(\rho(v-u(\rho)))}{\partial t} + \frac{\partial(\rho v(v-u(\rho)))}{\partial x} = b(\rho,v)\ \rho(u(\rho)-v)$$

pseudomomentum equation

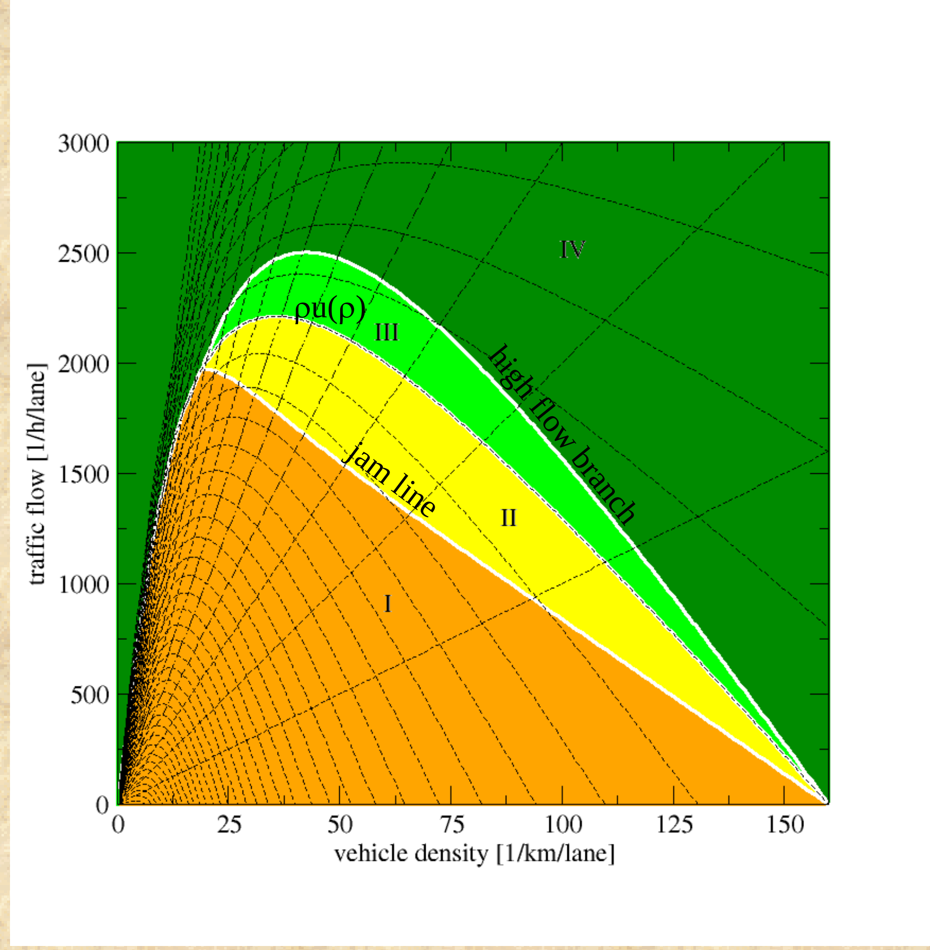
- Characteristic speeds
  - $\lambda_1 = v + \rho u'(\rho) \leq v$
  - $\lambda_2 = v$
- Effective relaxation coefficient b(ρ,v)
  - ARG: constant, inverse relaxation time
  - here: function of density ρ and velocity v
- Papers
  - F. Siebel, W. Mauser, *SIAP 66, 1150 (2006)*
  - F. Siebel, W. Mauser, PRE 73, 066108 (2006)



effective relaxation coefficient b(ρ,v)

Steady-state solutions

- Trivial steady-state solutions:
  - **equilibrium velocity** $v = u(\rho)$
  - **zeros** of the effective relaxation coefficient $b(\rho,v)$
    - **jam line**
    - **high flow branch**
- Non-trivial steady-state solutions:
  - lie on straight lines in the fundamental diagram
  - cover in particular regions II and III
- Characteristic curves:
  - $\lambda_1 = v+\rho u'(\rho) \leq v$
  - $\lambda_2 = v$



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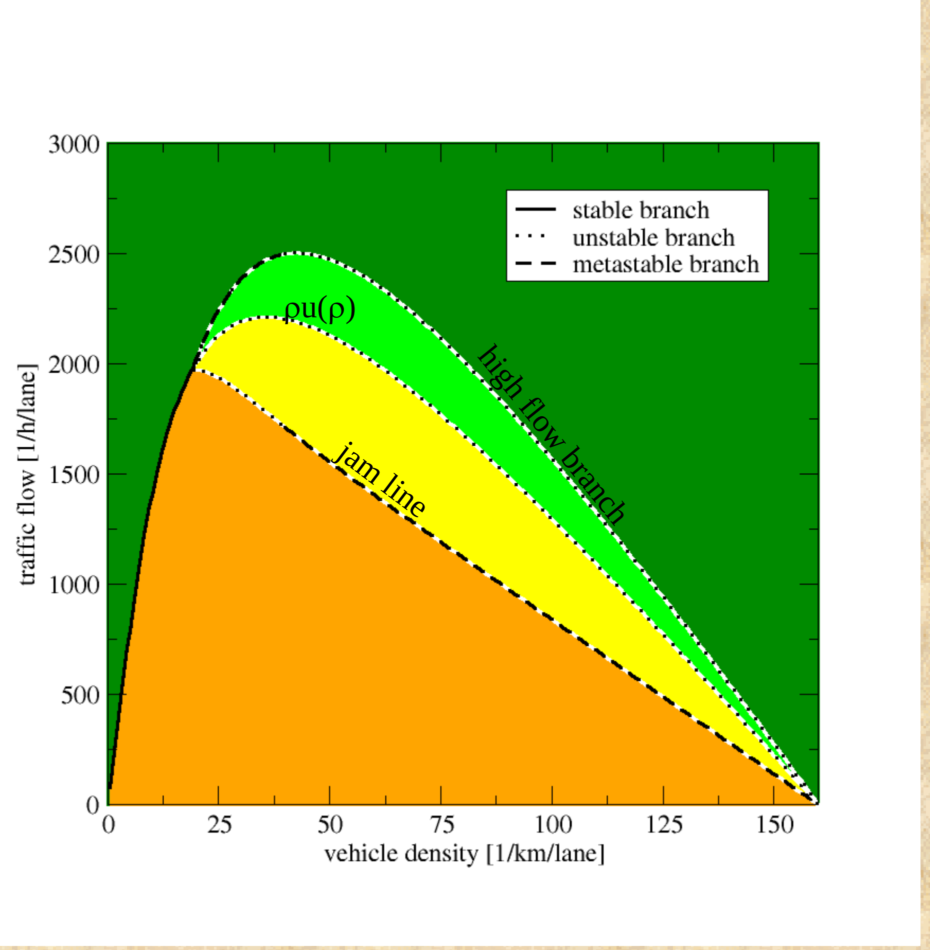
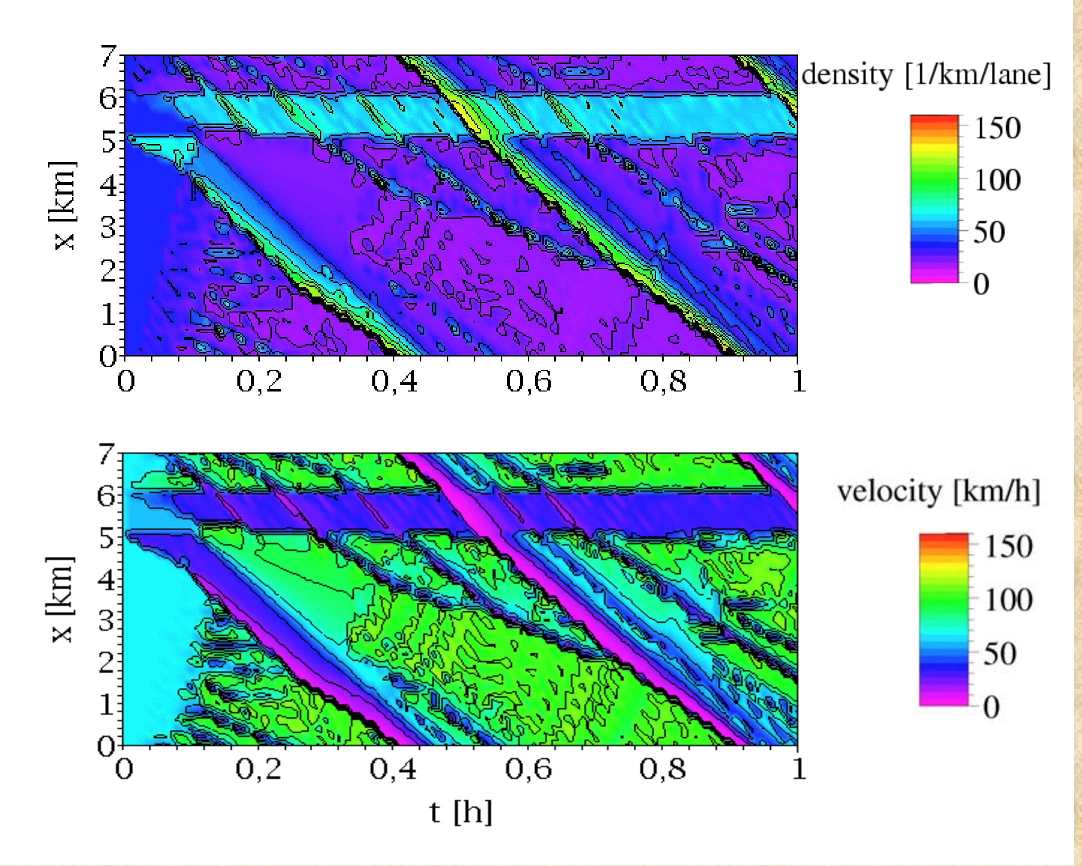
- Stability:
  - sub-characteristic condition (Whitham 1974)



Synchronized flow and wide moving jams

- Simulation setup:
  - periodic boundaries
  - speed limit between 5 and 6 km
  - initially free flow data
- Traffic dynamics:
  - synchronized flow:
    - bottleneck
    - narrow moving jams
      - pinch region
      - merging
      - catch effect
  - wide moving jam:
    - speed -15 km/h
    - robust



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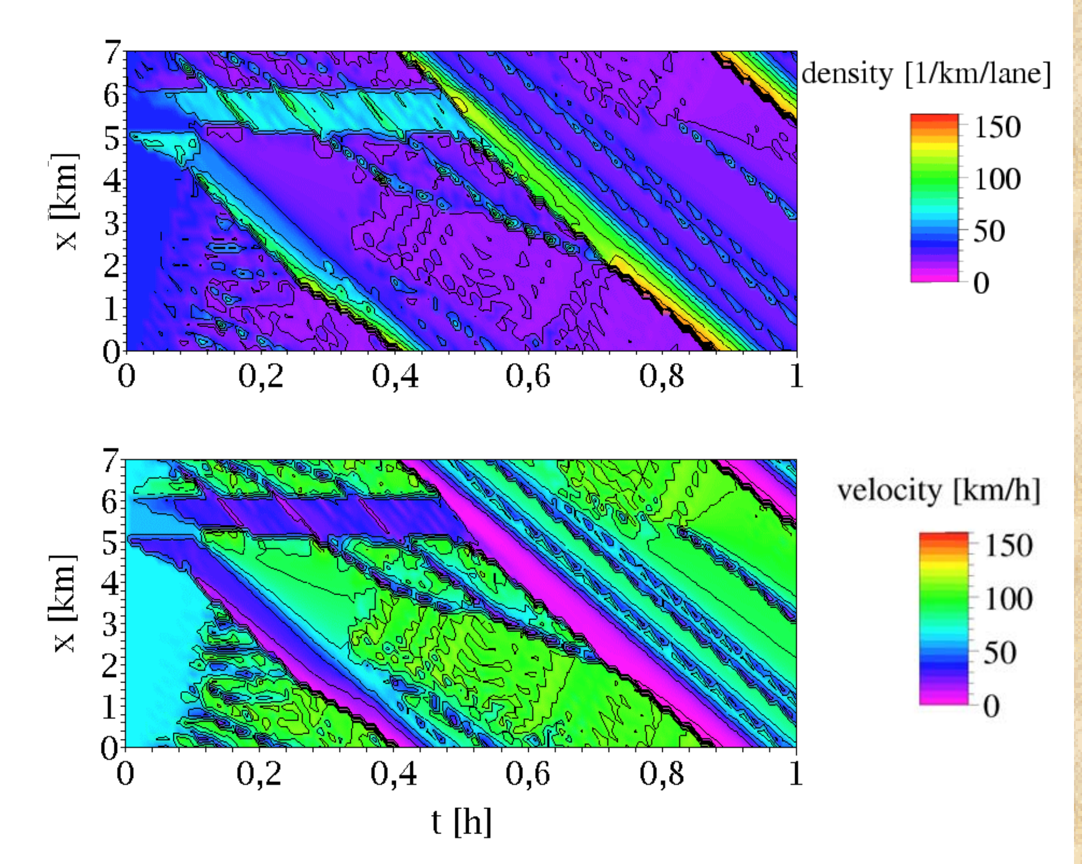
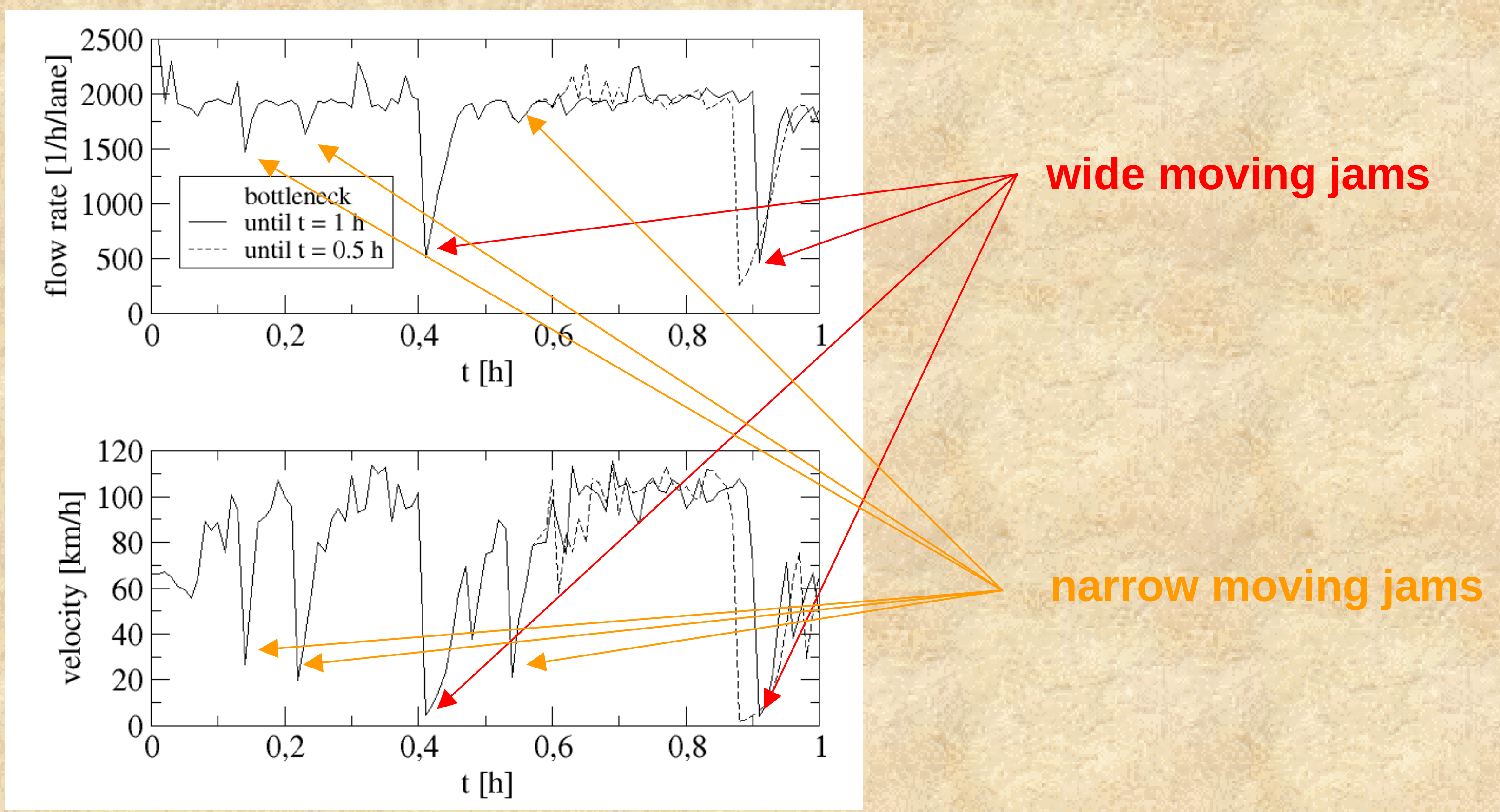


Synchronized flow and wide moving jams

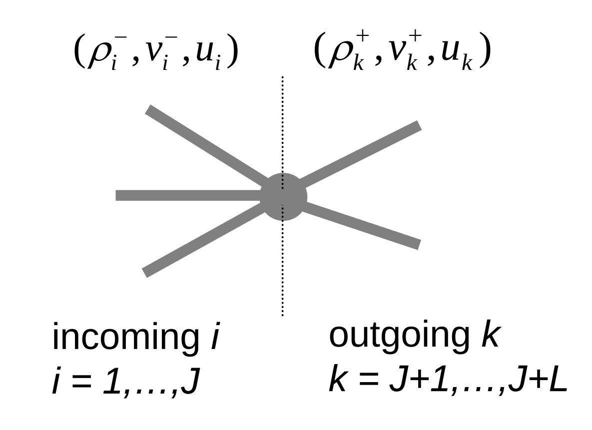
- Measurements of a virtual detector located at x=0 km:



Coupling conditions at intersections

Riemann problem at intersections

- Coupling conditions at the interface: Riemann problem for principal part
  - literature for the Aw, Rascle model:
    - Haut, Bastin (2005), Garavello, Piccoli (2006), Herty, Rascle (2006), Herty, Moutari, Rascle (2006), Haut, Bastin (2007)
  - boundary values for the fluxes at a junction
  - source term of BVT model treated separately
  - macroscopic description: lane changes neglected
  - generalization of the coupling conditions for the LWR model



- Solution of the Riemann problem:
  - shock / rarefaction waves and contact discontinuities
  - definitions:
    - $q_{i/k}$: outflow from / inflow to road section *i/k*
    - *distance from equilibrium* on road section *i/k*

$$w_{i/k}(\rho, v) = v - u_{i/k}(\rho)$$

Principles of the coupling conditions

1) Flow conservation

$$\sum_{i=1}^{J} q_i = \sum_{k=J+1}^{J+L} q_k$$

2) Conservation of *pseudomomentum* flow (no source term!)

$$\sum_{i=1}^{J} q_i w_i(\rho_i^-, v_i^-) = \sum_{k=J+1}^{J+L} q_k w_k$$

- define β_{ik} as the portion of the flow on the outgoing road *k* coming from road *i*

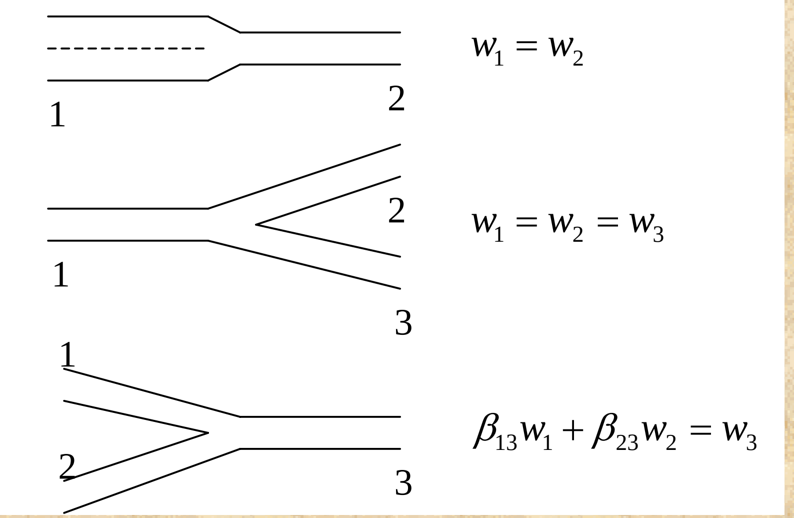
$$q_i = \sum_{k=J+1}^{J+L} \beta_{ik} q_k \ \forall i \quad \text{with} \ \sum_{i=1}^{J} \beta_{ik} = 1 \ \forall k$$

- conservation of *pseudomomentum* flow

$$\sum_{k=J+1}^{J+L} q_k \sum_{i=1}^{J} \beta_{ik} w_i(\rho_i^-, v_i^-) = \sum_{k=J+1}^{J+L} q_k w_k \quad \forall q_k$$

$$\Rightarrow \quad \sum_{i=1}^{J} \beta_{ik} w_i(\rho_i^-, v_i^-) = w_k \quad \forall k$$

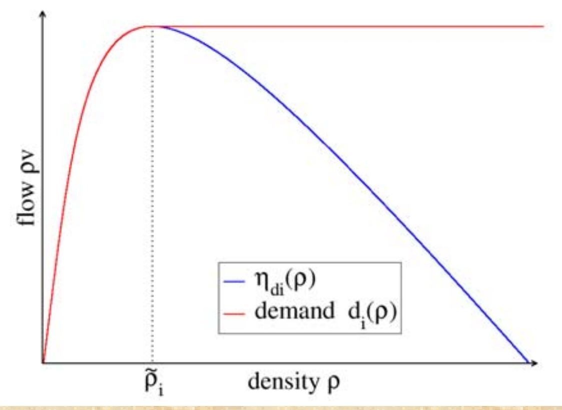
(homogenized distance from equilibrium)



Traffic demand (sending flow) on incoming roads

3) Demand functions on incoming roads *i*:

$$\text{let } w_i^- = w_i(\rho_i^-, v_i^-),$$
$$\eta_{\mathrm{di}}(\rho) = \rho u_i(\rho) + \rho w_i^- \text{ and } \tilde{\rho}_i = \arg\max_\rho(\eta_{di}(\rho))$$



demand functions:

$$d_i(\rho) = \begin{cases} \eta_{di}(\rho) & \text{for } \rho \le \tilde{\rho}_i \\ \eta_{di}(\tilde{\rho}_i) & \text{for } \rho > \tilde{\rho}_i \end{cases}$$

4) Prescription β_{ik} of the portion of the flow on road *k* from road *i*

- let α_{ik} the portion of the cars on road *i* intending to enter road *k*

$$\sum_{k=J+1}^{J+L} \alpha_{ik} = 1 \quad \forall i$$

(all cars on road *i* intend to go to one of the roads *k*)

- distribution according to demands (Haut, Bastin 2005)

$$\beta_{ik} = \frac{\alpha_{ik} d(\rho_i^-)}{\sum_{j=1}^{J} \alpha_{jk} d(\rho_j^-)}$$

(can be adjusted by traffic management)

Traffic supply (receiving flow) on outgoing roads

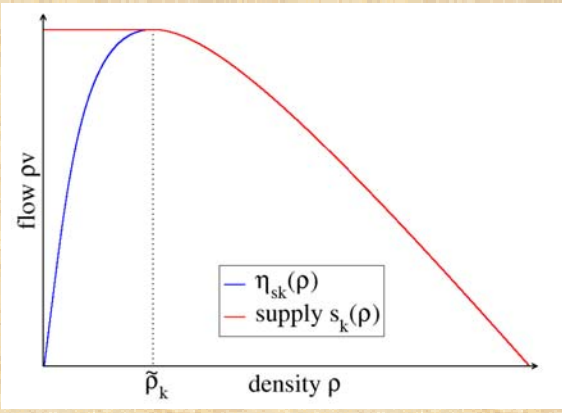
5) Supply functions on outgoing roads *k*:

let $\eta_{sk}(\rho) = \rho u_k(\rho) + \rho w_k$

and $\tilde{\rho}_k = \arg\max_\rho (\eta_{sk}(\rho))$

supply functions:

$$s_k(\rho) = \begin{cases} \eta_{sk}(\rho) & \text{for } \rho > \tilde{\rho}_k \\ \eta_{sk}(\tilde{\rho}_k) & \text{for } \rho \le \tilde{\rho}_k \end{cases}$$



6) Density on outgoing roads *k*, where supply function is evaluated:

solution $\rho_k^{\uparrow}$ of $v - u_k(\rho_k^{\uparrow}) = w_k, \quad v = v_k^{+}$

7) Optimization problem

maximize flow on the outgoing roads $\max \sum_{k=J+1}^{J+L} q_k$

under the boundary conditions

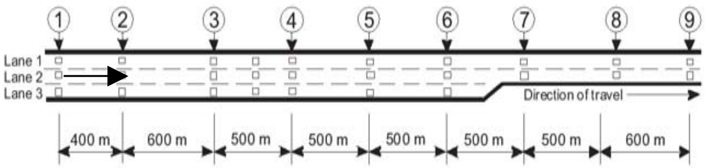
$$0 \le q_i \le d_i(\rho_i^{-}) \quad \forall i, \qquad \text{(flow from incoming road } i \text{ bounded by demand)}$$

$$0 \le q_k \le \min\left(s_k(\rho_k^{\uparrow}), \sum_{i=1}^{J} \alpha_{ik} d_i(\rho_i^{-}) \right) \quad \forall k, \qquad \text{(flow to outgoing road } k \text{ bounded by supply ... ... and by total demand for road } k\text{)}$$

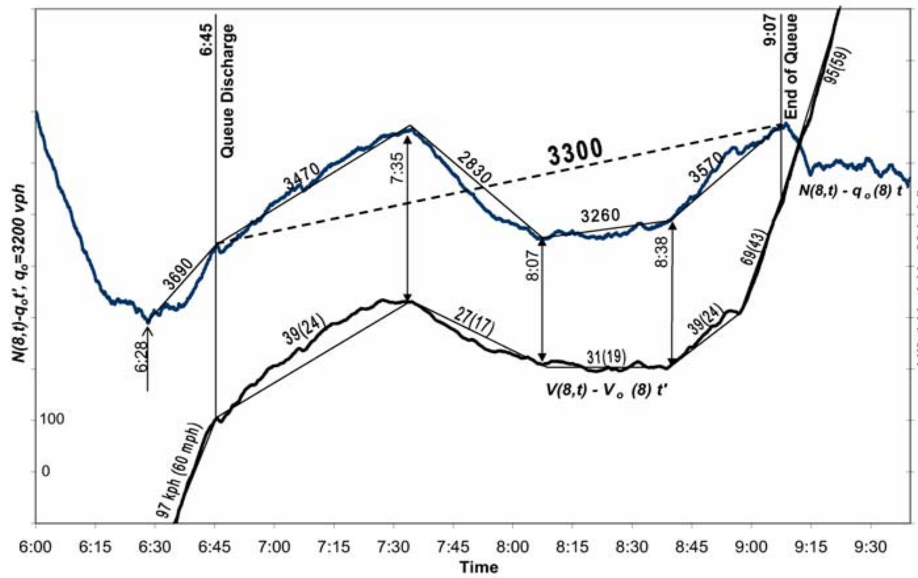
Modeling highway bottlenecks

Capacity drop at lane drop bottlenecks

- Measurements:
  (Bertini, Leal,
  Journal of Transportation
  Engineering 2005)



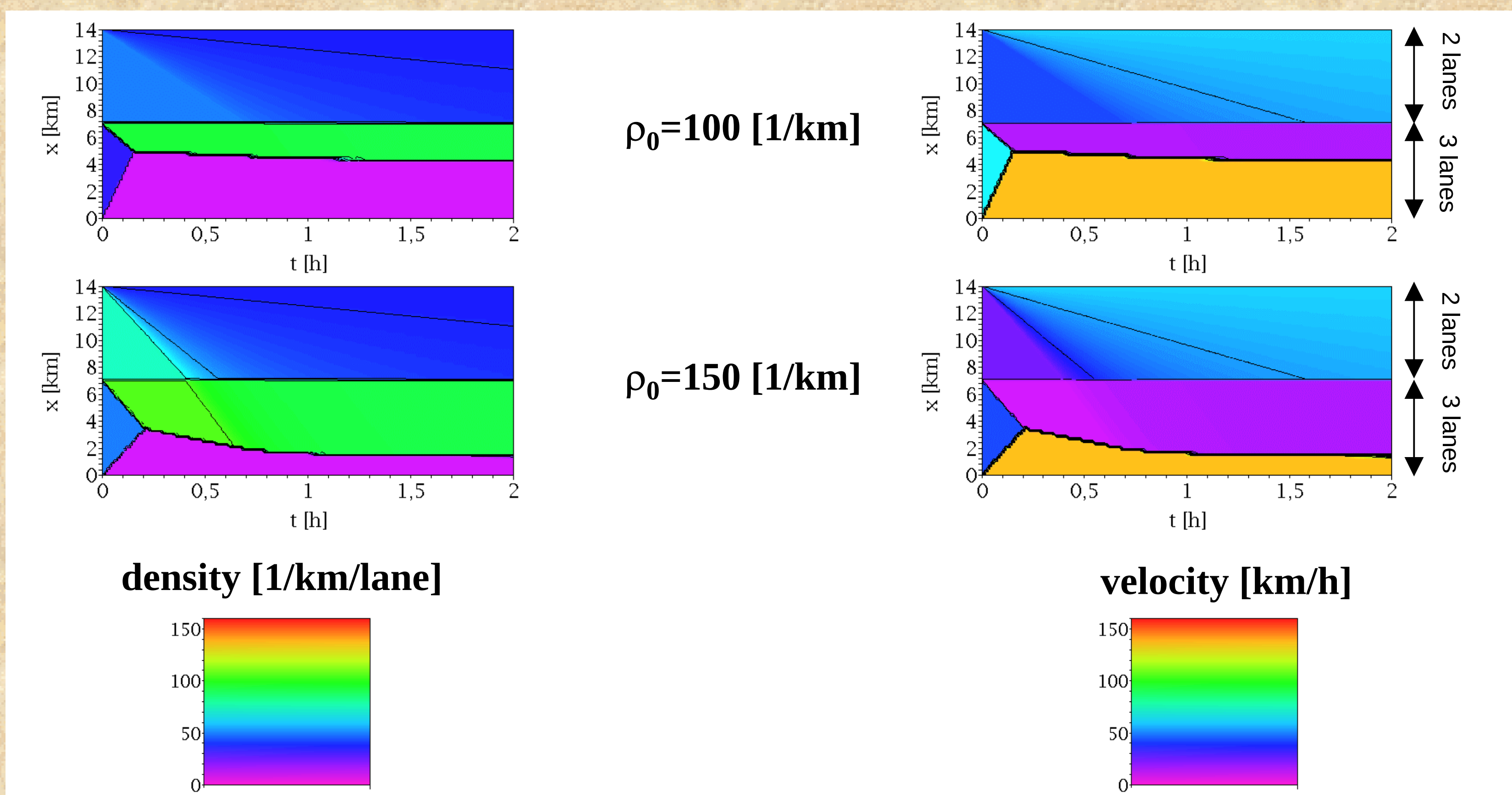
M4 Motorway, United Kingdom



- Capacity drop:
  - The outflow in the downstream section is below the maximum free flow of that section after synchronized flow has formed upstream of the bottleneck
- Simulation setup:
  - highway sections: three-lane section 1, two-lane section 2
  - periodic boundary conditions: simulation determined by initial data

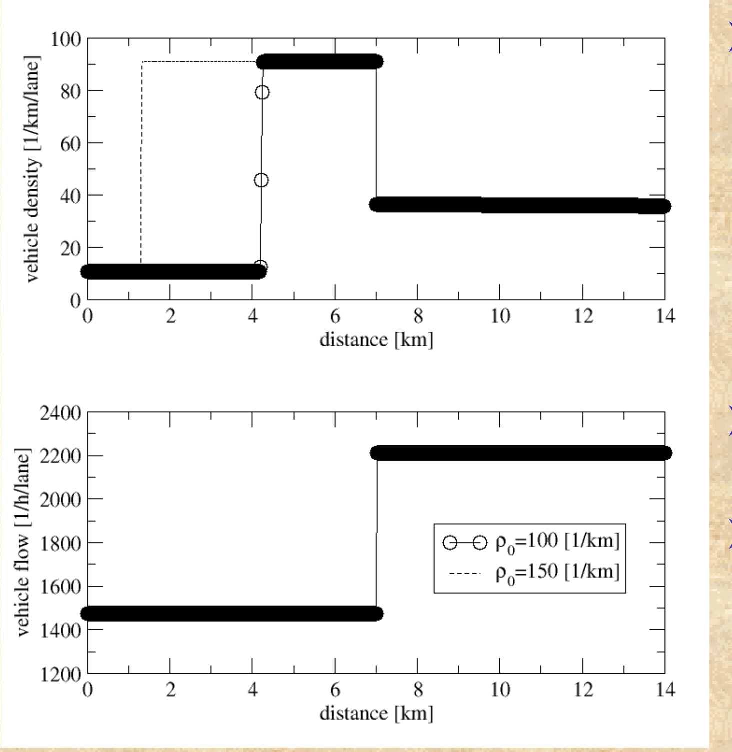
Lane drop bottleneck: Aw, Rascle, Greenberg model

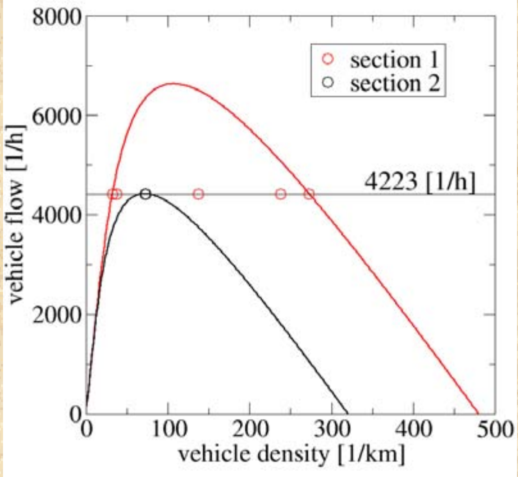
- Simulation results:



Lane drop bottleneck: Aw, Rascle, Greenberg model

- Static solution:
  - piecewise constant solution with constant total flow
  - shock discontinuity at about 4.2 km and 1.3 km

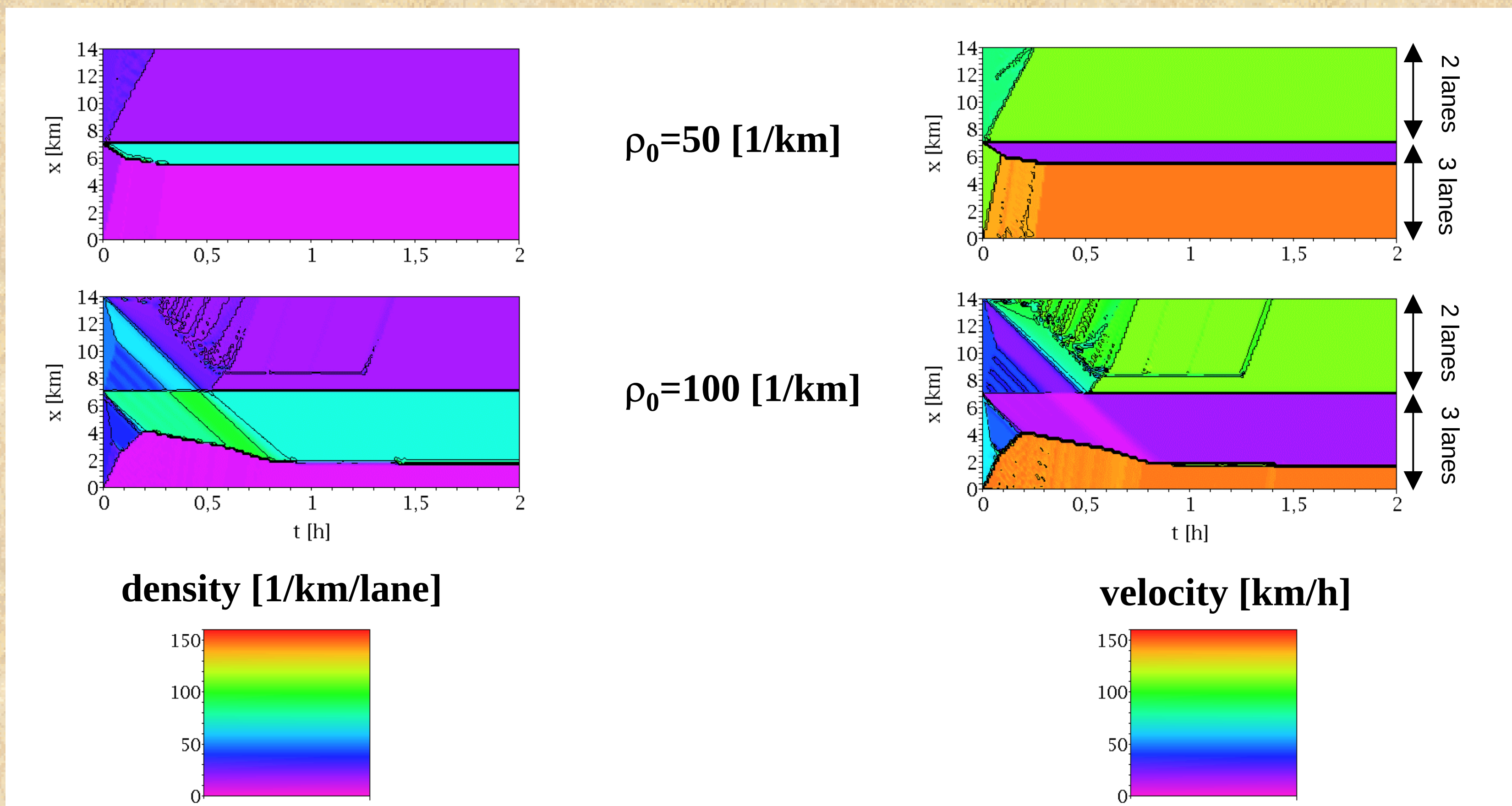




- Rankine-Hugoniot jump conditions:
  - flow ρv and distance from equilibrium w constant across the shock
- synchronized flow region in front of the bottleneck
- maximum outflow from the bottleneck region in section 2
  - no capacity drop

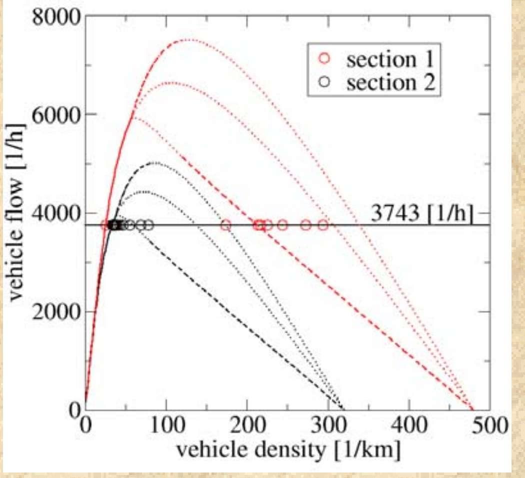
Lane drop bottleneck: BVT model

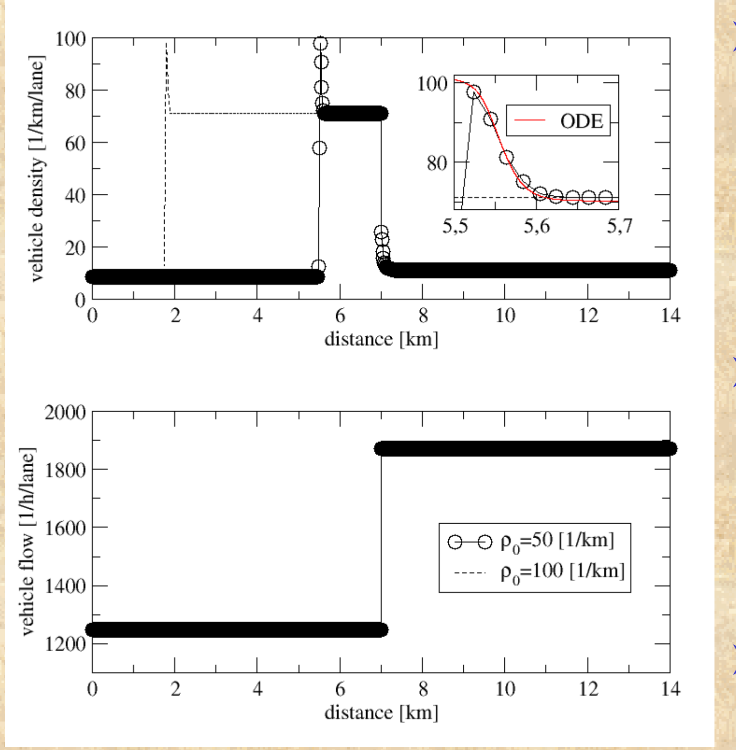
- Simulation results:



Lane drop bottleneck: BVT model

- Static solution:
  - ➢ von Neumann state downstream of the shock, followed by a section of a nontrivial steady-state solution





- ➢ **capacity drop**:
  - flow value below maximum in downstream section 2
- ➢ determined by the crossing of the static solutions with the jam line
  - similar to wide cluster solutions:
    - Zhang, Wong (2006), Zhang, Wong, Dai (2006)
- ➢ on/off-ramps:
  - http://arxiv.org/abs/physics/0609237

Conclusion

- Balanced vehicular traffic model
  - hyperbolic system of balance laws
    - macroscopic
    - deterministic
    - effective one lane
    - no distinction between different vehicle types
    - nonlinear dynamics
  - model results
    - multi-valued fundamental diagrams
    - metastability of free flow at the onset of instabilities
    - wide moving jams
    - synchronized flow
    - capacity drop

Michel Rascle, Salissou Moutari, Wolfram Mauser